비중복 고유치를 갖는 비비례 감쇠계의 고유치와 고유벡터의 민감도 해석법

Natural Frequency and Mode Shape Sensitivities of Non-Proportionally Damped Systems : Part I, Distinct Natural Frequencies

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요 지

본 연구에서는 중복되지 않는 고유치를 갖는 비비례 감쇠계의 고유치와 고유벡터의 민감도를 계산하는 새로운 방법을 제시하였다. 제안 방법에서는 $(n+1)$차의 대칭 행렬로 이루어진 대수방정식을 해석함으로써 n개의 자유도를 갖는 감쇠계의 고유치와 고유벡터의 설계변수에 대한 미분을 구한다. 제안 방법은 매우 간단하면서도 수치적 안정성이 보장되고 정확한 해를 주는 방법이다. 제안 방법의 검증을 위해 7자유도를 갖는 차량모델의 민감도해석을 예제에서 다루고 있다. 예제에서의 설계변수는 콘테이너의 질량으로 하였다.

핵심용어 : 비중복 고유치, 고유벡터, 민감도, 설계 변수, 부가조건

Abstract

A procedure for determining the sensitivities of the eigenvalues and eigenvectors of non-proportionally damped vibratory systems with distinct eigenvalues is presented. The eigenpair derivatives of the structural and mechanical damped systems can be obtained consistently by solving algebraic equations with symmetric coefficient matrix whose order is $(n+1)\times(n+1)$, where n is the number of coordinates. The algorithm of the method is very simple and compact. Furthermore, the method can find the exact solutions. One of the remarkable characteristics of the proposed method is that its numerical stability is established. As an example to verify the proposed method and its possibilities, 7-DOF half-car model is considered. The design parameter of the car model is the container mass.

Keywords : *distinct eigenvalue, eigenvector, sensitivity, design parameter, side condition*

1. INTRODUCTION

The dynamic responses of the structural or mechanical systems can be completely identified by obtaining the natural frequencies and mode shapes of the systems. Variations in

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• 이 논문에 대한 토론을 1999년 6월 30일까지 본 학회에 보내주시면 1999년 9월호에 그 결과를 게재하겠습니다.

system parameters lead to changes in these dynamic characteristics and hence in responses. The derivatives of the eigenpairs with respect to design parameters are useful in design trend studies and in gaining insight into the behavior of physical systems. Using these eigenpair derivatives in large systems can remarkably reduce the cost of reanalysis. In contrast to computing eigenvalue derivatives where preferred methods exist, there are a number of different methods for calculating mode shape derivatives. The different methods seek to overcome the practical difficulty of solving a singular matrix equation.

Methods for calculating mode shape derivatives include finite-difference method[1], iterative method[2,3], modal method[4], modified modal method[5], Nelson's method[6] and Lee and Jung's method[7,8]. The finite-difference method uses a difference formula to numerically approximate the derivative. This method is sensitive to round off and truncation errors associated with the step size used. The modal method approximates the mode shape derivatives as a linear combination of mode shapes. This method can be computationally expensive if a large number of modes are needed to accurately represent the mode shape derivative. The modified modal method was developed to reduce the number of modes needed to represent the derivative of mode shapes. Nelson's method is an exact analytical method for calculating mode shape derivatives. This method only requires the knowledge of the eigenvector to be differentiated and is recommended as an efficient solver for calculating the mode shape derivatives, however this method is lengthy and clumsy for programming and is restricted to the eigenvalue problem with only distinct natural frequencies. Nelson's method is extended to the eigenvalue problem with multiple natural frequencies by Dailey[9], however this method is lengthy and complicate too. Lee and Jung's method developed recently is an exact analytical method for calculating mode shape derivatives. Furthermore, it is very efficient and simple. For a thorough review of the research in sensitivity methods for finite-dimensional structural problems, refer to the excellent survey paper by Haftka and Adelman[10].

A number of the prescribed methods can be applied to the damped system; Pomazal and Snyder[11] extended the theory to the complex eigenvalue problem to analyze the effects of adding springs and dampers to viscously damped systems. Hallquist[12] presented for determining the effects of mass modification in viscously damped systems. Recently Zimoch[13] presented the sensitivity analysis method for determining the dynamic characteristics of mechanical systems to variations in the parameters. The method is applied to conservative as well as non-conservative systems, however it may be restricted to the mechanical systems (lumped systems) with only distinct natural frequencies ; it has some difficulties in applying to systems with multiple natural frequencies.

Generally, there are two types of damping matrices, such as proportional and non-proportional damping matrices, in structural systems. Proportional damping matrix of a structure is given by a linear superposition of the mass and stiffness matrices and it can be decoupled by the eigenvectors of the structure. However non-proportional damping matrix can not be decoupled and it makes many difficulties in structural analyses. In the sensitivity analysis, although the classical methods can be used easily in the case of the proportional damping, it is not easy to use them in the case of non-proportional damping.

The proposed method can find the eigenvalue and eigenvector derivatives of damped systems by solving the algebraic equations with symmetric coefficient matrix added with a side condition; the proposed algorithm can be applied consistently to the proportionally and non-proportionally damped systems. If the derivatives of the stiffness, mass and damping matrices are given, the proposed method can find the exact eigenpair derivatives. Also the proposed method can be extended for the eigenvalue problem with multiple eigenvalues (refer to Part II).

The second section of this paper presents the sensitivity analysis for finding eigenvalue and eigenvector derivatives of damped system. The third section presents the numerical stability of the proposed method for the eigenvalue problem with distinct eigenvalues, and the next section presents numerical examples.

2. SENSITIVITY ANALYSIS OF DAMPED SYSTEMS

The eigenvalue problem of a damped system can be written as

$$(\lambda^2\mathbf{M} + \lambda\mathbf{C} + \mathbf{K})\varphi = \mathbf{0} \tag{1}$$

where **M**, **C** and **K** are the matrices of mass, damping and stiffness respectively, and these are order n symmetric matrices. **M** is positive definite and **K** is positive definite or semi-positive definite. λ and φ are the eigenvalue and eigenvector and both are complex value in general. To determine eigenvalues and eigenvectors, for the case of non-proportionally damped system, one can use the $2n$-dimensional eigenvalue problem that is another form of equation (1) such as

$$\begin{bmatrix} -\mathbf{K} & \mathbf{0} \\ \mathbf{0} & \mathbf{K} \end{bmatrix} \begin{Bmatrix} \varphi \\ \lambda\varphi \end{Bmatrix} = \lambda \begin{bmatrix} \mathbf{C} & \mathbf{M} \\ \mathbf{M} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \varphi \\ \lambda\varphi \end{Bmatrix} \tag{2}$$

Note that $(2n)\times(2n)$ coefficient matrices in equation (2) are all symmetric, but not positive definite. One can normalize the eigenvectors such as

$$\begin{Bmatrix} \varphi_j \\ \lambda_j\varphi_j \end{Bmatrix}^T \begin{bmatrix} -\mathbf{K} & \mathbf{0} \\ \mathbf{0} & \mathbf{M} \end{bmatrix} \begin{Bmatrix} \varphi_j \\ \lambda_j\varphi_j \end{Bmatrix} = \lambda_j \quad \text{and} \tag{3}$$

$$\begin{Bmatrix} \varphi_j \\ \lambda_j\varphi_j \end{Bmatrix}^T \begin{bmatrix} \mathbf{C} & \mathbf{M} \\ \mathbf{M} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \varphi_j \\ \lambda_j\varphi_j \end{Bmatrix} = 1 \tag{4}$$

Before proceeding, we assume that eigenpairs and matrices $\partial K/\partial p$, $\partial K/\partial p$, and $\partial C/\partial p$ are known where p is a design parameter and that all eigenvalues are different.

Reconsidering the eigenvalue problem for the jth eigenmode, equation (1) can be rewritten as

$$(\lambda_j^2\mathbf{M}+\lambda_j\mathbf{C}+\mathbf{K})\varphi_j = \mathbf{0} \tag{5}$$

To obtain an equation for derivatives of eigenvalue and eigenvector, equation (5) is differentiated with respect to the design parameter p, then

$$(\lambda_j^2\mathbf{M}+\lambda_j\mathbf{C}+\mathbf{K})\frac{\partial\varphi_j}{\partial p} = -(2\lambda_j\mathbf{M}+\mathbf{C})\varphi_j\frac{\partial\lambda_j}{\partial p} - \left(\lambda_j^2\frac{\partial\mathbf{M}}{\partial p} + \lambda_j\frac{\partial\mathbf{C}}{\partial p} + \frac{\partial\mathbf{K}}{\partial p}\right)\varphi_j \tag{6}$$

Premultiplying each side of equation (6) by φ_j^T, the eigenvalue derivative can be obtained as

$$\frac{\partial \lambda_i}{\partial p} = -\varphi_i^T\left(\lambda_i^2\frac{\partial \mathbf{M}}{\partial p} + \lambda_i\frac{\partial \mathbf{C}}{\partial p} + \frac{\partial \mathbf{K}}{\partial p}\right)\varphi_i \tag{7}$$

The above equation gives derivative of eigenvalue directly, and now the right hand side of equation (6) is all known. However the eigenvector derivative $\partial\varphi_i/\partial p$ can not be found directly since the matrix $\lambda_i^2\mathbf{M}+\lambda_i\mathbf{C}+\mathbf{K}$ is singular. In this paper, to overcome this singularity problem and to find the eigenvector derivative, the algebraic method for calculating the eigenpair derivatives worked by Lee and Jung[7] is extended for the non-proportionally damped systems with distinct eigenvalues.

The proposed method solves a symmetric linear algebraic equation with side condition given by differentiating orthonormal condition. Rewriting the orthonormal condition, equation (4), in n order form gives

$$\varphi_i^T(2\lambda_i\mathbf{M} + \mathbf{C})\varphi_i = 1 \tag{8}$$

Differentiating equation (8) with respect to the design parameter yields

$$\varphi_i^T(2\lambda_i\mathbf{M}+\mathbf{C})\frac{\partial\varphi_i}{\partial p} + \frac{1}{2}\varphi_i^T\left[2\left(\frac{\partial\lambda_i}{\partial p}\mathbf{M}+\lambda_i\frac{\partial \mathbf{M}}{\partial p}\right)+\frac{\partial \mathbf{C}}{\partial p}\right]\varphi_i=0 \tag{9}$$

Equations (6) and (9) may be written as a single matrix equation as

$$\begin{bmatrix}\lambda_i^2\mathbf{M}+\lambda_i\mathbf{C}+\mathbf{K} & (2\lambda_i\mathbf{M}+\mathbf{C})\varphi_i \\ \varphi_i^T(2\lambda_i\mathbf{M}+\mathbf{C}) & 0\end{bmatrix}\begin{Bmatrix}\dfrac{\partial\varphi_i}{\partial p} \\ 0\end{Bmatrix}$$

$$=\begin{Bmatrix}-(2\lambda_i\mathbf{M}+\mathbf{C})\varphi_i\dfrac{\partial\lambda_i}{\partial p}-\left(\lambda_i^2\dfrac{\partial \mathbf{M}}{\partial p}+\lambda_i\dfrac{\partial \mathbf{C}}{\partial p}+\dfrac{\partial \mathbf{K}}{\partial p}\right)\varphi_i \\ -\dfrac{1}{2}\varphi_i^T\left[2\left(\dfrac{\partial\lambda_i}{\partial p}\mathbf{M}+\lambda_i\dfrac{\partial \mathbf{M}}{\partial p}\right)+\dfrac{\partial \mathbf{C}}{\partial p}\right]\varphi_i\end{Bmatrix} \tag{10}$$

Equation (10) is the key idea of the proposed method and the derivative of eigenvector can be found directly by solving the algebraic equation since the coefficient matrix in the left-hand side of the equation is always nonsingular and the nonsingularity is proved in the next chapter.

One can see that the algorithm of the proposed method is very simple and compact. The proposed method has desirable properties of preserving the band and symmetry of all matrices, and of requiring knowledge of only one eigenpair to be differentiated. Both properties are important in realistic structural problems where the stiffness and mass matrices are of very high-order, since these properties allow the use of efficient storage and solution techniques.

3. NUMERICAL STABILITY OF THE PROPOSED METHOD

To show that the coefficient matrix $\mathbf{A}^*$ is always nonsingular, consider another matrix such as $\mathbf{Y}^T\mathbf{A}^*\mathbf{Y}$ where $\mathbf{Y}$ is a nonsingular square matrix of order $(n+1)$.

$$\mathbf{A}^*=\left[\begin{array}{c:c}\lambda_i^2\mathbf{M}+\lambda_i\mathbf{C}+\mathbf{K} & (2\lambda_i\mathbf{M}+\mathbf{C})\varphi_i \\ \hdashline \varphi_i^T(2\lambda_i\mathbf{M}+\mathbf{C}) & 0\end{array}\right] \tag{11}$$

The determinant property, $\det(\mathbf{Y}^T\mathbf{A}^*\mathbf{Y})=\det(\mathbf{Y}^T)\det(\mathbf{A}^*)\det(\mathbf{Y})$, provides $\det(\mathbf{Y}^T\mathbf{A}^*\mathbf{Y})\neq 0$ if and only if $\det(\mathbf{A}^*)\neq 0$ and $\det(\mathbf{Y})\neq 0$. Therefor, if it is proved that the determinant of $\mathbf{Y}^T\mathbf{A}^*\mathbf{Y}$ is non-zero, then the determinant of matrix $\mathbf{A}^*$ may also be non-zero and $\mathbf{A}^*$ is nonsingular.

In this paper, the matrix $\mathbf{Y}$ is assumed as

$$\mathbf{Y}=\begin{bmatrix}\Psi & 0 \\ 0 & 1\end{bmatrix} \tag{12}$$

where Ψ is a $n \times n$ matrix having arbitrary independent vectors containing the j-th eigenvector of the system as its columns, as follows

$$\Psi = (\psi_1 \ \psi_2 \ \cdots \ \psi_{n-1} \ \varphi_j) \tag{13}$$

where ψ s are arbitrary independent vectors chosen to be independent of φ_j. The matrix $\mathbf{Y}$ is nonsingular and invertible since it is a set of $(n+1)$ independent vectors. Pre- and post-multiplying $\mathbf{Y}^T$ and $\mathbf{Y}$ to $\mathbf{A}^*$ yield

$$\mathbf{Y}^T\mathbf{A}^*\mathbf{Y} = \begin{bmatrix} \Psi^T(\lambda_j^2\mathbf{M}+\lambda_j\mathbf{C}+\mathbf{K})\Psi & \Psi^T(2\lambda_j\mathbf{M}+\mathbf{C})\varphi_j \\ \varphi_j^T(2\lambda_j\mathbf{M}+\mathbf{C})\Psi & 0 \end{bmatrix} \tag{14}$$

It is obvious that the last column and row of the matrix $\Psi^T(\lambda_j^2\mathbf{M}+\lambda_j\mathbf{C}+\mathbf{K})\Psi$ have all zero elements since $(\lambda_j^2\mathbf{M}+\lambda_j\mathbf{C}+\mathbf{K})\varphi_j=0$. That is,

$$\Psi^T(\lambda_j^2\mathbf{M}+\lambda_j\mathbf{C}+\mathbf{K})\Psi = \begin{bmatrix} \tilde{\mathbf{A}} & 0 \\ 0 & 0 \end{bmatrix} \tag{15}$$

where $\tilde{\mathbf{A}}$ is a non-zero $(n-1)\times(n-1)$ submatrix. The assumption that λ_j is a distinct eigenvalue of the system provides that the matrices $\lambda_j^2\mathbf{M}+\lambda_j\mathbf{C}+\mathbf{K}$ and $\Psi^T(\lambda_j^2\mathbf{M}+\lambda_j\mathbf{C}+\mathbf{M})\Psi$ of order n have rank of n-1 and they are singular. But $\tilde{\mathbf{A}}$ has a full rank and it is a nonsingular matrix since it is given by eliminating the last column and row having all zero elements from $\Psi^T(\lambda_j^2\mathbf{M}+\lambda_j\mathbf{C}+\mathbf{K})\Psi$. Therefore the determinant of $\tilde{\mathbf{A}}$ is non-zero, $\det(\tilde{\mathbf{A}})\neq 0$.

By the normalization condition, the last elements of the column vector $\Psi^T(2\lambda_j\mathbf{M}+\mathbf{C})\varphi_j$ and row vector $\varphi_j^T(2\lambda\mathbf{M}+\mathbf{C})\Psi$ are unity.

$$\Psi^T(2\lambda_j\mathbf{M}+\mathbf{C})\varphi_j = \begin{Bmatrix} \tilde{\mathbf{b}} \\ 1 \end{Bmatrix} \text{ and}$$

$$\varphi\mathrm{T}j(2\lambda_j\mathbf{M}+\mathbf{C})\Psi = \begin{Bmatrix} \tilde{\mathbf{b}} \\ 1 \end{Bmatrix}^T \tag{16}$$

where $\mathbf{b}$ is non-zero vector. Substituting equations (20) and (21) into equation (19) yields

$$\mathbf{Y}^T\mathbf{A}^*\mathbf{Y} = \begin{bmatrix} \tilde{\mathbf{A}} & \mathbf{0} & \tilde{\mathbf{b}} \\ \mathbf{0} & 0 & 1 \\ \tilde{\mathbf{b}} & 1 & 0 \end{bmatrix} \tag{17}$$

Applying the determinant property of partitioned matrices, the determinant of $\mathbf{Y}^T\mathbf{A}^*\mathbf{Y}$ can be written as follows

$$\det(\mathbf{Y}^T\mathbf{A}^*\mathbf{Y}) = \det\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$
$$\det\left(\tilde{\mathbf{A}} - [\mathbf{0} \ \tilde{\mathbf{b}}]\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}^{-1}\begin{bmatrix} \mathbf{0} \\ \tilde{\mathbf{b}}^T \end{bmatrix}\right) \tag{18}$$

or

$$\det(\mathbf{Y}^T\mathbf{A}^*\mathbf{Y}) = \det(\tilde{\mathbf{A}}) \neq 0 \tag{19}$$

The determinant of $\mathbf{A}^*$ thus is not equal to zero because $\det(\mathbf{Y}^T\mathbf{A}^*\mathbf{Y})\neq 0$. The nonsingularity of the matrix $\mathbf{A}^*$ is shown analytically ; the numerical stability of the proposed method in the case of distinct natural frequencies is established.

4. NUMERICAL EXAMPLES

To verify the proposed method and its possibilities, a numerical example is presented. A simple model of truck used in reference[13] is considered in this example problem for the non-proportionally damped system shown in Fig. 1. The truck is modeled as the lumped system with 7-DOF. Only the vibrations in vertical plane are considered; all the horizontal, rolling and yawing degrees of freedom are suppressed. Selected design parameter in this example is the mass of container m_5.

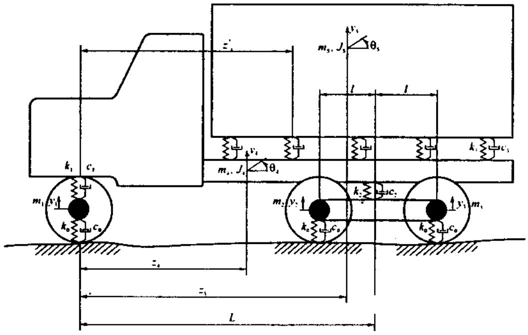

Fig. 1 The 7-DOF half-car model as non-proportionally damped system.

$z_1^*=1.8m$, $z_2^*=2.3m$, $z_3^*=3.8m$, $z_4^*=4.3m$, $z_5^*=5.0m$; $z_4=2.0m$, $z_5=3.0m$, $L=3.5m$, $l=0.85\ m$; $m_1=75kg$, $m_2=m_3=80kg$, $m_4=3500kg$, $m_5=1800kg$; $c_0=120Ns/m$, $c_1=150Ns/m$, $c_2=50Ns/m$, $c_3=80Ns/m$; $k_0=12\ 000N/m$, $k_1=14\ 000N/m$, $k_2=9500N/m$, $k_3=4000N/m$.

Some sensitivity results of the 7-DOF half-car model are represented in Tables 1. The lowest fourteen natural frequencies and their derivatives of the initial model are listed in the second and third columns of Table 1. The fourth and fifth columns of the table represent actual natural frequencies and approximated natural frequencies of the changed system of which the mass of the container is more massive than that of the initial model and the ratio of mass change to initial value is $\Delta m_5/m_5=0.01$. In what follows, $\bar{\lambda}_{changed}$ and $\bar{\varphi}_{changed}$ are approximated eigenvalue and eigenvector of the changed system respectively, and $\lambda_{changed}$ and $\varphi_{changed}$ are exact eigenvalue and eigenvector of it. The approximated eigenvalue is computed by

Table 1 The natural frequencies of the initial and changed half-car system, and results of the sensitivity analysis.

Mode Number	Initial System		Changed System		Variation of Eigenpair		Error of Approximation	
	Eigenvalue	Eigenvalue Derivative	Eigenvalue	Approximated Eigenvalue	Eigenvalue	Eigenvector	Eigenvalue	Eigenvector
1, 2	-6.9831×10^{-3} $\pm j1.3424\times10^{0}$	1.2809×10^{-6} $\mp j1.8533\times10^{-4}$	-6.9601×10^{-3} $\pm j1.3390\times10^{0}$	-6.9601×10^{-3} $\pm j1.3390\times10^{0}$	2.4773×10^{-3}	1.2777×10^{-3}	7.8769×10^{-6}	5.3031×10^{-6}
3, 4	-1.9389×10^{-2} $\pm j1.9168\times10^{0}$	7.7061×10^{-7} $\mp 5.9212\times10^{-5}$	-1.9375×10^{-2} $\pm j1.9157\times10^{0}$	-1.9375×10^{-2} $\pm j1.9157\times10^{0}$	5.5644×10^{-4}	1.6435×10^{-3}	3.8106×10^{-7}	5.8904×10^{-6}
5, 6	-1.3702×10^{-1} $\pm j3.8022\times10^{0}$	5.4075×10^{-5} $\mp 6.8017\times10^{-4}$	-1.3606×10^{-1} $\pm j3.7900\times10^{0}$	-1.3605×10^{-1} $\pm j3.7899\times10^{0}$	3.2023×10^{-3}	5.2593×10^{-3}	2.5847×10^{-5}	3.1281×10^{-5}
7, 8	-3.8500×10^{-1} $\pm j6.2351\times10^{0}$	1.6484×10^{-4} $\mp 1.2880\times10^{-3}$	-3.8207×10^{-1} $\pm j6.2121\times10^{0}$	-3.8204×10^{-1} $\pm j6.2119\times10^{0}$	3.7097×10^{-3}	5.1720×10^{-3}	3.1903×10^{-5}	2.3434×10^{-5}
9, 10	-7.5000×10^{-1} $\pm j1.2224\times10^{1}$	0.0000×10^{0} $\mp 0.0000\times10^{0}$	-7.5000×10^{-1} $\pm j1.2224\times10^{1}$	-7.5000×10^{-1} $\pm j1.2224\times10^{1}$	0.0000×10^{0}	0.0000×10^{0}	0.0000×10^{0}	0.0000×10^{0}
11, 12	-9.0807×10^{-1} $\pm j1.4490\times10^{1}$	8.1395×10^{-8} $\mp 1.8114\times10^{-7}$	-9.0807×10^{-1} $\pm j1.4490\times10^{1}$	-9.0807×10^{-1} $\pm j1.4490\times10^{1}$	2.4339×10^{-7}	1.9756×10^{-5}	2.8206×10^{-9}	2.3154×10^{-7}
13, 14	-1.8271×10^{0} $\pm j1.8661\times10^{1}$	1.3796×10^{-8} $\mp 2.0553\times10^{-8}$	-1.8271×10^{0} $\pm j1.8661\times10^{1}$	-1.8271×10^{0} $\pm j1.8661\times10^{1}$	2.3508×10^{-8}	6.0194×10^{-6}	2.5593×10^{-1}	6.5643×10^{-8}

$$\bar{\lambda}_{changed} = \lambda_{intial} + \frac{\partial \lambda}{\partial m_5} \Delta m_5 \tag{26}$$

and the approximated eigenvectors can be computed by the same way. The variations of exact natural frequencies and eigenvectors, which are calculated by $|\lambda_{initial}-\lambda_{changed}|/|\lambda_{initial}|$ and $\|\varphi_{initial}-\varphi_{changed}\|_2/\|\varphi_{initinl}\|_2$ respectively, are shown in the next two columns. The last two columns are errors of approximations calculated by $|\lambda_{changed}-\bar{\lambda}_{changed}|/|\lambda_{changed}|$ and $\|\varphi_{changed}-\bar{\varphi}_{changed}\|_2/\|\varphi_{changed}\|_2$ respectively. Considering the amount of variations of the eigenpair between initial and changed system, the errors of approximated eigenpair computed by using derivatives of eigenpair given by the proposed method are relatively quite small. And so one can say that the proposed method gives very good results.

5. CONCLUSIONS

This paper proposes an efficient numerical method whose stability is proved for calculation of the exact derivatives of vibration mode shapes of the non-proportionally damped system with distinct natural frequencies. The method has the desirable properties of preserving the band and symmetry of the system matrices and of requiring knowledge of only one eigenpair to be differentiated. The algorithm of the method can be added easily to the commercial FEM code because its numerical stability is guaranteed and gives exact solutions.

ACKNOWLEDGMENTS

This research was supported by the Korea Science and Engineering Foundation, and Korea Industrial Co., LTD. The support of the Korea Science and Engineering Foundation (203-014-2), and Korea Industrial Co., LTD. are greatly appreciated.

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(접수일자 : 199. 1. 13)