

# 회전자 시스템을 이용한 능동제어의 제어성능평가

## Control Performance of Active Vibration Control System using Gyroscope system

문영종<sup>1)</sup>

윤우현<sup>2)</sup>

이종현<sup>3)</sup>

이인원<sup>4)</sup>

*Moon, Yeong-Jong*

*Yoon, Woo-Hyun*

*Lee, Jong-Heon*

*Lee, In-Won*

**Abstract:** This paper investigates the possibility and effectiveness of gyroscope system for seismic protection of flexible structures employing LQG control algorithms. The gyroscope system is commonly used for the attitude control of an unstructured object in navigational, aeronautical and space engineering. The gyroscope system also can be used for vibration control of structured objects like buildings, towers and bridges. The system is effective for bending modes rather than shear modes, because the system utilizes the gyroscopic moment. The results of the numerical simulation show that gyroscope system has the possibility to reduce the vibration.

Keyword: Active control, Gyroscope system

### 1. Introduction

In recent years, many advanced techniques have been made in modern control theory, and they have been successfully applied to the study of control of a wide variety of engineering systems. Then, control of tall buildings and towers, long bridges and other flexible civil engineering structures has generated much interest. The necessity of considering control for these structures is due to the fact that, with the trend toward using lighter and more flexible construction materials, excess sways can occur under environmental loads. Environmental loads on flexible civil structures, such as those stemming from wind and earthquake can cause human discomfort,

motion sickness and sometimes endanger structural safety and integrity.

Passive and active control schemes are becoming an integral part of the structural systems over the last two decades. A passive control system does not require an external power source. Passive devices, such as base isolation system, viscoelastic dampers and tuned mass damper, are widely accepted by the engineering community as a means for mitigating the effect of dynamic loading on structures. However, these passive device methods are unable to adapt to structural changes and to varying usage patterns and loading conditions.

An active control system is one in which an external source powers control actuators that apply forces to the

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1) 한국과학기술원 건설환경공학과, 박사과정

2) 경원대학교 산업환경대학원, 교수

3) 경일대학교 토목환경공학과, 교수

4) 한국과학기술원 건설환경공학과, 교수

structures in a manner calculated by control algorithms. These forces can be used to both add and dissipate energy in the structure. In an active feedback control system, the signals sent to the control actuators are a function of the response of the system measured with physical sensors.

In the last two decades, many other active control devices have been developed and conducted by many researchers for civil engineering applications. But most of these devices are based on the active mass damper, applied shear forces to the structure. With distinctive properties, the popular one of active control devices used in navigational, aeronautical and space engineering is gyroscope system.

The gyroscope system is commonly used for the attitude control of an unstructured object. For example, when we need to lift up or carry a beam with crane, the attitude of beam turn aside or may be rotated while the crane is moving. Another case is attaching a big sign board on the wall. These need the attitude control to reject the effect due to the unwanted external force like winds. The gyroscope system also can be used for vibration control of structured objects like buildings, towers and bridges. The system is effective for bending modes rather than shear modes, because the system utilizes the gyroscopic moment. High-rise buildings and tall towers are generally flexible and therefore vibrations can be easily induced by external forces. The predominant dynamic deformation mode of tower-like structures is the bending mode rather than the shear mode. For this reason, the gyroscopic moment is effective in controlling vibrations in such structures. Furthermore, this system is more compact and has smaller mass than other control devices with the same ability to control.

## 2. Gyroscope system

A gyroscope system consists of a rotor which can spin freely about its geometric axis. When mounted in a Cardan's suspension (Fig. 1), a gyroscope can assume any orientation, but its mass center must remain fixed in space. The position of the gyroscope at any given instant can be

characterized by following angles.

- 1) a rotation of the outer gimbal through an angle  $\phi$  about the axis  $AA'$ ,
- 2) a rotation of the inner gimbal through  $\theta$  about  $BB'$ ,
- and 3) a rotation of the rotor through  $\psi$  about  $CC'$ .

The angles  $\phi$ ,  $\theta$  and  $\psi$  are called the Eulerian angles. Their derivatives  $\dot{\phi}$ ,  $\dot{\theta}$  and  $\dot{\psi}$  define, respectively, the rate of precession, the rate of nutation and the rate of spin of the gyroscope at the instant considered.

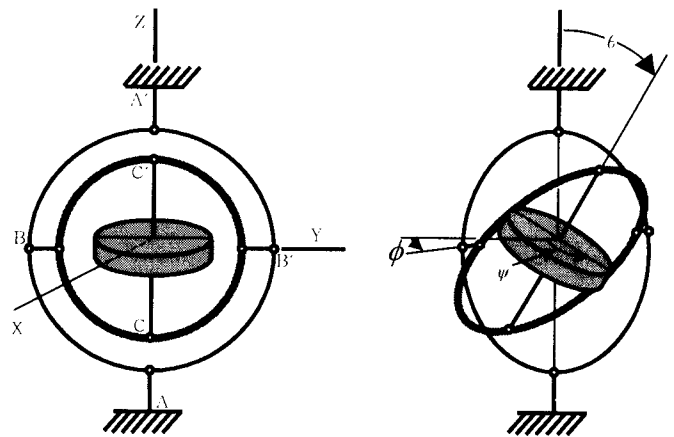


Fig. 1 Gyroscope system

The angular velocity  $\omega$  of the gyroscope will now be expressed as the sum of three partial angular velocities corresponding, respectively, to the precession, the nutation and the spin of the gyroscope. Denoting by  $i$ ,  $j$  and  $k$  the unit vectors along the rotating axes and by  $K$  the unit vector along the fixed  $Z$  axis, we have

$$\omega = \dot{\phi}K + \dot{\theta}j + \dot{\psi}k \quad (2.1)$$

Since the vector components obtained for  $\omega$  in equation (2.1) are not orthogonal, the unit vector  $K$  will be resolved into components along the  $x$  and  $z$  axes. We write

$$K = -\sin \theta i + \cos \theta k \quad (2.2)$$

and substituting for  $K$  into equation (2.1)

$$\omega = -\dot{\phi} \sin \theta i + \dot{\theta} j + (\dot{\psi} + \dot{\phi} \cos \theta) k \quad (2.3)$$

The components of the angular momentum  $H_O$  can be obtained by multiplying the components of  $\omega$  by the moments of inertia of the rotor about the  $x$ ,  $y$  and  $z$  axes,

respectively. Denoting by  $I$  the moment of inertia of the rotor about its spin axis, by  $I'$  its moment of inertia about a transverse axis through  $O$ , and neglecting the mass of the gimbals, we write

$$H_O = -I'\dot{\phi}\sin\theta\mathbf{i} + I'\dot{\theta}\mathbf{j} + I(\dot{\psi} + \dot{\phi}\cos\theta)\mathbf{k} \quad (2.4)$$

The rotating axes are attached to the inner gimbal and thus do not spin, we express their angular velocity as the sum

$$\Omega = \dot{\phi}\mathbf{K} + \dot{\theta}\mathbf{j} \quad (2.5)$$

or, substituting for  $\mathbf{K}$  from equation (2.2)

$$\Omega = -\dot{\phi}\sin\theta\mathbf{i} + \dot{\theta}\mathbf{j} + \dot{\phi}\cos\theta\mathbf{k} \quad (2.6)$$

Substituting for  $H_O$  and  $\Omega$  from (2.4) and (2.6) into the equation

$$\sum M_O = (\dot{H}_O)_{r.f.} + \Omega \times H_O \quad (2.7)$$

where  $r.f.$  means the rotating frame.

We can obtain the three differential equations

$$\sum M_x = -I'(\ddot{\phi}\sin\theta + 2\dot{\theta}\dot{\phi}\cos\theta) + I\dot{\theta}(\dot{\psi} + \dot{\phi}\cos\theta) \quad (2.8)$$

$$\sum M_y = I'(\ddot{\theta} - \dot{\phi}^2\sin\theta\cos\theta) + I\dot{\phi}\sin\theta(\dot{\psi} + \dot{\phi}\cos\theta) \quad (2.9)$$

$$\sum M_z = I\frac{d}{dt}(\dot{\psi} + \dot{\phi}\cos\theta) \quad (2.10)$$

The equations (2.8), (2.9) and (2.10) define the motion of a gyroscope subjected to a given system of forces when the mass of its gimbals is neglected.

We investigate the particular case of gyroscopic motion in which the angle  $\theta$ , the rate of precession  $\dot{\phi}$  and the rate of spin  $\dot{\psi}$  remain constant. Instead of applying the general equations, we will determine the sum of the moments of the required forces by computing the rate of change of the angular momentum of the gyroscope in the particular case considered. The angular velocity  $\omega$  of the gyroscope, its angular momentum  $H_O$ , and the angular velocity  $\Omega$  of the rotating frame reduce, respectively, to

$$\omega = -\dot{\phi}\sin\theta\mathbf{i} + (\dot{\psi} + \dot{\phi}\cos\theta)\mathbf{k} \quad (2.11)$$

$$H_O = -I'\dot{\phi}\sin\theta\mathbf{i} + I(\dot{\psi} + \dot{\phi}\cos\theta)\mathbf{k} \quad (2.12)$$

$$\Omega = -\dot{\phi}\sin\theta\mathbf{i} + \dot{\phi}\cos\theta\mathbf{k} \quad (2.13)$$

Since  $\theta$ ,  $\dot{\phi}$  and  $\dot{\psi}$  are constant, the sum of the moments of the required forces reduce to

$$\sum M_O = (I(\dot{\psi} + \dot{\phi}\cos\theta) - I'\dot{\phi}\cos\theta)\dot{\phi}\sin\theta\mathbf{j} \quad (2.14)$$

In the particular case when the precession axis and the spin axis are at a right angle to each other, we have  $\theta = 90^\circ$  and equation (2.14) reduces to

$$\sum M_O = I\dot{\psi}\dot{\phi}\mathbf{j} \quad (2.15)$$

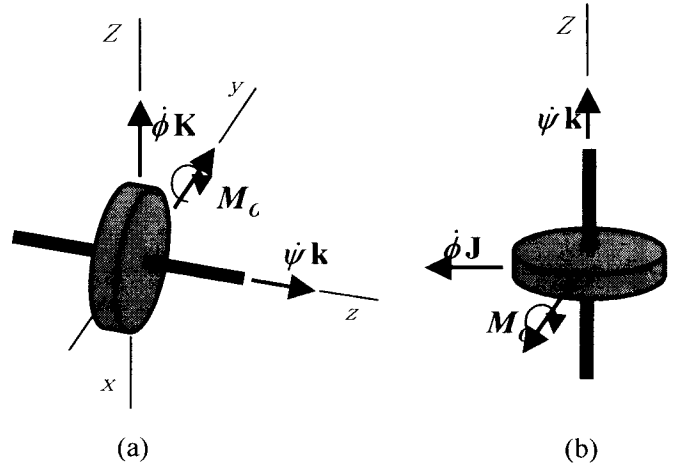


Fig. 2 Particular case of gyroscope system

In actual implementation of gyroscope to the structure, the axis of precession of gyroscope is changed as shown in Fig. 2 (b). Then, equation (2.15) should be changed to

$$M_y = I\dot{\psi}\dot{\phi}\cos\phi \quad (2.16)$$

$$M_z = I\dot{\psi}\dot{\phi}\sin\phi \quad (2.17)$$

The moment  $M_y$  is able to control the bending response of the structure of which this actuator is placed, while  $M_z$  acts as a torsional moment on the structure. It is to be noted, however, that it is also possible to eliminate the effect of this torsional moment by using a pair of gyroscope system. The equation of motion can be expressed as

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = \Gamma u(t) + \Lambda f(t) \quad (2.18)$$

$$u(t) = GM_y(t) \quad (2.19)$$

where  $G$  is the transformation matrix from the control moment to the equivalent shear force. The transformation matrix  $G$  can be obtained by static condensation of stiffness matrix. A static condensation to the full model of

structure is used as a model reduction scheme. First, the stiffness matrix is partitioned into transverse and rotational DOFs as

$$K = \begin{bmatrix} K_{aa} & K_{ad} \\ K_{da} & K_{dd} \end{bmatrix} \quad (2.20)$$

The system equation for static equilibrium is written as

$$\begin{bmatrix} K_{aa} & K_{ad} \\ K_{da} & K_{dd} \end{bmatrix} \begin{Bmatrix} u_a \\ u_d \end{Bmatrix} = \begin{Bmatrix} F_a \\ F_d \end{Bmatrix} \quad (2.21)$$

or

$$K_{aa}u_a + K_{ad}u_d = F_a \quad (2.22)$$

$$K_{da}u_a + K_{dd}u_d = F_d \quad (2.23)$$

where  $u_a$  and  $u_d$  are the transverse displacement and rotational angle, respectively.  $F_a$  and  $F_d$  are the applied force and moment, respectively.

Equation (2.23) can be changed as

$$u_d = -K_{dd}^{-1}K_{da}u_a + K_{dd}^{-1}P_d \quad (2.24)$$

Substituting for equation (2.24) into equation (2.22)

$$(K_{aa} - K_{ad}K_{dd}^{-1}K_{da})u_a = P_a - K_{ad}K_{dd}^{-1}P_d \quad (2.25)$$

So we obtain the transformation matrix  $G$

$$G = -K_{ad}K_{dd}^{-1} \quad (2.26)$$

### 3. Numerical Simulation Results

To evaluate the control performance of gyroscope system, numerical examples are considered in which a model of 2.5m cantilever beam is controlled with AMD or gyroscope system. The model consists of 5 beam elements, 5 nodes with two degrees of freedom, lateral and rotation displacements, for each node. Then, 5 degrees of freedom for rotation displacements are removed by static condensation. Therefore, 5 degrees of freedom are included in the model shown in Fig. 3. The stiffness of the beam element was obtained from the classical bending theory or Euler-Bernoulli theory. Mass was represented in the model of the cantilever beam as lumped mass element.

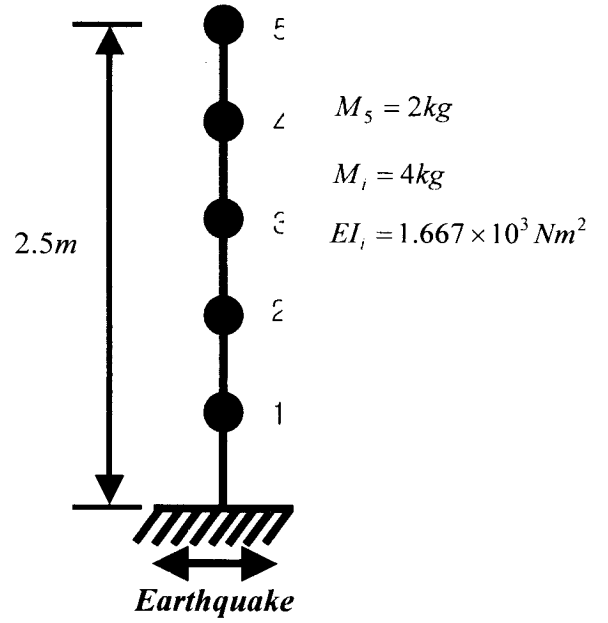


Fig. 3 Numerical Model for Cantilever Beam

In simulation, the model of the structure is subjected to the four earthquakes, El Centro, Hachinohe, Gebze and Mexico earthquake. Because the model considered is a scaled model, the amplitude of the earthquakes was scaled to ten percent of the full scale earthquakes.

To systematically evaluate the control performance of each controller, the four evaluation criteria defined. : normalized peak displacement ( $J_1$ ), normalized peak inter node drift ( $J_2$ ), normalized peak acceleration ( $J_3$ ), normalized peak force generated by all control devices ( $J_4$ ).

The control performance of the LQG control algorithm in reducing the responses of the cantilever beam will be demonstrated through numerical simulation. A 1kg AMD (mass ratio of 5%) will be installed on the top of the structure. A tuned mass damper with no damping ratio and the frequency tuned to the first mode will be used in the simulation. An AMD can be simply modeled by connecting actuator between the tuned mass damper and the top of the structure to provide control force acting on the structure. For the actuators, the maximum force capacity and the maximum stroke are unlimited. The variations of each evaluation criteria for increasing weighting parameters are studied. When  $J_7$  is the

summation of evaluation criteria,  $J_1$ ,  $J_2$  and  $J_3$ , we can find the weighting for reduction of overall structural responses from the variations of  $J_T$  whereas from  $J_1$ ,  $J_2$  and  $J_3$ , we can find the weighting for reduction of related responses. The measurement noise is assumed to be identically distributed, statistically independent Gaussian white noise processes and  $S_{\ddot{x}_g, \ddot{x}_g} / S_{v, v} = 50$ .

The response quantities of the structure with the LQG controller are presented in Table 1. As shown Table 1, the AMD system reduces the maximum displacements and internode drifts of the structure by approximately 75-90% of the uncontrolled values. The maximum accelerations is also approximately 65-80% smaller than the uncontrolled values.

The control performance of gyroscope system in reducing the responses of the cantilever beam will be demonstrated through numerical simulation. Two gyroscope systems will be installed on the top of the structure. A pair of gyroscope systems, whose flywheels rotate in the opposite directions, are used to eliminate torsional moment. The gyroscope system has a flywheel rotating speed of 1500rpm and a mass of 0.5kg. The moment of inertia of a flywheel is 0.0025kg·m<sup>2</sup>.

The response quantities of the structure with LQG controller are presented in Table 2. As shown Table 2, the gyroscope system reduces the maximum displacements and inter-node drifts of the structure by approximately 70-85% of the uncontrolled values. The maximum accelerations is also approximately 65-80% smaller than the uncontrolled values. The overall performance of gyroscope is less than that of the AMD. But this results show that gyroscope system has the possibility to reduce the vibration.

## 6. Conclusions

Characteristics of a vibration control system utilizing gyroscope system and its effectiveness in the control of earthquake and wind induced vibration were studied. Numerical analysis for a scaled cantilever beam was

carried out. The numerical example shows that the gyroscope system has the possibility to reduce the vibration. The performance of the gyroscope system is proportional to rotating speed and moment of inertia of flywheel. The moment of inertia of flywheel is proportional to square of radius of flywheel if flywheel is circular disk. Therefore, mass of control system can be reduced by increasing radius of flywheel.

This system has a potential for application to towers, cranes, high-rise buildings and other structures with prominent bending modes. Furthermore, this system is more compact and has smaller mass than other control devices with the same ability to control

Table 1 Evaluation criteria of AMD

Earthquake	El centro	Hachinohe	Gebze	Mexico
$J_1$	0.1139	0.1566	0.2128	0.2059
$J_2$	0.1206	0.1690	0.2415	0.2040
$J_3$	0.2805	0.1903	0.2913	0.3552
$J_4$	0.0175	0.0090	0.0179	0.0056

Table 2 Evaluation criteria of gyroscope system

Earthquake	El centro	Hachinohe	Gebze	Mexico
$J_1$	0.1317	0.1526	0.2837	0.2059
$J_2$	0.1500	0.1690	0.3250	0.2375
$J_3$	0.3227	0.1977	0.3489	0.3014
$J_4$	0.0222	0.0121	0.0122	0.0048

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## References

1. Kobori, T. (1994), "Future Direction on Research and Development of Seismic-Response-Controlled Structure," *Proc. 1st World Conf. On Struct. Control, LA, California*, Panel:19-31.
2. Sakamoto, M., Kobori, T., Yamada, T. and Takahashi, M., (1994), "Practical Application of Active and Hybrid Response Control Systems and Their Verifications by Earthquake and Strong Wind Observations," *Proc. 1st World Conf. On Struct. Control, LA, California*, pp. WP2:90-99.
3. Petersen, N. R. (1980), "Design of Large Scale Tuned mass Dampers", *Proc. First Int. KUTAM Symposium on Structural Control, University of Waterloo, Canada*, pp. 581-596.
4. Yao, J. T. P., (1972), "Concept of Structural Control", *ASCE Journal of Structural Division*, Vol. 98, pp. 1567-1574
5. Brogan, W.L. (1991). *Modern Control Theory*, Prentice Hall, Englewood Cliffs, New Jersey.
6. Bryson, A. E. Jr, (1985), "New Concepts in Control Theory, 1959-1984," *J. Guidance, Control and Dyn.*, 8(4), 417-425.
7. Chang, J. C. H. and Soong, T. T.,(1980), "The Use of Aerodynamic Appendages for Tall Building Control", *Proc. First Int. KUTAM Symposium on Structural Control, University of Waterloo, Canada*, pp. 199-210.
8. Dyke, S.J., Spencer Jr., B.F., Quast, P., Sain, M.K., Kaspari Jr., D.C. and Soong, T.T. (1996d). "Acceleration Feedback Control of MDOF Structures," *J. of Engrg. Mech., ASCE*, Vol. 122, No. 9, pp. 907-918.
9. Dyke, S.J., Spencer Jr., B.F., Quast, P., Kaspari Jr., D.C., and Sain, M.K. (1996e). "Implementation of an AMD Using Acceleration Feedback Control," *Micro-computers in Civil Engrg.*, Vol. 11, pp. 305-323.
10. Indrawan, B. and Higashihara, H., (1993). "Active Vibration Control with Explicit treatment of actuator's limit," *Proc., ATC-17-1 Seminar on Seismic Isolation, Passive Energy Dissipations, and Active Control. Aplied Tech. Council, Redwood City, Calif.*, pp. 715-726
11. Soong, T. T. (1990), "Active structural control," *Longman's, London, and John Wile and Sons, Inc., Newyork.*
12. Chang, J. C. H and Soong, T. T., (1980), " Structural Control using Active Tuned mass Dampers", *ASCE Journal of Engineering Mechanics*, Vol. 106, pp. 1091-1098.
13. NcNamara, R. J., (1977), "Tuned Mass Damper for Buildings", *ASCE Journal of Structural Division*, Vol. 103, pp. 1789-1798.
14. Lund, R. A., (1980), "Active Damping of Large Structures in Winds", *Structural Control, North Holland, Amsterdam*, pp. 459-470.
15. Higashiyama, H., Yamada, M., Kazao, Y. and Namiki, M., (1998), "Characteristics of Active Vibration Control System using Gyro-stabilizer", *Engineering Structure*, Vol. 20, No. 3, pp. 176-183.