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비비례 감쇠시스템의 고유치 해석

Solution of Eigenvalue Problems for Nonclassically Damped Systems

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요 지

본 논문에서는 비비례 감쇠 시스템의 고유치 해석 방법을 제안하였다. 2차 고유치 문제의 행렬 조합을 통한 선형 방정식에 수정된 Newton-Raphson 기법과 고유벡터의 직교성을 적용하여 제안방법의 알고리즘을 유도하였다. 벡터 반복법 또는 부분공간 반복법과 같은 기존의 반복법에서는 수렴성을 향상시키기 위해 변위법을 적용하였으며, 이 값이 시스템의 고유치에 근사하게 되면 행렬분해 과정에서 특이성이 발생한다. 그러나 제안방법은 구하고자 하는 고유치가 중복근이 아닐 경우에, 변위값이 시스템의 고유치 일지라도 항상 정칙성을 유지하며, 이것을 해석적으로 증명하였다. 제안방법은 수정된 Newton-Raphson 기법을 이용하기 때문에 초기값을 필요로 한다. 제안방법의 초기값으로는 반복법의 중간결과나 근사법의 결과를 사용할 수 있다. 이들 방법중 Lanczos 방법이 가장 효율적으로 좋은 초기값을 제공하기 때문에 Lanczos 방법의 결과를 제안방법의 초기값으로 사용하였다. 제안방법의 효율성을 증명하기 위하여 두가지 예제 구조물에 대해 해석시간 및 수렴성을 가장 많이 사용하고 있는 부분공간 반복법과 Lanczos 방법의 결과와 비교하였다.

핵심용어 : 고유치해석, 비비례 감쇠 시스템, 수렴성, Newton-Raphson 기법

Abstract

A solution method is presented to solve the eigenvalue problem arising in the dynamic analysis of nonclassically damped structural systems. The proposed method is obtained by applying the modified Newton-Raphson technique and the orthonormal condition of the eigenvectors to the linear eigenproblem through matrix augmentation of the quadratic eigenvalue problem. In the iteration methods, such as the inverse iteration method and the subspace iteration method, singularity may be occurred during the factorizing process when the shift value is close to an eigenvalue of the system. However, even through the shift value is an eigenvalue of the system, the proposed method provides nonsingularity, if the desired eigenvalue is not multiple, and that is analytically proved. Since the modified Newton-Raphson technique is adopted to the proposed

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method, initial values are need. The initial values of the proposed method can be obtained by the intermediate results of iteration methods or results of approximate methods. Because the Lanczos method effectively produces better initial values than other methods, the results of the Lanczos method are taken as the initial values of the proposed method. Two numerical examples are presented to demonstrate the effectiveness of the proposed method and the results are compared with those of the well-known subspace iteration method and the Lanczos method.

Keywords : *eigenanalysis, nonclassically damped system, modified Newton-Raphson technique*

1. Introduction

In the dynamic analysis of structures, the eigenvalue problem of the system is to be solved a priori in order to avoid a resonance or to define the natural vibration characteristics. The process is low in cost, and straightforward if the damping is proportional, or more generally, if the mass, damping and stiffness matrices satisfy a condition developed by Caughey and O'Kelly.[1] If the system is nonclassical, the eigenanalysis becomes relatively expensive because it is conventionally performed in a space of twice the system's dimension, and because complex computation is required.

The high relative cost of the eigenanalysis of nonclassically damped systems is possibly a motivation for investigators who explored efficient means for the analysis, and many methods have been proposed as a result of it. Of these, transformation methods such as QR,[2] LZ[3] or Jacobi[4] determine all the eigenvalues and the associated eigenvectors in an arbitrary sequence. This is not very efficient in situations where only the lowest frequencies are of interest and there is a large number of degrees of freedom. Also transformation methods by their nature modify the initial matrices during the solution process and can not take full advantage of the sparseness of these matrices. Perturbation method[5-9] is used for the eigenvalue problem of lightly damped systems. Since weak damping implies that the eigensolution of the damped system will differ only slightly from that of the corresponding undamped system, it is to set the eigensolution of the undamped system as the zero order approximation of that of the damped system and let the higher order terms account for the slightly damping effect.

The classical inverse iteration method[10-12] is commonly used to solve for only a small number of desired modes. However, the method requires a great deal of complex arithmetic for each eigenvalue sought. The subspace iteration method[13,14] is a more efficient alternative than the inverse iteration method. It yields all modes requested simultaneously and does not have the drawback that the higher modes are less accurate than the lower modes because it avoids the round-off errors of the inverse iteration method due to the deflation process. However, as in the inverse iteration method, a large number of complex arithmetic are required in the iteration process for general structural systems. Furthermore, when the shift value becomes close to an eigenvalue of the system, singularity may be encountered during triangularization process.

In recent years there has been considerable interest in the Lanczos algorithm and its applications. The Lanczos algorithm for the computation of eigenvalues and eigenvectors of a real symmetric matrix was

presented in reference 15 and improved in references 16~20. The Lanczos algorithm to solve the eigenvalue problem of nonclassically damped system is dealt with in references 21~26. Two sided-Lanczos algorithm[21-24] requires the generation of two sets of Lanczos vectors, left and right, and the symmetric Lanczos algorithm[25,26] uses a set of Lanczos vectors to reduce a large eigenvalue problem in a much smaller one. Although only real arithmetic is solved during the solution process, in contrast to the case of real symmetric eigenproblems, there will be a possibility of serious breakdown and the accuracy of the solutions obtained is low.[27]

In this paper, an efficient method for solving the eigenvalue problem of nonclassically damped systems is developed. In the second section, the basic concept of the proposed method which applies the modified Newton-Raphson technique to an eigenvalue problem is presented and the nonsingularity of the proposed method is established. In the third section, two numerical examples are presented to identify the efficiency of the proposed method and the results are compared with those of the well-known subspace iteration method[14] and the Lanczos method.[25]

2. Method of Analysis

2.1 Problem Definition

The equation for free vibration of the nth order dynamic system may be written as

$$M\ddot{u}+C\dot{u}+Ku=0 \tag{1}$$

where M, C and K are the (n×n) mass, damping and stiffness matrices, respectively, and u is the (n×1) vector of system displacements. For symmetric and positive definite matrices, the eigenvalues and the associated eigenvectors of the system in Eq. (1) may be determined in a straightforward and efficient manner provided that the damping matrix is proportional or, more generally, provided that

$$CM^{-1}K=KM^{-1}C \tag{2}$$

Systems, which do not satisfy Eq. (2), are called the nonclassically damped. The eigenanalysis for such systems is traditionally performed in the space extended to 2n dimension such as

$$\begin{bmatrix}-K & 0\\ 0 & M\end{bmatrix}\begin{Bmatrix}\phi\\ \lambda\phi\end{Bmatrix}=\lambda\begin{bmatrix}C & M\\ M & 0\end{bmatrix}\begin{Bmatrix}\phi\\ \lambda\phi\end{Bmatrix} \tag{3}$$

where λ and ϕ are the eigenvalue and the associated eigenvector of the system. Eq. (3) may be written as

$$A\psi=\lambda B\psi \tag{4}$$

with

$$A=\begin{bmatrix}-K & 0\\ 0 & M\end{bmatrix},\quad B=\begin{bmatrix}C & M\\ M & 0\end{bmatrix}\ \text{and}\ \psi=\begin{Bmatrix}\phi\\ \lambda\phi\end{Bmatrix} \tag{5}$$

Since both the matrices A and B are not positive definite although are symmetric, the eigenvalues and the associated eigenvectors are complex values in general.

2.2 Proposed Eigenanalysis

Suppose that initial approximate solutions $\lambda_j^{(0)}$, and $\psi_j^{(0)}$ of the jth eigenvalue and the associated eigenvector of Eq. (4) are known, where the superscript denotes the iteration number. Denoting the approximate eigenvalue and the associated eigenvector after k iterations by $\lambda_j^{(k)}$ and $\psi_j^{(k)}$, the residual vector becomes as follow:

$$r_j^{(k)}=A\psi_j^{(k)}-\lambda_j^{(k)}B\psi_j^{(k)} \tag{6}$$

The residual vector $r_j^{(k)}$ is not generally zero because the approximate eigenpairs are substituted into Eq. (4). $\psi_j^{(k)}$ satisfies the orthonormal condition with respect to matrix B such as

$$(\psi_j^{(k)})^T B\psi_j^{(k)}=1 \tag{7}$$

In order to get a solution converged to the

eigenvalue and the associated eigenvector of the system, the residual vector should be removed. Applying the Newton-Raphson technique for this purpose to the eigenproblem, the residual vector can be written as follow:

$$r_j^{(k+1)} = A\psi_j^{(k+1)} - \lambda_j^{(k+1)} B\psi_j^{(k+1)} = 0 \tag{8}$$

where $\psi_j^{(k+1)}$ also satisfies the orthonormal condition with respect to matrix B as follows:

$$(\psi_j^{(k+1)})^T B\psi_j^{(k+1)} = 1 \tag{9}$$

and then

$$\lambda_j^{(k+1)} = \lambda_j^{(k)} + \Delta\lambda_j^{(k)} \quad \text{and} \tag{10}$$

$$\psi_j^{(k+1)} = \psi_j^{(k)} + \Delta\psi_j^{(k)} \tag{11}$$

Substitutuing Eqs. (6), (7), (10) and (11) into Eqs. (8), (9) and neglecting their higher order terms $\Delta\lambda_j^{(k)}B\Delta\psi_j^{(k)}$ and $(\Delta\psi_j^{(k)})^T B\Delta\psi_j^{(k)}$, it yields

$$(A-\lambda_j^{(k)}B)\Delta\psi_j^{(k)} - \Delta\lambda_j^{(k)} B\psi_j^{(k)} = -r_j^{(k)} \tag{12}$$

and

$$(\psi_j^{(k)})^T B\Delta\psi_j^{(k)} = 0 \tag{13}$$

where $\Delta\lambda_j^{(k)}$ and $\Delta\psi_j^{(k)}$ are unknown incremental values corresponded to $\lambda_j^{(k)}$ and $\psi_j^{(k)}$. Writing Eqs. (12) and (13) in matrix form as following:

$$\begin{bmatrix} A-\lambda_j^{(k)}B & -B\psi_j^{(k)} \\ (-B\psi_j^{(k)})^T & 0 \end{bmatrix} \begin{Bmatrix} \Delta\psi_j^{(k)} \\ \Delta\lambda_j^{(k)} \end{Bmatrix} = -\begin{Bmatrix} r_j^{(k)} \\ 0 \end{Bmatrix} \tag{14}$$

The symmetric coefficient matrix of the above linear algebraic equation is of order (2n+1). If all eigenvalues are distinct, $\lambda_i \neq \lambda_j$ ($i \neq j$), then the coefficient matrix is nonsingular. Nonsingularity of the method will be proved in the following section. If the shift is near to an eigenvalue, singularity in the iteration methods may be encountered. In the proposed method, the singularity problem, however, can be solved by means of including a side condition $(\psi_j^{(k)})^T B\Delta\psi_j^{(k)} = 0$ as shown in Eq. (14). This is the main difference compared with the iteration methods with shift. The above method applying the Newton-Raphson technique, despite of its rapid convergence, is not efficient, since the new coefficient matrix has to be reformed and refactorized in each iteration step.

The complicated elimination procedure in each iteration may be overcome by applying the modified Newton-Raphson technique as follows:

$$\begin{bmatrix} A-\lambda_j^{(0)}B & -B\psi_j^{(k)} \\ (-B\psi_j^{(k)})^T & 0 \end{bmatrix} \begin{Bmatrix} \Delta\psi_j^{(k)} \\ \Delta\lambda_j^{(k)} \end{Bmatrix} = -\begin{Bmatrix} r_j^{(k)} \\ 0 \end{Bmatrix} \tag{15}$$

Eqs. (15), (10), (11) and (6) are the basic concept of the proposed method. The symmetric coefficient matrix of Eq. (15) is also nonsingular. Once the submatrix $A-\lambda_j^{(0)}B$ is decomposed into LDL^T(L: lower triangular matrix, D: diagonal matrix), a small number of operations are required to solve Eq. (15), since only the vector $B\psi_j^{(k)}$ in the coefficient matrix is changed in each iteration. The complete procedure of the proposed method for calculating the eigenpairs is summarized in Table 1.

2.3 Nonsingularity of the Proposed Method[28]

In the iteration methods such as the inverse and the subspace iteration methods, the shifting algorithm is adopted to improve the convergence. However, singularity may be occurred during the factorizing process when the shift value is close to an eigenvalue of the system. One of the characteristics of the proposed method is that its nonsingularity is also guaranteed in this situation. If the proposed method is nonsingular when the shift values is an eigenvalue itself, the coefficient matrix encountered in the iteration process must

Table 1. Algorithm of the Proposed Method

I. Calculate initial eigenvalues $\lambda_j^{(0)}$ and eigenvectors $\psi_j^{(0)}$ for j=1, 2, ···, p
where p is the number of eigenvalues to be sought.

II. Iterate the following procedure for each eigenvalue and the associated eigenvector.
a) For j=1
b) For k=0
c) Define $\begin{bmatrix} A-\lambda_j^{(0)}B & -B\psi_j^{(k)} \\ (-B\psi_j^{(k)})^T & 0 \end{bmatrix}$
d) Define $-\begin{Bmatrix} r_j^{(k)} \\ 0 \end{Bmatrix}$
where $r_j^{(k)} = A\psi_j^{(k)} - \lambda_j^{(k)} B\psi_j^{(k)}$
e) Solve $\begin{bmatrix} A-\lambda_j^{(0)}B & -B\psi_j^{(k)} \\ (-B\psi_j^{(k)})^T & 0 \end{bmatrix} \begin{Bmatrix} \Delta\psi_j^{(k)} \\ \Delta\lambda_j^{(k)} \end{Bmatrix} = \begin{Bmatrix} r_j^{(k)} \\ 0 \end{Bmatrix}$ for $\begin{Bmatrix} \Delta\psi_j^{(k)} \\ \Delta\lambda_j^{(k)} \end{Bmatrix}$
f) Compute $\lambda_j^{(k+1)} = \lambda_j^{(k)} + \Delta\lambda_j^{(k)}$ and $\psi_j^{(k+1)} = \psi_j^{(k)} + \Delta\psi_j^{(k)}$
g) If the norm of the residual vector does not satisfy the predetermined error limit, then go to b) with k=k+1, otherwise go to a) with j=j+1

necessarily be nonsingular. Let the coefficient matrix of Eq. (15) be denoted by $E_j^{(k)}$ to prove the nonsingularity of the proposed method as follow:

$$E_j^{(k)} = \begin{bmatrix} A-\lambda_j^{(0)}B & -B\psi_j^{(k)} \\ (-B\psi_j^{(k)})^T & 0 \end{bmatrix} \tag{16}$$

and replace the approximate values $\lambda_j^{(0)}$ and $\psi_j^{(k)}$ in $E_j^{(k)}$ by the exact ones. The resulting matrix E_j becomes as follows:

$$E_j = \begin{bmatrix} A-\lambda_j B & -B\psi_j \\ (-B\psi_j)^T & 0 \end{bmatrix} \tag{17}$$

Consider the following new eigenproblem to prove the nonsingularity of E_j.

$$E_j u_i = \gamma_i F u_i \quad i = 1, 2, \cdots, 2n, 2n+1 \tag{18}$$

where γ_i and u_i are the ith eigenvalue and the associated eigenvector of the new eigenproblem, and F is as follows:

$$F = \begin{bmatrix} B & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} C & M & 0 \\ M & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{19}$$

And Eq. (18) can be written the following matrix form:

$$E_j U = FU\Gamma \tag{20}$$

where

$$\Gamma = \mathrm{diag}(\gamma_1, \gamma_2, \cdots, \gamma_{2n+1}) \quad \text{and} \tag{21}$$

$$U = [u_1, u_2, \cdots, u_{2n+1}] \tag{22}$$

The eigenvalues and eigenvectors of the new eigenproblem are as follows:
Eigenvalues γ:

$$1, \ -1, \ \lambda_k - \lambda_j \quad k = 1, 2, \cdots, 2n, \ k \neq j \tag{23}$$

Eigenvectors u:

$$\begin{Bmatrix} \psi_j \\ -1 \end{Bmatrix}, \ \begin{Bmatrix} \psi_j \\ 1 \end{Bmatrix}, \ \begin{Bmatrix} \psi_k \\ 0 \end{Bmatrix} \quad k = 1, 2, \cdots, 2n, \ k \neq j \tag{24}$$

The determinant of Eq. (20) is as follows:

$$\det[E_j]\det[U] = \det[F]\det[U]\det[\Gamma] \tag{25}$$

or

$$\det[E_j] = \det[F]\det[\Gamma] = -\det[F]\prod_{\substack{k=1 \\ k\neq j}}^{2n}(\lambda_k - \lambda_j) \tag{26}$$

where

$$\det[F] = \begin{vmatrix} C & M & 0 \\ M & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} = \begin{vmatrix} C & M \\ M & 0 \end{vmatrix} = -\det[M]\det[M] \tag{27}$$

The determinant of F is not equal to zero because the determinant of M is non-zero by definition. If all eigenvalues are distinct, the determinant of E_j is not equal to zero since the determinant of F is non-zero by Eq. (27). The

proof of the nonsingularity is completed.

2.4 Starting Values of the Proposed Method

Initial values of the proposed method can be obtained as the intermediate results of iteration methods[10-14] or results of approximate methods.[21-26] In this paper, the starting values are taken as the results of the symmetric Lanczos method[25] with selectively reorthogonalization process[19,20] because the method does not need complex arithmetic in Lanczos recursive process and effectively produces the good approximate values of the systems. If the lowest p eigenvalues and the corresponding eigenvectors are desired, the eigenvalue problem reduced by 2p Lanczos vectors is solved, and then the p approximate eigenpairs are improved by the proposed method until the predetermined error norm is satisfied. In the Lanczos method, the initial Lanczos vectors is set equal to $A^{-1}<1\cdots1>^T$ and normalized with respect to matrix B.

3. Numerical Examples

Two test problems with nonclassical damping are presented to show the effectiveness of the proposed method. The CPU time spent for the first ten eigenvalues and the corresponding eigenvectors and the convergence of the proposed method are compared with those of the subspace iteration method.[14] The number of simultaneously iterated vectors of the subspace iteration method is 20. Also, the variation of the error norm of each eigenpair by the Lanczos method[25] is concerned as the number of Lanczos vectors increases. The predetermined error norm is 10^{-6}, which yields a stable eigensolution and sufficient accuracy in the calculated eigenvalues and eigenvectors for practical analysis.[29] The error norm[29] is defined as

$$\text{error norm} = \frac{\| A\psi_j^{(k)} - \lambda_j^{(k)} B\psi_j^{(k)} \|_2}{\| A\psi_j^{(k)} \|_2} \qquad (28)$$

All executions are computed on the CONVEX C 3420 with 100 MIPS and 200 MFLOPS.

3.1 Cantilever Beam with Multi-Lumped Dampers

The first example is a cantilever beam with multi-lumped tangential viscous-dampers attached at each node. The geometric configuration is shown in Fig. 1. The structural model could be considered as a representative of a soil-structure interaction. The structure is modeled by 100 equal elements and has 200 degrees of freedom. The order of the associated (A, B) is 400. Dimensionless value of Young's modulus for the beam material is taken as 1,000, while mass density, section area and inertia are specified to be of unit value. The mass matrix is consistent one. The damping matrix C consists of the Rayleigh damping and concentrated dampers. The Rayleigh damping is a linear combination of the stiffness and the mass matrices such as

$$C = \alpha M + \beta K \qquad (29)$$

where the Rayleigh coefficients α and β are 0.001. On the other hand, the damping coefficient of each tangential damper is 0.3.

The computed results are shown in Table 2. The variation of the error norm for each method is plotted in Figs. 2 to 4. The error norms of the initial values of the proposed method which are calculated by the symmetric Lanczos method[25] with selectively reorthogonalization are about 0.3 to 10^{-4}. Fig. 2 shows that only one iteration is needed to obtain the eigenpair within the error norm of 10^{-6}. For example, in spite of the initial value of the nineth mode having the large

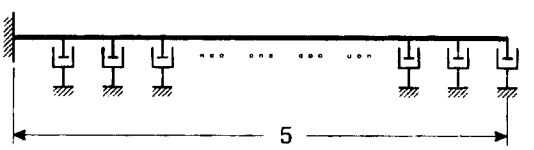

Fig. 1. Cantilever Beam with Multi-Lumped Dampers.

Table 2. Eigenvalues of the Cantilever Beam with Multi-Lumped Dampers

Mode Number	Eigenvalues
1	- 2.57457 + j 3.17201
2	- 2.57457 - j 3.17201
3	- 1.53800 + j 18.3566
4	- 1.53800 - j 18.3566
5	- 1.69581 + j 39.6477
6	- 1.69581 - j 39.6477
7	- 2.43492 + j 61.0104
8	- 2.43492 - j 61.0104
9	- 3.78360 + j 82.3222
10	- 3.78360 - j 82.3222

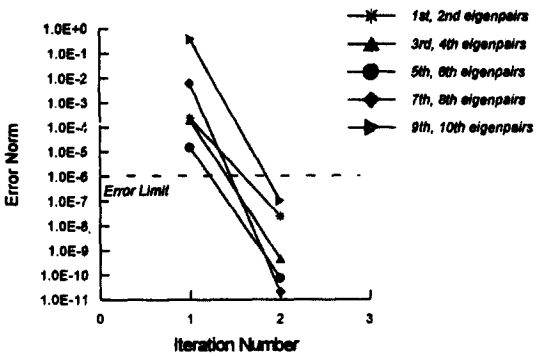

Fig. 2. Variation of the Error Norm of the Beam Model by the Proposed Method.

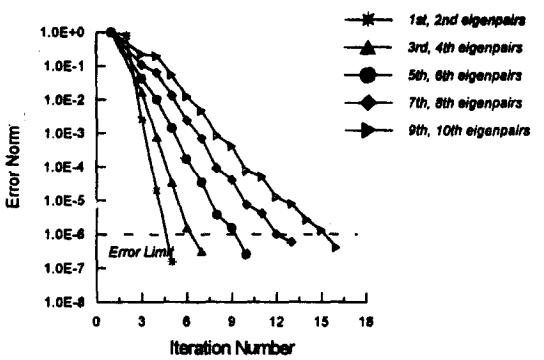

Fig. 3. Variation of the Error Norm of the beam Model by the Subspace Iteration Method.

error norm of 0.3, the proposed method gets it satisfied the predetermined error norm after only one iteration. In the figures, it denotes that the bigger the slope of the line is, the better the convergence is. Thus, it can be seen that the proposed method has the

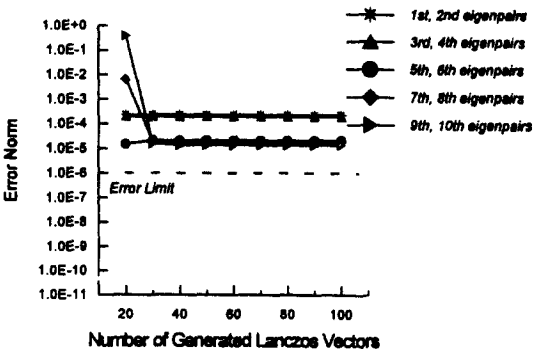

Fig. 4. Variations of the Error Norm of the Beam Model by the Lanczos Method.

Table 3. CPU Time Spent for the First Ten Eigenvalues of the Cantilever Beam with Multi-Lumped Dampers

Methods	CPU Time in seconds (Ratio)
Proposed Method	76.10(1.00)
Subspace Iteration Method	100.94(1.33)

Table 4. CPU Time for the Lanczos Method vs. the Number of Generated lanczos Vectors

Number of Generated Lanczos Vectors	CPU Time in seconds
20	13.55
30	21.15
40	29.57
50	37.67
60	48.16
70	56.54
80	69.72
90	79.81
100	93.78

better convergence than the subspace iteration method. The error norm of each eigenpair by the Lanczos method is not improved within a certain value in spite of the increase of the Lanczos vectors.

The CPU time for the proposed method is compared with that of the subspace iteration method in Table 3. If we let the solution time for the proposed method be 1, it takes 1.33 times for the subspace iteration method.

3.2 Three-Dimensional Frame Structure with Concentrated Dampers

A three-dimensional frame structure with concentrated dampers is presented. Two layers of the foundation are damped as shown in Fig. 5(b). The system could be considered as a representative of a control system or a passively damped space structure. The model has 1,008 degrees of freedom. The associated (A, B) is of order 2,016. The material and cross-sectional properties are $E=2.1\times10^{11}$, $I=8.3\times10^{-6}$, Area=0.01 and $\rho=7,850$. The consistent mass matrix is used to define M. The damping matrix C is consists of the Rayleigh damping and concentrated dampers. The Rayleigh coefficient α is -0.92 and β is 0.1016. The damping for each concentrated damper is 1,000 as before.

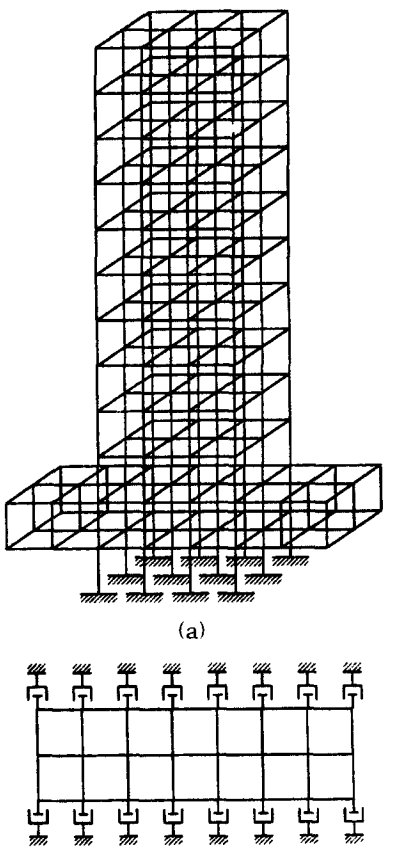

Fig. 5. (a) 3-Dimensional Frame Structure, (b) damping from Two-Layer Foundation.

The results are shown in Tables 5 to 7 and Figs. 6 to 8. It can be seen that the results are similar to ones of the previous example. The error norms of the initial values obtained by using the symmetric Lanczos method with selectively reorthogonalization are about 0.6 to 10^{-5}. The number of iterations needed to satisfy the error norm of 10^{-6} in the proposed method is one or two. The results of the subspace iteration method are shown in Fig. 7. The con-

Table 5. Eigenvalues of the 3-Dimensional Frame Structure with Concentrated Dampers

Mode Number	Eigenvalues
1	- 0.01503 + j 3.03037
2	- 0.01503 - j 3.03037
3	- 0.02478 + j 3.09011
4	- 0.02478 - j 3.09011
5	- 0.24376 + j 3.65157
6	- 0.24376 - j 3.65157
7	- 3.83006 + j 7.78173
8	- 3.83006 - j 7.78173
9	- 3.42807 + j 8.04793
10	- 3.42807 - j 8.04793

Table 6. CPU Time Spent for the First Ten Eigenvalues of the 3-Dimensional Frame Structure with Concentrated Dampers

Methods	CPU Time in seconds (Ratio)
Proposed method	12,238.46(1.00)
Subspace iteration method	27,004.22(2.21)

Table 7. CPU Time for the Lanczos Method vs. the Number of Generated Lanczos Vectors

Number of Generated Lanczos Vectors	CPU Time in seconds
20	425.22
30	570.72
40	770.26
50	974.02
60	1,162.96
70	1,396.65
80	1,591.55
90	1,825.67

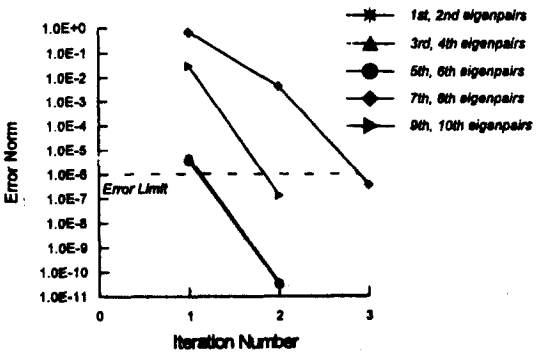

Fig. 6. Variation of the Error Norm of the 3-D. Frame by the Proposed Method.

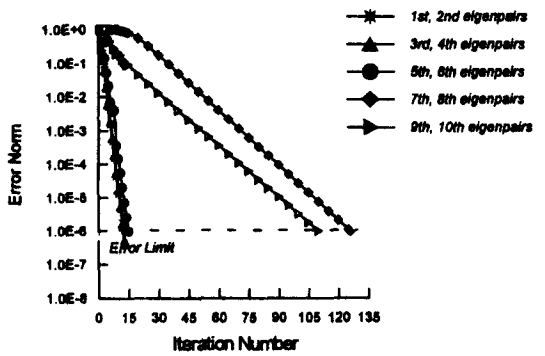

Fig. 7. Variation of the Error Norm of the 3-D. Frame by the Subspace Iteration Method.

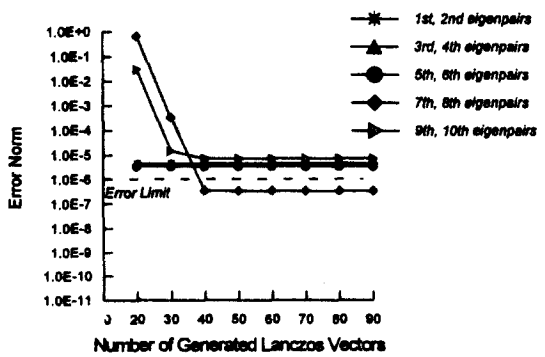

Fig. 8. Variation of the Error Norm of the 3-D. Frame by the Lanczos Method.

vergence of the higher modes is very low, while the lower modes satisfy the error norm of 10^{-6} in 15 iterations. The results in Figs. 6 to 7 imply that the proposed method has the better convergence than the subspace iteration method. The results of the Lanczos method in Fig. 8 are not improved in spite of the increase of the number of the Lanczos vectors.

The CPU time spent for the proposed method is 12,238.46 seconds and for the subspace iteration method 27,004.22 seconds. The ratio is 1.00 to 2.21.

4. Conclusions

The method for finding the eigenpairs of the nonclassically damped systems is proposed. As shown in numerical analysis, characteristics of the proposed method are identified as follows:

1. The convergence of the proposed method is better than that of the subspace iteration method.
2. The proposed method guarantees the nonsingularity, even if the shift value is an eigenvalue of the system, which is proved analytically. Therefore, if the exact eigenvalues of the system are known, the proposed method can effectively calculate the corresponding eigenvectors.
3. The algorithm of the proposed method is simple.

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