

# MR댐퍼를 이용한 진동의 모드제어

## Modal Control of Vibration Using MR Damper

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### ABSTRACT

본 연구에서는 지진하중을 받는 구조물의 MR감쇠기를 이용한 모드제어를 구현하고자 한다. 모드제어는 저차의 모드만을 이용하여 구조물의 진동을 제어한다. 따라서 토목구조물과 같은 자유도가 많은 대형구조물의 제어에 적합하며, 제어기의 설계역시 다른 제어이론에 비해 간편하다. 모드제어에 대한 앞선 연구들이 많이 있음에도 불구하고, 반능동제어, 특히 MR감쇠기에 대한 모드제어의 성능에 대한 연구가 없었다. 본 연구에서는 MR댐퍼에 대한 효율성을 검토하기 위해서, 모드제어이론을 지진하중을 받는 구조물에 구현하였다. 센서로부터 측정된 응답을 이용하여 모드응답을 예측하기 위해 칼만필터를 구현하였으며, Spillover problem을 해결하기 위해 low-pass filter가 적용되었다. 수치예제를 통해 저차의 모드만을 제어함으로써, 구조물의 거동이 효과적으로 줄어드는 것을 확인하였다. 그러나 줄어드는 정도는 제어기의 가중치행렬과 칼만필터에 사용되는 센서에 의해 측정된 응답에 따라 다르다.

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### 1. Introduction

Magnetorheological (MR) dampers are one of semi-active control devices, which use MR fluids to provide controllable damping forces. A number of control algorithms have been adopted for semi-active systems including the MR damper. Jansen and Dyke (2000) discussed recently proposed semi-active control algorithms including the decentralized bang-bang controller (MaClamroch and Gavin 1995), the controller based on Lyapunov stability theory (Brogan 1991; Leitmann 1994), the clipped-optimal controller (Sack et al. 1994; Dyke 1996), the modulated homogeneous friction controller (Inaudi 1997), and the maximum energy dissipation algorithm. They, also, formulated these algorithms for use with MR dampers and evaluated and compared

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the performance of each algorithm.

Modal control represents one control class, in which the motion of a structure is reshaped by merely controlling some selected vibration modes. Modal control is especially desirable for the vibration control of civil engineering structure may involve hundred or even thousand degrees of freedom, its vibration is usually dominated by the first few modes. Therefore, the motion of the structure can be effectively suppressed by merely controlling these few modes. A modal control scheme, which uses modal state estimation, is desirable (Meirovitch 1990).

The purpose of this study is to implement modal control for seismically excited structures that use MR dampers and to compare the performance of the proposed method with that of other control algorithms previously studied. A modal control scheme with a Kalman filter and a low-pass filter is applied. A Kalman filter is included in a control scheme to estimate modal states from measurements by sensors. Three cases of the structural measurement are considered by a Kalman filter to verify the effect of each measurement; displacement, velocity, and acceleration, respectively. Moreover, a low-pass filter is applied to eliminate the spillover problem.

## 2. Modal Control Scheme for MR dampers

### 2.1 Modal Control

Consider a seismically excited structure controlled with  $m$  MR dampers. In modal control, only a limited number of lower modes are controlled. Hence,  $l$  controlled modes can be selected with  $l < n$  and the displacement may be partitioned into controlled and uncontrolled parts as  $x(t) = x_c(t) + x_R(t)$  where  $x_c$  and  $x_R$  represent the controlled and uncontrolled displacement vector, respectively. We refer to the uncontrolled modes as residual. Considering orthogonal condition between eigenvectors, we obtain

$$\ddot{\eta}_r + 2\zeta_r \omega_r \dot{\eta}_r + \omega_r^2 \eta_r = \phi_r^T \Lambda f - \phi_r^T \Gamma \ddot{x}_g = 1, 2, \dots, l \quad (1)$$

where  $\eta_r(t)$  is  $r$  th modal displacement;  $\phi_r$  is the  $r$  th natural frequency;  $\phi$  is a eigenvector set;  $\eta$  is a modal displacement;  $\zeta_r$  are modal damping ratios; and  $\omega_r$  is a natural frequency;  $f = [f_1, f_2, \dots, f_m]^T$  is the vector of measured control forces generated by  $m$  MR dampers;  $\ddot{x}_g$  is ground acceleration;  $\Gamma$  is the column vector of ones; and  $\Lambda$  is the matrix determined by the placement of MR dampers in the structure. Then, Equation (1) can be rewritten in state-space form such as

$$\dot{w}_c(t) = A_c(t) w_c(t) + B_c f(t) + E_c \ddot{x}_g, y_c(t) = C_c w_c(t) \quad (2)$$

where  $w_c$  is a  $2l$ -dimensional modal state vector by the controlled modes and

$$A_c = \begin{bmatrix} 0 & I_c \\ -\Omega_c^2 & -\Delta_c \end{bmatrix}, B_c = \begin{bmatrix} 0 \\ B'_c \end{bmatrix}, E_c = \begin{bmatrix} 0 \\ E'_c \end{bmatrix} \quad (3)$$

are the  $2l \times 2l$ ,  $2l \times m$  matrixes and a  $2l \times 1$  vector, respectively, and  $\Delta_c$  is the diagonal matrix listing  $2\omega_r \zeta_r$ ;  $\Omega_c^2$  is the diagonal matrix listing  $w_1^2, \dots, w_n^2$ ;  $B_c' = \Phi^T A$ ; and  $E_c' = \Phi^T \Gamma$ . For feedback control, the control vector is related to the modal state vector according to

$$f(t) = -K_c w_c(t) \quad (4)$$

where  $K_c$  is an  $m \times 2l$  control gain matrix. Because the force generated in the  $i$  th MR damper depends on the responses of the structural system, the MR damper cannot always produce the desired optimal control force  $f_{ci}$ . Thus, the strategy of a clipped-optimal control (Dyke et al. 1996) is used. Referring to the discussions in above section, control gain matrix  $K_c$  should be decided. Although a variety of approaches may be used to design the optimal controller, H2/LQG (Linear Quadratic Gaussian) methods are advocated because of their successful application in previous studies (Dyke et al. 1996).

## 2.2 Modal State Estimation

To estimate the modal state vector  $w_c(t)$  from the measured output  $y(t)$ , we consider a Kalman-Bucy filter as an observer (Meirovitch, 1990). Not only, in this paper, the state feedback including velocities or displacements is considered, but also the acceleration feedback is implemented for the modal state estimation using a Kalman-Buch filter. Then we can write the observer equation in the form

$$\dot{\hat{w}}_c(t) = (A_c - B_c K_c) \hat{w}_c(t) + LC_c(w_c - \hat{w}_c) + LC_R w_R(t) + E_c \ddot{x}_g \quad (5)$$

where  $w_c(t)$  is the estimated controlled modal state and  $L$  is the optimally chosen observer gain matrix by solving a matrix Riccati equation, which assumes that the noise intensities associated with earthquake and sensors are known.  $C_c$  is changeable according to the signals which are used for the feedback and  $D_c$  is generally zero except the acceleration feedback. The error vector is defined such as  $e_c(t) = \hat{w}_c(t) - w_c(t)$ . Then the Equations can be written in the matrix form

$$\begin{bmatrix} \dot{w}_c(t) \\ \dot{w}_R(t) \\ \dot{e}_c(t) \end{bmatrix} = \begin{bmatrix} A_c - B_c K_c & 0 & -B_c K_c \\ -B_R K_c & A_R & -B_R K_c \\ 0 & LC_R & A_c - B_c K_c \end{bmatrix} \begin{bmatrix} w_c(t) \\ w_R(t) \\ e_c(t) \end{bmatrix} + \begin{bmatrix} E_c \\ E_R \\ E_c \end{bmatrix} \ddot{x}_g \quad (6)$$

Note that the term  $-B_R K_c$  in Eq. (6) is responsible for the excitation of the residual modes by the control forces and is known as control spillover. If  $C_R$  is zeros, which means the sensor signal only include controlled modes, the term  $-B_R K_c$  has no effect on the eigenvalues of the closed-loop system. Hence, we conclude that control spillover cannot destabilize the system. Normally, however, the above system can not satisfy the separate principle because the term  $LC_R$  affects eigenvalues of the controlled system by the observer. This effect is observation

spillover and can produce instability in the residual modes. However, a small amount of damping inherent in the structure is often sufficient to overcome the observation spillover effect.

### 3. Numerical Example

To evaluate the proposed modal control scheme for use with the MR damper, a numerical example is considered in which a model of a six-story building is controlled with four MR dampers (Fig. 1). This numerical example is the same with that of Jansen and Dyke (2000) and is adopted for direct comparisons between the proposed modal control scheme and other control algorithms. In simulation, the model of the structure is subjected to the NS component of the 1940 El Centro earthquake. Because the building system considered is a scaled model, the amplitude of the earthquake was scaled to ten percent of the full-scale earthquake. The various control algorithms were evaluated using a set of evaluation criteria based on those used in the second generation linear control problem for buildings (Spencer and Sain 1997) such as

$$J_1 = \max \left( \frac{|x_i(\dot{t})|}{x_{\max}} \right), \quad J_2 = \max \left( \frac{|d_i(\dot{t})/h_i|}{d_n^{\max}} \right), \quad J_3 = \max \left( \frac{|\dot{x}_{at}(\dot{t})|}{\dot{x}_a^{\max}} \right), \quad J_4 = \max \left( \frac{|f_i(\dot{t})|}{W} \right)$$

The resulting evaluation criteria are presented in Table 1 for the control algorithms previously studied (Jansen and Dyke, 2000). The numbers in parentheses indicate the percent reduction as compared to the best passive case. To compare the performance of the semiactive system to that of comparable passive systems, two cases are considered in which MR dampers are used in a passive mode by maintaining a constant voltage to the devices. The results of passive-off (0V) and passive-on (5V) configurations are included.

For modal control, three cases of the structural measurements are considered; displacements, velocities and accelerations. Using each structural measurement, a Kalman filter estimates the modal states. Fig. 2 represents the variations of each evaluation criteria for increasing weighting parameters in a 3-dimensional plot.  $J_T$  is the summation of evaluation criteria,  $J_1$ ,  $J_2$  and  $J_3$ . We can find the weighting for reduction of overall structural responses from the variations of  $J_T$ , whereas we can find the weighting for reduction of related responses from  $J_1$ ,  $J_2$  and  $J_3$ . Designer can decide which to use according to control objectives. By using the controller (H2/LQG) with designed weighting matrices from Fig. 2, we can get the results in Table 2.

### 4. Conclusion

In this paper, modal control was implemented to seismically excited structures using MR dampers. To this end, a modal control scheme was applied together with a Kalman filter and a low-pass filter. A Kalman filter considered three cases of the structural measurement to estimate modal states: displacement, velocity, and acceleration, respectively. Moreover, a

low-pass filter was used to eliminate spillover problem. In designing phase of controller, the size of weighting matrix  $Q$  was reduced because the lowest one or two modes were controlled. Therefore, it is more convenient to design the smaller weighting matrix of modal control. This is one of the important benefits of the proposed modal control scheme. The numerical results show that the motion of the structure was effectively suppressed by merely controlling a few lowest modes, although resulting responses varied greatly depending on the choice of measurements available and weightings. The modal controller  $A$  and  $V$  achieved significant reductions in the responses. The modal controller  $A_{J2}$ ,  $V_{J2}$  and  $V_{J3}$  achieve reductions (39%, 30%, 30%) in evaluation criteria  $J_1$ ,  $J_2$  and  $J_3$ , respectively, resulting in the lowest values of all cases considered here. The modal controller  $A_{JT}$  and  $V_{JT}$  fail to achieve any lowest value of evaluation criteria, but have competitive performance in all evaluation criteria. Based on these results, the proposed modal control scheme is found to be suited for use with MR dampers in a multi-input control system.

Table 1\* Normalized Controlled Maximum Responses due to the Scaled El Centro Earthquake

Control strategy	$J_1$	$J_2$	$J_3$	$J_4$
Passive-off	0.862	0.801	0.904	0.00292
Passive-on	0.506	0.696	1.41	0.0178
Lyapunov controller A	0.686(+35)	0.788(+13)	0.756(16)	0.0178
Lyapunov controller A	0.326(35)	0.548(21)	1.39(+53)	0.0178
Decentralized bang-bang	0.449(11)	0.791(+13)	1.00(+11)	0.0178
Maximum energy dissipation	0.548(+8)	0.620(11)	1.06(+17)	0.0121
Clipped-optimal A	0.631(+24)	0.640(8)	0.636(29)	0.01095
Clipped-optimal B	0.405(20)	0.547(21)	1.25(+38)	0.0178
Modified homogeneous friction	0.421(17)	0.599(20)	1.06(+17)	0.0178

(\*Jansen and Dyke 2000)

Table 2 Normalized Controlled Maximum Responses of the Various Feedback due to the Scaled El Centro Earthquake

Control strategy	Weighting parameters	$J_1$	$J_2$	$J_3$	$J_4$
Modal control AJ1	qmd=400, qmv=1500	0.310(-39)	0.529(-24)	1.07(+18)	0.0178
Modal control AJ2	qmd=1, qmv=500	0.398(-21)	0.485(-30)	0.870(-4)	0.0178
Modal control AJ3	qmd=2200, qmv=100	0.549(+8)	0.618(-11)	0.697(-23)	0.0176
Modal control AJT	qmd=500, qmv=600	0.380(-25)	0.488(-30)	0.823(-9)	0.0178
Modal control DJ1	qmd=100, qmv=4900	0.403(-20)	0.560(-20)	0.765(-15)	0.0177
Modal control DJ2	qmd=100, qmv=4900	0.403(-20)	0.560(-20)	0.769(-15)	0.0178
Modal control DJ3	qmd=200, qmv=4900	0.702(+39)	0.728(+5)	0.671(-26)	0.0178
Modal control DJT	qmd=3300, qmv=4700	0.408(-19)	0.566(-19)	0.721(-20)	0.0178
Modal control VJ1	qmd=700, qmv=800	0.327(-35)	0.554(-20)	1.06(+17)	0.0178
Modal control VJ2	qmd=1, qmv=400	0.383(-24)	0.487(-30)	0.874(-3)	0.0177
Modal control VJ3	qmd=1300, qmv=100	0.541(+7)	0.611(-12)	0.632(-30)	0.0178
Modal control VJT	qmd=600, qmv=500	0.354(-30)	0.502(-28)	0.825(-9)	0.0176

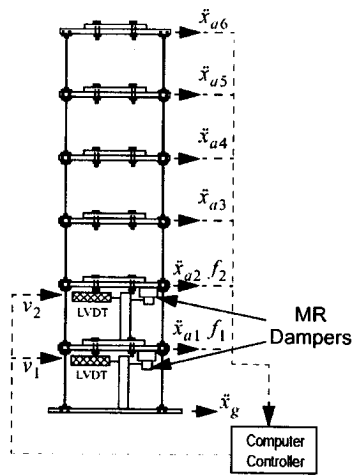


Fig. 1 예제구조물 (Jansen and Dyke 2000)

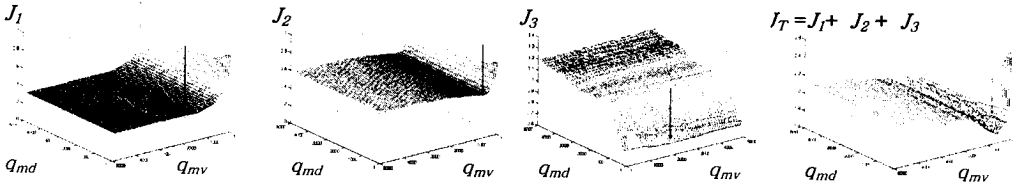


Fig. 2 Variations of Evaluation Criteria with Weighting Parameters

### 감사의 글

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