기하학적 비선형을 고려한 지하매설 복합재료 파이프의 해석

GEOMETRIC NONLINEAR ANALYSIS OF UNDERGROUND LAMINATED COMPOSITE PIPES

김 덕 현*
Kim, Duk-Hyun
이 인 원**
Lee, In-Won
변 문 주***
Byeon, Moon-Joo

요 약

우리는 생활주변에서 파이프의 사용을 흔히 볼 수 있다. 그 만큼 파이프의 소요량은 우리 생활에서 엄청난 양이라 할 수 있는데 그것이 기존 재료로는 콘크리트나 철강제품이 대부분을 차지하고 있다. 요즈음 대체 재료로써 복합재료가 여러 산업 분야에서 각광을 받고 있다. 처음 항공분야에서 사용이 시작되어 제품의 우수성 뿐 아니라 그 값이 점차 낮아짐에 따라 여러 분야에서 사용되고 있다. 복합재료는 내구성, 내열성, 내부식성 등 다른 어느 재료보다 좋은 성질을 가지고 있으며 특히 중량이 가볍다. 파이프 매설 공사에 있어서 운반비의 비중은 전체 공사비에 약 20−40%에 달할 만큼 크다. 따라서 복합재료의 선택은 그 비용을 감소시킬 수 있을 뿐 아니라 내구성, 내열성 등 복합재료의 여러가지 우수성을 동시에 가질 수 있다. 그리고 재료의 발달이 가속되고 있어 앞으로 유용성은 더욱 커질 것이다.

지하매설 파이프로써 반경에 비해 두께가 얇은 관인 경우 큰 변형이 발생할 것이다. 따라서 기하학적 비선형성을 고려하여야 한다. 이를 위해 변형 후의 형상에 대해 평형방정식을 세웠으며 이를 Galerkin's method에 의해 풀었다. 하중 조건은 파이프가 땅속에 묻히게 되므로 수직하중은 매설 깊이에 비례하며 수평하중은 수평변위에 비례하게 가정하였다. 복합재료로 만들어진 파이프는 층(layer)수와 fiber 방향등에 따라 강성이 틀려지며 또한 흙의 종류와 발생되는 변위에 따라 파이프−흙간의 상호작용이 달라진다. 본 연구에서는 복합재료로 만들어진 파이프가 지하에 매설된 경우 기하학적 비선형성을 고려한 해석방법을 제시하며 파이프 강성에 미치는 여러 인자에 대해 고찰해 보았다. 결과가 유한요소법에 의해 검증되었다.

* 선경 주식회사 기술고문
** 한국과학기술대학 기계, 재료공학부 CAD/CAM 전공 부교수
*** 한국과학기술대학 기계, 재료공학부 CAD/CAM 전공 조교

□ 이 논문에 대한 토론을 1989년 6월30일까지 본학회에 보내주시면 1989년 12월호에 그 결과를 게재하겠읍니다.

Abstract

An analytical study was conducted using the Galerkin technique to determine behaviour of thin fibrereinforced and laminated composite pipes under soil pressure. Geometric nonlinearity and material linearity have been assumed. It is assumed that vertical and lateral soil pressure are proportional to the depth and lateral displacement of the pipe respectively. It is also assumed that radial shear stress is negligible because the ratio of thickness to the radius of pipe is very small. The above results are verified by the finite element analysis.

INTRODUCTION

Filament wound composite pipes have been used for small diameter water pipes, large diameter sewage pipes, grain storage tanks,rocket propellent containers, flow lines, salt water handlings, petrochemical plant facilities, structural frames of large structures, pressure vessels for military and aerospace applications, and many others. Compared with metal pipes, composite pipes have the following advantages.

Corrosion resistance – Maintenance free
Lightweight – Economical to install
Excellent flow properties
Paraffin build – up resistance
Scale build – up resistance
Nonconductive
Low thermal conductivity

In case of line pipe construction, cost of transporting pipes can be significant : it could be as much as or more than the cost of pipes, depending on the length of the pipeline. This transporation effort could become the major concern of the project if the size of the pipes is large. Because of its lightweight nature, composite pipes can be extensively used for pipeline construction. When pipe is buried, the surrounding soil and the pipe interact structurally, and the forces acting on the pipe are functions of the medium that surrounds it as well as the stiffness of the pipe itself. The contribution of the surrounding soil in resisting external loads can be very important and can provide considerable saving in pipe material. The conventional recommendations for design and construction of underground pipes are based on the extensive work of Marton and Spangler[1]. However, the use of this method for flexible pipes may cause high safety factors[2]. Earlier works on this subject by others include these of Meyerhof [3], Timmers[4], Brockenbrough[5], Valentine [6], et al. Costes[7] suggests to use arching theory to reduce the soil load directly above the pipe. Richards and Agrawal[8] applied Barjansky's tunnel solution by transforming the pipe section to an equivalent ring of soil. The loading on the cross section of the buried pipes is assumed to come from two sources : (A) The weight of backfill soil and any top loading present. The magnitude of this vertical forces, q_v, is the weight of backfill soil, modified by the effect of shear stress due to settlement of backfill, plus the pressure due to any line load present, usually calculated by Boussinesq solution. (B) The force of the soil against pipe sides which tends to prevent deformations caused by the (A) type force(see Figure 2). The assumption generally made here is that this force, q_v, is proportional to the horizontal pipe deflection, δ_h, and can be expressed and $K\delta_h$ where K is called as subgrade reaction coefficient of modulus of the foundation. Defining the value of K correctly is rather complex. Rubin[9], using inextensional cylindrical shell theory, and assuming that the vertical deformation is approximately equal to the negative of the horizontal deformation, obtained the value of K as the function of rigidity and diameter of the pipe. However, according to Valentine's experiment[6], the magnitude of the horizontal deformation is between 0 to 80 percent of the vertical deformation. Furthermore, it has to be noted that K value is related to soil properties. Molin[10] adds the horizontal earth pressure at rest to the Spangler's concept. This may be acceptable if the surrounding media is clay. However, it is general practice to put granular material around the pipes and to compact it. If the pipe deflects to "near" maximum under above condition, when the horizontal thrust from soil becomes signficant, the $K\sigma_h$ value can be assumed to be "close" enough to the passive earth pressure of the soil. Szyszkoski and Glockner[12] reported result of nonlinear analysis of buried aluminum tubes. The model used is similar to of Spangler with $K\sigma_h$ value as passive earth pressure. In this paper, underground pipes made of laminated composited are considered. Because of the flexible nature of thin composite walls, the problem is geometrically nonlinear. Assuming the pipe section under consideration is sufficiently "far" from end or bent, the problem is considered as that of plane strain. Equilibrium equations are obtained from the deformed shape and are solved by Galerkin's method. It is assumed that the vertical load on the pipe is proportional to the depth of the backfill and horizontal load is proportional to the horizontal deformation. Any modification of loading can be made easily depending on actual soil condition and live load, if necessary. Given constant value of such surrounding condition, a composite pipe has different stiffness depending on number of layers and fiber directions and so on. This results in different amount of deflection and differect soil-structure interaction. This paper presents a method of nonlinear analysis of underground composite pipes and effect of variable factors, such as fiber orientations and different values of subgrade reaction coefficients, on soil-structure interaction.

ANALYSIS METHOD

1. Displacement Relation

Equations of strain and displacement with respect to orthogonal curvilinear coordinates are as follows [13] [14].

$$\begin{aligned}
\varepsilon_x &= \frac{1}{\alpha}\Big[u_x+\frac{\alpha_y v}{\beta}+\frac{\alpha_z w}{\gamma}+\frac{1}{2\alpha}\Big(u_x+\frac{\alpha_y v}{\beta}+\frac{\alpha_z w}{\gamma}\Big)^2 \\
&\quad +\frac{1}{2\alpha}\Big(v_x-\frac{\alpha_y u}{\beta}\Big)^2+\frac{1}{2\alpha}\Big(w_x-\frac{\alpha_y u}{\gamma}\Big)^2\Big] \\
\varepsilon_y &= \frac{1}{\beta}\Big[v_y+\frac{\beta_z w}{\gamma}+\frac{\beta_x u}{\alpha}+\frac{1}{2\beta}\Big(v_y+\frac{\beta_y w}{\gamma}+\frac{\beta_x u}{\alpha}\Big)^2 \\
&\quad +\frac{1}{2\beta}\Big(w_y-\frac{\beta_z v}{\gamma}\Big)^2+\frac{1}{2\beta}\Big(u_y-\frac{\beta_x v}{\alpha}\Big)^2\Big] \\
\varepsilon_z &= \frac{1}{\gamma}\Big[w_z+\frac{\gamma_z u}{\alpha}+\frac{\gamma_y v}{\beta}+\frac{1}{2\gamma}\Big(w_z+\frac{\gamma_y u}{\alpha}+\frac{\gamma_y v}{\beta}\Big)^2 \\
&\quad +\frac{1}{2\gamma}\Big(u_z-\frac{\gamma_z w}{\alpha}\Big)^2+\frac{1}{2\gamma}\Big(v_z-\frac{\gamma_y w}{\beta}\Big)^2\Big] \qquad (1)
\end{aligned}$$

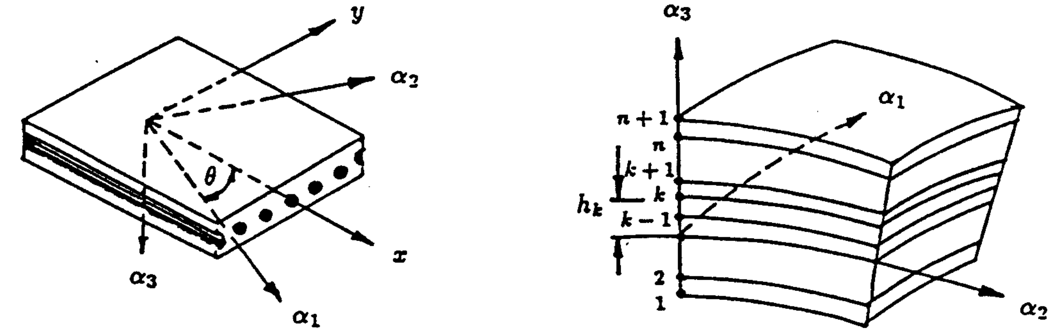
And where u, v, w are displacement with repect to axes and α, β, γ, are Lame coefficients. When we ignore square terms except term about w and transfer these to cylinderical coordinate, we can obtain the following relation between circumferential strain and displacements.

$$\varepsilon_\phi = \frac{1}{R}\Big[\Big(\frac{dv}{d\phi}-w\Big)-\frac{1}{2R}\Big(\frac{dw}{d\phi}\Big)^2\Big] \qquad (2)$$

where v, w are tangential and radial displacement and we assume that w is positive with repect to center. According to inextensional deformations, the displacements due to extension of the center line of a cylinder section are very small in comparison with the displacements due to bending and can usually be neglected[15].

$$v = \int\Big[w+\frac{1}{2R}\Big(\frac{dw}{d\phi}\Big)^2\Big]d\phi \qquad (3)$$

2. Composite-pipe stiffness



(a) lamina (b) laminate

Fig. 1 Element of composit pipe

Consider an element of the composite pipe shown Fig. 1. It is assumed that the thickness of the pipe is small relsative to the radius and that the deformation in the transverse direction varies linearly. The constitutive equations for moment are as follows [16].

$$\begin{Bmatrix} M_{11}\\ M_{22}\\ M_{12}\end{Bmatrix} = [B]\begin{Bmatrix}\varepsilon^0_{11}\\ \varepsilon^0_{22}\\ \varepsilon^0_{12}\end{Bmatrix} + [D]\begin{Bmatrix}\kappa_{11}\\ \kappa_{22}\\ \kappa_{12}\end{Bmatrix}$$

where $B_{ij} = \frac{1}{2}\sum_{k=1}^{n}(\bar{Q}_{ij})_k(h^2_{k+1}-h^2_k)$

$$D_{ij} = \frac{1}{3}\sum_{k=1}^{n}(\bar{Q}_{ij})_k(h^3_{k+1}-h^3_k) \qquad (4)$$

$$\bar{Q}_{11} = U_1 + U_2\cos 2\theta + U_3\cos 4\theta$$
$$\bar{Q}_{22} = U_1 - U_2\cos 2\theta + U_3\cos 4\theta$$
$$\bar{Q}_{12} = U_4 - U_3\cos 4\theta = \bar{Q}_{21}$$
$$\bar{Q}_{33} = U_5 - U_3\cos 4\theta$$
$$\bar{Q}_{13} = -\frac{1}{2}U_2\sin 2\theta - U_3\sin 4\theta = \bar{Q}_{31}$$
$$\bar{Q}_{23} = -\frac{1}{2}U_2\sin 2\theta + U_3\sin 4\theta = \bar{Q}_{32}$$
$$U_1 = \frac{1}{8}(3Q_{11}+3Q_{22}+2Q_{12}+4Q_{33})$$
$$U_2 = \frac{1}{2}(Q_{11}-Q_{22})$$
$$U_3 = \frac{1}{8}(Q_{11}+Q_{22}-2Q_{12}-4Q_{33})$$
$$U_4 = \frac{1}{8}(Q_{11}+Q_{22}+6Q_{12}-4Q_{33})$$
$$U_5 = \frac{1}{8}(Q_{11}+Q_{22}-2Q_{12}+4Q_{33})$$
$$Q_{11} = \frac{E_{xx}}{1-\mu_{xy}\mu_{yx}}$$
$$Q_{22} = \frac{E_{yy}}{1-\mu_{xy}\mu_{yx}}$$
$$Q_{12} = \frac{\mu_{yx}E_{xx}}{1-\mu_{xy}\mu_{yx}}$$
$$Q_{21} = \frac{\mu_{yx}E_{yy}}{1-\mu_{xy}\mu_{yx}}$$
$$Q_{33} = G_{xy}$$

If the pipe section is made such that it is exectly symmetric about its middle surface, all components of the bending streching coupling matrix, [B], are vanished.

$$\begin{aligned} M_{11} &= D_{11}k_{11} + D_{12}k_{22}\\ M_{22} &= D_{21}k_{11} + D_{22}k_{22}\\ M_{12} &= D_{33}k_{12}\end{aligned} \qquad (5)$$

3. Equilibrium Equation

$$\begin{aligned} \delta_h &= -v\sin\phi - w\cos\phi\\ \delta_v &= -v\cos\phi + w\sin\phi\\ x &= (R+\delta_A) - R\cos\phi - \delta_h\\ &= R(1-\cos\phi) + \delta_A + v\sin\phi + w\cos\phi\end{aligned} \qquad (6)$$

In Fig. 2(b) we established equilibrium equation on the defomed shape.

$$\Sigma M_c' = 0$$

$$-M_\phi + M_A + q_v x\frac{x}{2} - q_v(R+\delta_A)x + \int_0^\phi \delta_h K (R d\varphi\cos\phi)[R(\sin\phi - \sin\varphi)] = 0 \qquad (7)$$

Substituting Eq.(6) into Eq.(7)

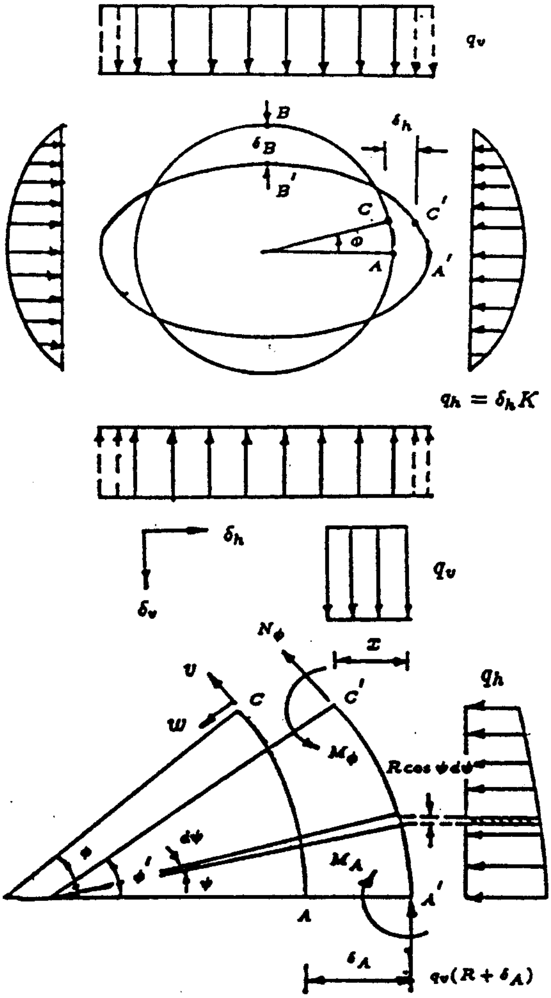


Fig. 2 Definition of loading and sign convention for conduit

$$-M_\phi + M_A - \frac{q_v}{2}[R(1-\cos\phi)+\delta_A+v\sin\phi + w\cos\phi][R(1+\cos\phi)+\delta_A - v\sin\phi - w\cos\phi] + \int_0^\phi \delta_h K R^2(\sin\varphi - \sin\phi)]\cos\varphi\, d\varphi = 0 \tag{8}$$

Since the problem is assumed to be that of plane strain, the following relation can be obtained,

$$M_\phi = D\kappa_\phi$$

$$\text{where } \kappa_\phi = \frac{1}{R^2}\left(\frac{dv}{dw} - \frac{d^2 w}{d\phi^2}\right) \tag{9}$$

$$D = D_{22} - \frac{1}{3}\sum_{k=1}^{n}(\bar{Q}_{22})_k(h_{k+1}^3 - h_k^3)$$

Since Eq.(8) is a differential equation about w, we solved using Galerkin's method.

$$\bar{w} = \frac{w}{R} = A_1\cos 2\phi + A_2\cos 4\phi \tag{10}$$

$$\int_0^{\frac{\pi}{2}} X\cos 2n\phi = 0 \quad n = 1,\ 2 \tag{11}$$

And where X is left side of Eq.(8). Substituting Eqs. (3), (8), (9) and (10) into (11) we can obtain nonliner simultaneous equations.

$$\bar{q}\left[(1-A_1)^2 - \frac{5}{4}A_2 - \frac{3}{16}\left(\frac{A_1^2}{3} - 6A_1A_2 - \frac{5}{2}A_2^2\right)\right] = -12A_1\left(1+\frac{38.3}{210}\frac{\beta}{\pi}\right) - A_2\frac{96.4}{360}\beta$$

$$\bar{q}\left[\frac{A_1}{8} - 2A_2 + \frac{3}{32}A_1^2 + \frac{3}{8}A_1A_2\right] = -15A_2\left(1+\frac{11.9}{1920}\frac{\beta}{\pi}\right) + \frac{12.6}{385}A_1\frac{\beta}{\pi} \tag{12}$$

$$\text{where } \bar{q} = \frac{q_v R^3}{D};\ \beta = \frac{KR^4}{D}$$

If we solve above equation using Taylor's series, we can finally obtain the continuous displacement function equation expressed with trigonometric terms.

NUMERICAL EXAMPLE

A composite pipe made of Glass / Epoxy which has the following material property and geometry

$E_1 = 0.55\times 10^6$ kg/cm²
$E_2 = 0.18\times 10^6$ kg/cm²
$G_{12} = 0.091\times 10^6$ kg/cm²
$\nu = 0.25$
$R = 12.7$cm
4 layer with same thickness
total thickness $= 0.203$cm
soil unit weight $(\gamma) = 1995$kg/cm³

is analyzed. As results, Fig. 3 appears an effect which soil have against pipe and show the nonlinearity in the displacement according to depth. Fig. 4 displays the verification of this buried composite pipe analysis method by general purpose FEM package, COSMOS / M.

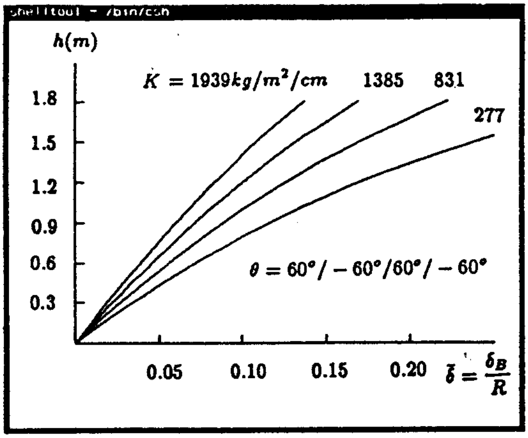


Fig. 3 Displacement according to depth and soil

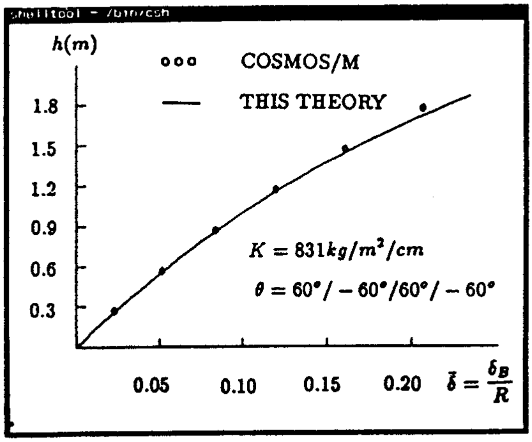


Fig. 4 Comparison with FEM

CONCLUSION

This paper presented geometric nonlinear analysis method of underground laminated composite pipes. It is much simpler than FEM in discussing fiber orientation, soil−pipe interaction and geometric nonlinearity. Just solving Eq. (12), we can obtain the pipe displacement in practical instance. Stress can be obtained from moment equstion, Eq.(10). So it takes a less time to analysis than FEM. We think that method may be used to analysis the practical problem. For more accracy material nonlinearity and nonlinearity of soil−pipe interaction need to be developed later on.

REFERENCES

1. Spangler, M.G. and Handy, R.L., "Soil Engineering", 4th ed., Harper and Row Publishers, New York, N.Y., 1982.
2. Watkins, R.K., "Structural Design of Buried Circular Conduits", HRB No. 145, 1966, pp 1−16.
3. Meyerhof, G.G., "Strength of Steel Culvert Sheets Bearing against Compacted Sand Backfill", HRR No. 30, 1963.
4. Timmers, J.H., "Load Study of Flexible Pipes under High Fills", HRB Bul No. 125, 1956.
5. Brockenbrough, R.L., "Influence of Wall Stiffness on Corrugated Metal Culvert Design", HRR No. 56,1968.
6. Valentine, H.E., "Structural Performance and Load Reaction Patterns of Flexible Aluminum Culvert", HRR No. 56, 1964.
7. Costes, N.C., "Factors Affecting Vertical Loads on Underground Ducts Due to Arching", HRB Bull. 125, 1956.
8. Richards, R., and Agrawal, J.S., "Stresses on Shallow Circular Pipe by Transformed Section", Jour. of Geotechnical Engineering Div., A.S.C.E., GT6, June 1974.
9. Rubin, C., "Buried Pipe Deflections Using Elastic Theory", Jour. of Structural Division, A.S.C.E., ST 12, Dec., 1975.
10. Molin J., "Flexible Pipes Buried in Clay", Proc. of th International Conference on Underground Plastic Pipe, New Orleans, Louisiana, U.S.A., Mcrch 30~April 1, 1981.
11. Galili, N. and Shmulevich, I., "A Refined Elastic Model for Soil−Pipe Interaction", Proc. of the International Conference on Underground Plastic Pipe, New Orleans, Louisiana, U.S.A., March 30~April 1, 1981.
12. Szyszkowski, W. and Glockner, P.G., "Large−−deformation and Collapse Behaviour of Underground Aluminum Piping", Thin−Walled Structures, Vol.5, No. 1, 1987.
13. Henry L. Langhaar, "Energy Methods in Applied Mechanics", John Wiley and Sons, 1962.
14. V.V. Novozillov, "Foundations of Nonlinear Theory of Elasticity", Graylock Press, 1953.
15. S.P. Timoshenko and J.M. Gere, "Theory of Elastic Stability", McGraw−Hill, 1961.
16. Robert M. Jones, "Mechanics of Composite Materials", Scripa Book Company, 1975.

(접수일자 1988. 12. 21)