

모달 퍼지 이론을 이용한  
지진하중을 받는 구조물의 능동제어  
**Active control for Seismic Response Reduction  
using Modal-fuzzy Approach**

최강민\*      박규식\*\*      김춘호\*\*\*      이인원\*\*\*\*  
Choi, Kang-Min      Park, Kyu-Sik      Kim, Chun-Ho      Lee, In-Won

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**ABSTRACT**

An active modal-fuzzy control method using hydraulic actuators is presented for seismic response reduction. In the proposed control system, a new fuzzy controller designed in the modal space produces the desired active control force. This type controller has all advantages of the fuzzy control algorithm and modal approach. Since it is very difficult to select input variables used in fuzzy controller among an amount of state variables in the active fuzzy control system, the presented algorithm adopts the modal control algorithm which is able to consider more easily information of all state variables in civil structures that are usually dominated by first few modes. In other words, all information of the whole structure can be considered in the control algorithm evaluated to reduce seismic responses and it can be efficient for especially civil structures. In addition, the presented algorithm is expected to magnify utility and performance caused by efficiency that the fuzzy algorithm can handle complex model more easily. An active modal-fuzzy control scheme is applied together with a Kalman filter and a low-pass filter to be applicable to real civil structures. A Kalman filter is considered to estimate modal states and a low-pass filter was used to eliminate spillover problem. The results of the numerical simulations for a wide amplitude range of loading conditions show that the proposed active modal-fuzzy control system can be beneficial in reducing seismic responses of civil structures.

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**1. Introduction**

In the field of civil engineering, many different techniques for actively controlling vibrations of seismically excited structure have been developed and applied to civil structures. Active control system reduces the structural response by using external energy supplied by actuators to impart forces on the structures, generally depending on a sizeable power supply. It is considerably more flexible to reduce the structural responses for a wide variety of loading conditions. Although in the past researches of civil structures were often treated separately, the fields

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\* 한국과학기술원 건설및환경공학과 박사과정  
\*\* 한국과학기술원 건설및환경공학과 박사후연구과정  
\*\*\* 정회원·중부대학교 토목공학과 교수  
\*\*\*\* 정회원·한국과학기술원 건설및환경공학과 교수

have now been interfaced to combine two or more algorithms for reducing effectively and treating more realistically.

Fuzzy theory has been recently proposed for the active structural control of civil engineering systems. Active vibration control of structural systems using fuzzy set theory has been widely investigated in the past years [1-7]. The fuzzy set theory was introduced by Zadeh [1] in 1965. In 1974 Mamdani [2], by applying Zadeh's theories of linguistic approach and fuzzy inference, successfully used the 'IF-THEN' rule on the automatic operating control of steam generator. In civil engineering, the fuzzy set theory was applied by Brown and Yao [3], Juang and Elton [4], Faravelli et al. [5], Teng and Peng [6] and Wang [7]. Since especially buildings in civil engineering are getting so higher and bridges are getting so longer that those structures are very complex systems of multi-degree of freedom, it is very difficult to find an exact mathematical model to describe the behavior of the structures. Because the fuzzy controller does not rely on the analysis and synthesis of the mathematical model of the process, the uncertainties of input data from the external loads and structural responses sensors are treated in a much easier way by the fuzzy controller than by classical control theory. Moreover, it offers a simple and robust structure for the specification of nonlinear control laws that can accommodate uncertainty and imprecision (Subramanian et al. 1996 [8]).

Modal control algorithm represents one control class in which the vibration behavior is reshaped by merely controlling some selected vibration modes. Modal control approach has been demonstrated to have advantages over the design in physical space, in that it demands far less computer storage, reduces the computational effort significantly, and allows a larger choice of control algorithms, including nonlinear control. Moreover, because civil structures has hundred or even thousand degrees of freedom and its vibration is usually dominated by first few modes, modal control algorithm is especially desirable for reducing vibration of civil engineering structure. In other words, all information of the whole structure can be considered in the control algorithm evaluated to reduce seismic responses and it can be efficient for especially civil structures.

In this work reported here, a fuzzy control approach designed in the modal space is presented. The design of the fuzzy controller began to select the response quantities to be used as inputs to the fuzzy controller and then what control functions are needed is defined as output variable. However, for civil structures having hundred or even thousand degrees of freedom, it is very difficult to select input variables used in fuzzy controller among an amount of state variables. This can be only selected by expert's experience. However, in the case of combination of fuzzy and modal approach, an active modal-fuzzy control algorithm proposed can be magnified efficiency caused by belonging their' own advantages together.

## 2. Active Modal-fuzzy Control Strategy

### 2.1 Modal control system

Consider a seismically excited structure controller with  $m$  control devices. Assuming that the forces provided by the control devices are adequate to keep the response of the primary structure from exiting the linear region, the equations of motion can be written as

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = \Gamma f(t) - M\Lambda\ddot{x}_g \quad (1)$$

where  $M$ ,  $C$ , and  $K$  are the ( $n \times n$ ) mass, damping, and stiffness matrices, respectively;  $x$  is the  $n$ -dimensional vector of the displacements of the floors of the structure relative to the ground;  $f$  is the vector of measured control forces generated by  $m$  control devices;  $\ddot{x}_g$  is ground acceleration;  $\Gamma$  is the matrix determined by the placement of control devices in the structure;  $\Lambda$  is the column vector of ones. This equation can be

written in state-space form as

$$\dot{z} = Az + Bf + E\ddot{x}_g \quad (2a)$$

$$y = Cz + Df + v \quad (2b)$$

where  $z$  is a state vector;  $y$  is the vector of measured outputs; and  $v$  is a measurement noise vector. The next step is to transform Eq. (2) into a set of modal equations. Let us use the expansion theorem and express the solution of Eq. (2) as a linear combination of the right eigenvectors multiplied by time-dependent modal coordinates as follows:

$$x(t) = \sum_{i=1}^{2n} \eta_i(t)\phi_i = \Phi\eta(t), \quad i = 1, 2, \dots, 2n \quad (3)$$

where  $\eta_i(t)$  ( $i = 1, 2, \dots, 2n$ ) are the modal coordinates;  $\eta(t)$  is the corresponding vector;  $\phi_i$  is the  $i$  th right eigenvector;  $\Phi$  is an right eigenvector set. The eigenvectors are orthogonal and they are assumed to be normalized so as to satisfy the orthonormality relations. The orthonormality relations can be written in the compact form

$$\phi_s^T M \phi_r = \delta_{sr}, \quad \phi_s^T K \phi_r = \omega_r^2 \delta_{sr} \quad (4a)$$

$$\Phi^T \Phi = I, \quad \Phi^T A \Phi = \Lambda \quad (4b)$$

where  $\Phi = [\phi_1 \phi_2 \dots \phi_n]$  and  $\Lambda = \text{diag } \lambda_i$  is the diagonal matrix of the eigenvalues.

where  $\delta_{sr}$  is the Kronecker delta and  $\omega_r$  is the natural frequency.

Inserting Eq. (3) into Eq. (1), multiplying on the left by  $\phi_r^T$  and considering Eq. (4), we obtain the modal equation

$$\ddot{\eta}_r + 2\zeta_r \omega_r \dot{\eta}_r + \omega_r^2 \eta_r = \phi_r^T \Gamma f - \phi_r^T M \Lambda \ddot{x}_g \quad (5)$$

where  $\zeta_r$  are modal damping ratios.

Modal equations that is similar form to Eq. (5) for whole system can be written in the matrix form as

$$\eta + \Delta \dot{\eta} + \Omega \eta = \bar{\Gamma} f(t) - \bar{\Lambda} \ddot{x}_g \quad (6)$$

where  $\Delta$  is the diagonal matrix listing  $2\zeta_r \omega_r$ ;  $\Omega$  is the diagonal matrix listing  $\omega_1^2, \omega_2^2, \dots, \omega_n^2$ ;  $\bar{\Gamma} = \Phi^T \Gamma$ ; and  $\bar{\Lambda} = \Phi^T M \Lambda$ . This equation can be written in state-space form as

$$\dot{w}(t) = \bar{A} w(t) + \bar{B} f(t) + \bar{E} \ddot{x}_g(t) \quad (7a)$$

$$y(t) = \bar{C} w(t) \quad (7b)$$

where  $w(t) = [\eta^T \dot{\eta}^T]^T$  is the modal state vector and

$$\bar{A} = \begin{bmatrix} \mathbf{0} & I \\ -\Omega & -\Delta \end{bmatrix}, \quad \bar{B} = \begin{bmatrix} \mathbf{0} \\ \bar{\Gamma} \end{bmatrix}, \quad \text{and} \quad \bar{E} = \begin{bmatrix} \mathbf{0} \\ -\bar{\Lambda} \end{bmatrix} \quad (8)$$

For linear feedback control, the control vector is related to the modal state vector according to

$$F(t) = -G w(t) \quad (9)$$

where  $G$  is an  $m \times 2n$  control gain matrix. Determination of infinite-dimensional gain matrices is not possible, so the control of the entire infinity of modes is not feasible, nor is it necessary. Indeed, higher modes have only minimal participation in the motion and especially the motion of civil structure with hundred or even thousand DOFs is usually dominated by first few modes, as they are difficult to excite. Practically in modal control, only a limited number of lower modes are controlled. In view of the above, we propose to control  $l$  modes only. The  $l$  controller modes can be selected with  $l < n$  and the displacement may be partitioned into controller and uncontrolled parts. Retracing the steps leading to Eq. (7), we obtain

$$\dot{w}_c(t) = \bar{A}_c w_c(t) + \bar{B}_c f(t) + \bar{E}_c \ddot{x}_g \quad (10a)$$

$$y_c(t) = \bar{C}_c w_c(t) \quad (10b)$$

where  $w_c$  is a  $2l$ -dimensional modal state vector by the controller modes and

$$\bar{A}_c = \begin{bmatrix} \mathbf{0} & I_c \\ -\Omega_c & -\Delta_c \end{bmatrix}, \bar{B}_c = \begin{bmatrix} \mathbf{0} \\ \bar{\Gamma}_c \end{bmatrix}, \text{ and } \bar{E}_c = \begin{bmatrix} \mathbf{0} \\ -\bar{\Lambda}_c \end{bmatrix} \quad (11)$$

are  $2l \times 2l$  and  $2l \times m$  matrices and  $2l \times 1$  vector, respectively. In this case Eq. (9) must be replaced by

$$F(t) = -G_c w_c(t) \quad (12)$$

where  $G_c$  is an  $m \times 2l$  control gain matrix. Note that, in using the control law given by Eq. (12), the closed-loop modal equations are not independent, so that this procedure represents coupled control (Meirovitch 1990).

## 2.2 Active modal-fuzzy control system

The strategy of the active modal-fuzzy control algorithm for seismic protection is presented in Fig. 1. Though it is difficult to select input variables used in active fuzzy controller among an amount of state variables, the proposed active modal-fuzzy algorithm uses only modal coordinates corresponding selected first few modes as input variables and produces the desired control force. It is very effective for civil structures usually dominated by just first few modes. The proposed method has advantages over the design in physical space, in that it demands far less computer storage, reduces the computational effort significantly, and handle more easily. In other words, in this case of combination of fuzzy and modal approach, a modal-fuzzy control algorithm proposed can be magnified the efficiency caused by belonging their' own advantages together.

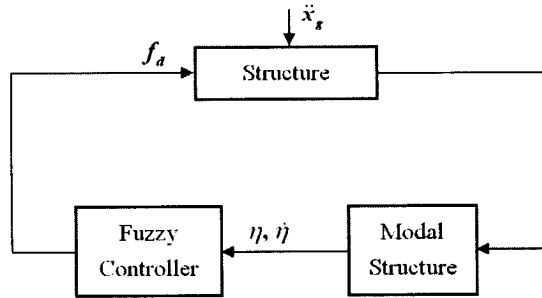


Fig. 1. Control diagram for the active modal-fuzzy control

## 3. Numerical Example

A model of a six-story shear building that is controlled with two hydraulic actuators is performed. This system is a simple model of the scaled, six-story, test structure adopted by Jansen and Dyke [12]. One device is rigidly connected between the ground and the first floor, and the other device is rigidly connected between the first and second floors, as shown in Fig. 2. The governing equations can be written in the form of Eq. (1) by defining the mass of each floor  $m_i$  as  $0.0227 \text{ N}/(\text{cm}/\text{s}^2)$ , the stiffness of each floor  $k_i$  as  $297 \text{ N}/\text{cm}$ , and a damping ratio for each mode of 0.5%.

An optimal control system is performed to compare with proposed algorithm and to use as reference of this simulation. The LQR (Linear Quadratic Regulator) control system with output weighting is very efficient and traditional linear optimal controller. The optimal control approach is to design a linear optimal controller gain vector that calculates a vector of desired control forces  $f_c = [f_{c1} \ f_{c2}]^T$  based on the measured structural

responses and the measured control force vector  $f$  applied to the structure.

For the control design, an infinite horizon performance index is chosen

$$J = \lim_{\tau \rightarrow \infty} \frac{1}{\tau} E \left[ \int_0^{\tau} \{y^T Q y + f_c^T R f_c\} dt \right]. \quad (13)$$

A wide variety of controllers were evaluated. The best results through many iterations were obtained using  $R=I_{2 \times 2}$  and placing a weighting 9000(cm<sup>-2</sup>) on the relative displacements of all floors.

The usual LQR is very efficient classical control algorithm. However, it is difficult to select weighting matrices (Q and R) and to design control system. And although it gives good performance, it is not guaranteed robustness.

The design of the active modal-fuzzy controller began to select the quantities to be used as input to the fuzzy controller. This needs not to select input variables among an amount of state variables. Especially, in civil structure like this example, it is possible to reduce the responses that control using the lowest one or two modes.

In this active modal-fuzzy case, two kinds of control design are considered. One (i.e., type A) is based a focus to reduce the structural displacement, and the other (i.e., type B) is based a focus to reduce the structural acceleration.

Type A controller is designed using two input variables (i.e. one is first mode displacement coordinate and the other is first mode velocity coordinate), each one having five membership functions, and one output variable (i.e. desired control force) with five membership functions. The membership functions chosen for the input and output variables are triangular shaped. Type B controller is similar to Type A except the number of membership functions output variable, having seven membership functions.

Fuzzy inference rule is completely based on the structural first mode displacement coordinate and first mode velocity coordinate. The fuzzy inference rules are shown in Table 2 for type A controller and the same as that of active fuzzy control system.

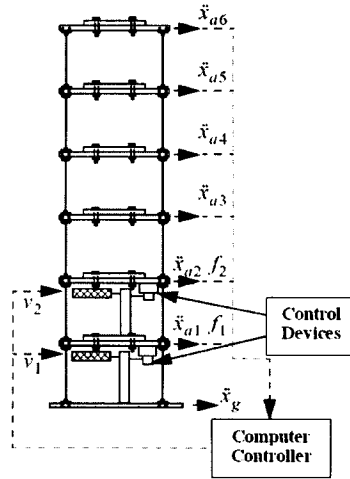


Fig. 2. Schematic diagram of the control devices implementation

Table 1 summarizes the results of the high(120%), medium(100%) and low(80%) scaled El Centro earthquake excitation simulations. Additionally, to compare the performance of the control algorithms considered the peak of the interstory drift and absolute acceleration responses for all floors were examined.

In the medium excitation simulation, the ratios of the normalized maximum responses in the optimal control system, active fuzzy control system and active modal-fuzzy control system (Type B) are 0.48, 0.60, 0.55 for the

displacement ( $J_1$ ), 0.63, 0.76, 0.64 for the interstory drift ( $J_2$ ) and 0.69, 0.66, 0.60 for the acceleration ( $J_3$ ), respectively. As seen the results, the overall performance of the system employing the active modal-fuzzy control system (Type B) is slightly better than the active fuzzy system and comparable to the optimal control system. However, Even though the active modal-fuzzy control system based a focus to reduce the structural displacement performs significantly better than other systems restricted within the displacement and interstory drift, the performance of the normalized peak floor acceleration are not good. In addition, Fig. 3 also shows similar phenomenon. This occurs because a trade-off is established between the various control objectives. It means that the building behaves in a more rigid manner, consequently decreasing the amount of interstory drift of the structure, and at the same time, increased rigidly results in higher floor accelerations within the building. However, a well designed active modal-fuzzy control system (Type B) balances the benefits of the different objectives within the requirements of the specific design scenario.

At high and low excitations, the performance results show similar trend to the medium case. It is demonstrated that the active modal-fuzzy control system that information of all state variables and the whole structure system is considered, without difficulty to select input variables used in fuzzy controller among numerous state variables is very effective in reducing the structural responses due to the earthquake excitation and is applicable to real civil structures.

Consequently, the results of the numerical simulations for a wide amplitude range of loading conditions and for historic earthquake show that the proposed active modal-fuzzy control system can be beneficial in reducing seismic responses of civil structures. The proposed algorithm gives comparable performances to active fuzzy controller and optimal controller and moreover, it is much easier to design control system than those controllers.

Table 1. Normalized controlled maximum responses due to high(120%), medium(100%), and low(80%) amplitude scaled El Centro earthquake.

Input excitation	Control strategy	$J_1$	$J_2$	$J_3$	$J_4$
High amplitude (120%)	LQR	0.479	0.610	0.912	0.0178
	Active fuzzy	0.745	0.885	0.939	0.0178
	Modal-fuzzy A	0.449	0.727	1.856	0.0178
	Modal-fuzzy B	0.729	0.762	0.842	0.0134
Medium amplitude (100%)	LQR	0.479	0.626	0.685	0.0178
	Active fuzzy	0.600	0.756	0.660	0.0178
	Modal-fuzzy A	0.343	0.562	1.186	0.0178
	Modal-fuzzy B	0.548	0.635	0.601	0.0134
Low amplitude (80%)	LQR	0.474	0.657	0.586	0.0178
	Active fuzzy	0.473	0.640	0.531	0.0178
	Modal-fuzzy A	0.231	0.467	1.110	0.0178
	Modal-fuzzy B	0.403	0.509	0.619	0.0134

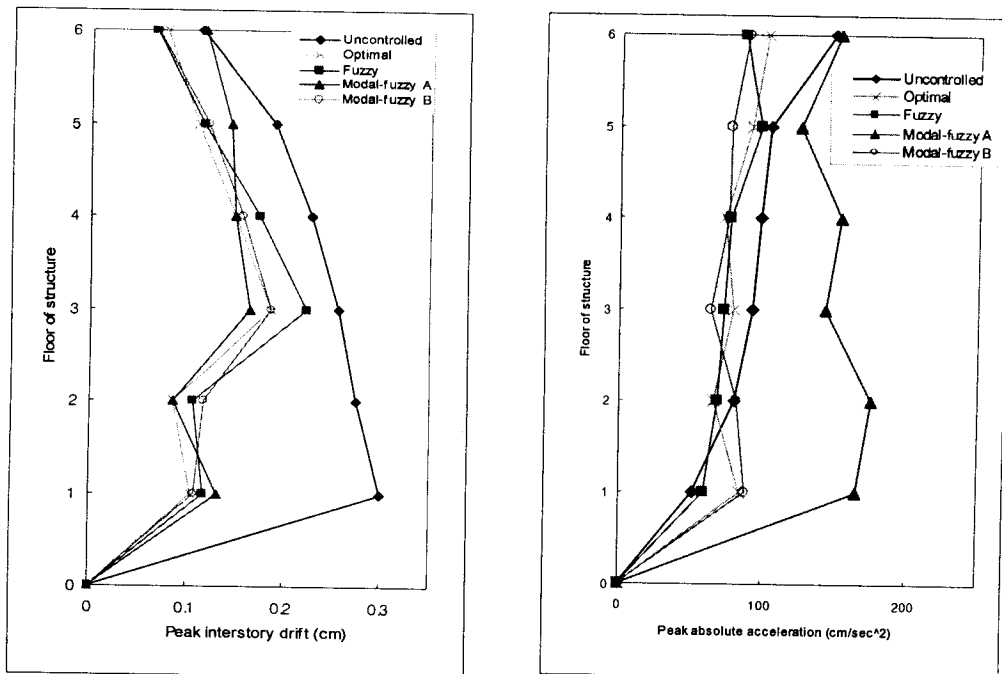


Fig. 3. Peak responses of each floor of structure to scaled El Centro earthquake

#### 4. Conclusions

An active modal-fuzzy control method is presented for seismic response reduction. The major issue in this study is based on the fuzzy algorithm adding modal approach for seismic response reduction. In the case of only active fuzzy control system, it is very difficult to select input variables used in fuzzy controller among an amount of state variables. However, the fuzzy theory has advantages to treat in a much easier way is the uncertainties of input data from the external loads and structural responses sensors and offer a simple and robust structure for the specification of nonlinear control laws. The modal approach has advantages over the design in physical space, in that it demands far less computer storage, reduces the computational effort significantly, allows a larger choice of control algorithms, including nonlinear control and is especially desirable for handling of civil structures that dominated by first few modes. In the case of combination of fuzzy and modal approach, therefore, a modal-fuzzy control algorithm proposed can be magnified the efficiency caused by belonging their' own advantages together. To this end, a modal-fuzzy control scheme is applied together with a Kalman filter and a low-pass filter to be applicable to real civil structures. A Kalman filter is considered to estimate modal states and a low-pass filter was used to eliminate spillover problem. The effectiveness of the proposed method in reducing the structural responses for a wide amplitude range of loading conditions has been demonstrated via a six-story building structure with hydraulic actuators. Numerical simulation results show that the proposed algorithm is quite effective to reduce seismic responses. The results of this investigation, therefore, indicate that the active modal-fuzzy control strategy could be used for control of seismically excited structures.

## Acknowledgement

This research was supported by the National Research Laboratory (NRL) program (Grant No.: 2000-N-NL-01-C-251) from the Ministry of Science and Technology in Korea. The financial support is gratefully acknowledged.

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