Robust Analysis of a Hybrid System Controlled by a μ-Synthesis Method

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ABSTRACT

This paper presents an extensive robust analysis of hybrid system combining lead rubber bearings and hydraulic actuators controlled by a μ-synthesis method for seismic protection of a cable-stayed bridge. Typically, passive control devices do not add mechanical energy to structural system, therefore bounded-input bounded-output stability is guaranteed. So, the passive components, i.e., lead rubber bearings, in the hybrid system provide the reliability of the control system whereas the active components, i.e., hydraulic actuators, improve the control performance. However, the overall system robustness may be deteriorated by introducing active control devices in the hybrid system, and active control devices may cause instability due to the small margins. Therefore, a μ-synthesis method that simultaneously guarantees the performance and stability of the closed-loop system (robust performance) is considered for hydraulic actuators in the hybrid system to improve the overall robustness without loss of the control performance. The performances of the proposed control system are compared with those of the performance-oriented hybrid system controlled by a LQG algorithm, and control system robustness is investigated with respect to various perturbations in the stiffness matrix, mass matrix, time delay of control command signal and combinations thereof. Furthermore, additional input earthquakes not used in the controller design are considered to investigate the controller robustness to different input excitations. Numerical simulation results show that the responses of bridge are much reduced in the proposed control system compared to uncontrolled case, and the proposed control system shows similar control performance with the performance-oriented hybrid system under earthquakes considered in this study. Furthermore, the proposed control system does not show the instability under the perturbations considered in this study except -20% stiffness perturbation with additional snow load in the deck. Therefore, the hybrid system controlled by a μ-synthesis method could be proposed as an improved control strategy for a seismically excited cable-

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stayed bridge containing many uncertainties.

INTRODUCTION

The civil engineering structures have distinctive features compared to other structures, such as the statically stable, high uncertainties of external loads with respect to magnitude and arrival times as well as structure itself, and so on. Furthermore, various uncertainties in structures as well as external loads may degrade the control performance and even cause the instability of the controlled structural system. Therefore, the robustness of control system is important in order to guarantee the stability of the controller with respect to various uncertainties as well as to increase the possibility of real application of control system in civil engineering structures.

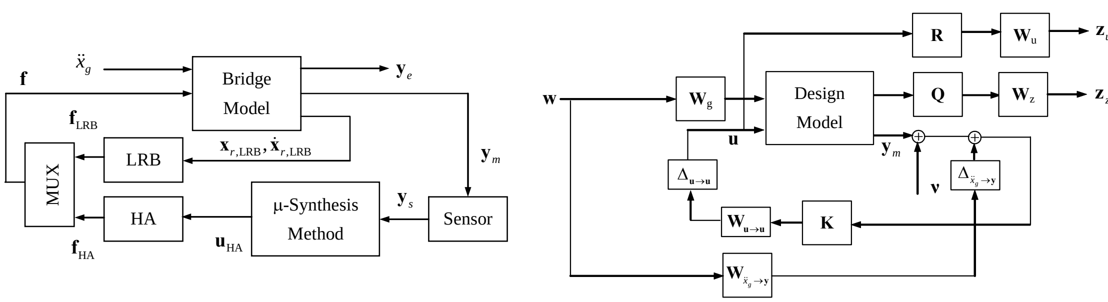
However, there are few results about the robustness of hybrid system controlled by robust algorithms for seismically excited cable-stayed bridge. Furthermore, extensive robust analysis could be used as an alternative method to verify the applicability of control systems. Therefore, the objective of this study is to investigate the control performances and controller robustness of hybrid system controlled by a μ-synthesis method with respect to various uncertainties.

CONTROL STRATEGY

Control Devices

The hybrid control system used in this study is presented in Park *et al.* (2003). This hybrid system combines lead rubber bearings (LRBs) designed by using some results of Ali and Abdel-Ghaffar (1995) and hydraulic actuators (HAs) used in benchmark control problem (Dyke *et al.* 2003). A detailed description of the hybrid control system can be found in Park *et al.* (2003).

The schematic block diagram of the proposed hybrid control system is shown in Fig. 1(a).



(a) proposed hybrid control system (b) μ-controller with various filters

Fig. 1. Block diagrams of control system

In the Fig. 1(a), $\ddot{x}_g$ is the input ground excitation, $\mathbf{y}_e, \mathbf{y}_m$, and $\mathbf{y}_s$ are evaluation outputs, measured outputs, and outputs from the sensor including noise, $\mathbf{v}$, respectively. And $\mathbf{u}_{\mathrm{HA}}$ is the control command signal that is determined by a μ-synthesis method in Volts, $\mathbf{x}_{r,\mathrm{LRB}}$ and $\dot{\mathbf{x}}_{r,\mathrm{LRB}}$ are relative displacement and velocity of LRBs. As can be seen in Fig. 1(a), in this study the energy dissipated by LRBs is expressed as external input force to consider the nonlinear behavior of LRBs.

μ-Synthesis Method

The μ-synthesis method is used for HAs to improve the control system robustness by guaranteeing the robust performance. Robust performance representing the performance and stability of the closed-loop system simultaneously can be evaluated using the structured singular value (SSV) for systems containing both structured and unstructured perturbations. The system with normalized performance criteria and perturbations performs robustly if and only if the SSV is bounded as follows (Burl 1999).

$$\sup_{\omega}\left\{\mu_{\bar{\Delta}}\left[\mathbf{N}\left(j\omega\right)\right]\right\}<1 \tag{1}$$

where $\mu_{\bar{\Delta}}(\bullet)$ is the SSV, and $\mathbf{N}(j\omega)$ is the nominal closed-loop transfer function (TF) formed by combining the system TF, $\mathbf{P}(j\omega)$, and controller TF, $\mathbf{K}(j\omega)$. $\bar{\Delta}$ is perturbation block related to stability and performance. Therefore, the SSV can be used as the objective function in the μ-synthesis method, however the direct computation of the SSV is intractable in all but the simplest case. Therefore, it is reasonable to minimize the upper bound on SSV as an alternative. During the minimization, **D-K** iteration (Burl 1999) is used to seek the solution by alternatively performing ∞-norm optimization of closed-loop system and **D**-scale (frequency dependent scale used to find upper bound of the SSV) optimization. Fig. 1(b) shows the block diagram of the μ-controller incorporated with frequency dependent weighting and uncertainty filters.

Two perturbations are implemented into the controller design model to account for differences between design (reduced-order model for controller design) and evaluation (full-order model) models and uncertainties of control command signal. First, there is an additive perturbation, $\Delta_{\ddot{x}_g\to\mathbf{y}}$, on the TF between the earthquake excitation and measurements to account for the difference between the evaluation and design models. Therefore, the weight, $\mathbf{W}_{\ddot{x}_g\to\mathbf{y}}$, on this perturbation is included in the system chosen based on this uncertainty, so that the additive uncertainty could account for unmodelled dynamics in the system. Furthermore, the multiplicative uncertainty is considered to account for the interaction between the bridge model and control device, difference of input signal due to the error of mechanical and electrical models, and so on. The weighting function, $\mathbf{W}_{\mathbf{u}\to\mathbf{u}}$, on this perturbation is simply chosen to include 1% uncertainty.

In addition two uncertainty filters, three frequency dependent weighting filters are considered to improve the control performance and controller robustness. In the μ-synthesis method, the earthquake excitation is taken to be a stationary white noise. However, the energy of earthquake is concentrated in the low frequency range in general. Therefore, *Kanai-Tajimi* filter is considered to capture the characteristics of input earthquakes. In addition, a high-pass filter is adopted for control inputs to prevent spillover effects of controller caused by difference error of the design and evaluation models. Finally, a low-pass filter is used for regulated outputs to control low frequency responses effectively.

NUMERICAL EXAMPLES

Benchmark Cable-Stayed Bridge

The bridge considered in this study is that of a benchmark control problem (Dyke *et al.* 2003) which is provided as a testbed structure for the development of strategies for the control of cable-stayed bridges. This bridge is the Missouri 74-Illinois 146 bridge spanning the Mississippi River near Cape Girardeau, MO. A total number of 24 LRBs and 24 HAs is used in the bridge and 4 displacement sensors and 5 accelerometers are used for feedback.

The detailed description of the benchmark control problem for cable-stayed bridge including the bridge model, evaluation criteria, input earthquake excitation, and control system constraints can be found in Dyke *et al.* (2003) or *http://wusceel.cive.wustl.edu/quake/bridgebenchmark.*

Analysis Results

Control performance   Fig. 2 shows the base shear force record of uncontrolled (UNCONSYS, there are 16 shock transmission devices between deck and tower connections) and proposed control system (RHCS) at pier 2 under Gebze earthquake. It is clear that the controlled system undergoes less force than the uncontrolled system. Fig. 3 shows the maximum and minimum cable tension as a function of cable number. The gray region provides the acceptable range of cable tension as specified in the control constraints ($0.2T_{fi}$ ~ $0.7T_{fi}$, where T_{fi} is failure tension of each cable), and black region provides a graphical description of the actual minimum and maximum cable tension. Note that cable tension in the uncontrolled system falls below the lower bound in cables near the tower when subjected to El Centro earthquake, however in each controlled cable tension is well within the bounds and the variation of cable tension is smaller than that of uncontrolled case.

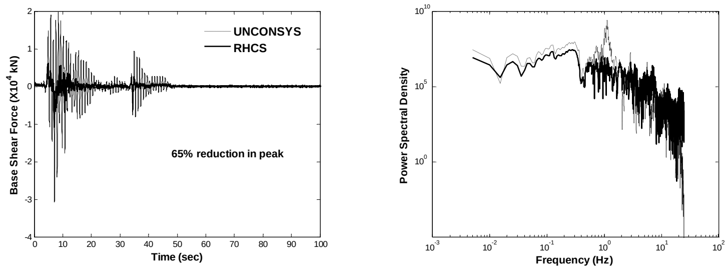
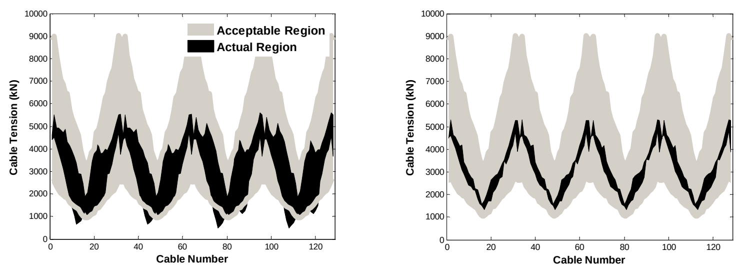


Fig. 2. Time history and power spectral density of base shear at pier 2 under Gebze earthquake



(a) Uncontrolled system (b) Proposed control system

Fig. 3. Deviation of cable tension under El Centro earthquake

In general, proposed hybrid control system shows similar control performance with conventional hybrid system controlled by a LQG algorithm (CHCS, Park *et al.* 2003) in the sense of evaluation criteria as shown in Fig. 4. Note that the shear at deck level (J_2) could be increased compare to uncontrolled case, because the control devices are installed between the deck and towers and the deck displacement (J_6) could be also increased in the controlled case compare to uncontrolled case, because all connections between the deck and towers are fixed in the uncontrolled case. Furthermore, all the control system constraints are satisfied in the proposed control system

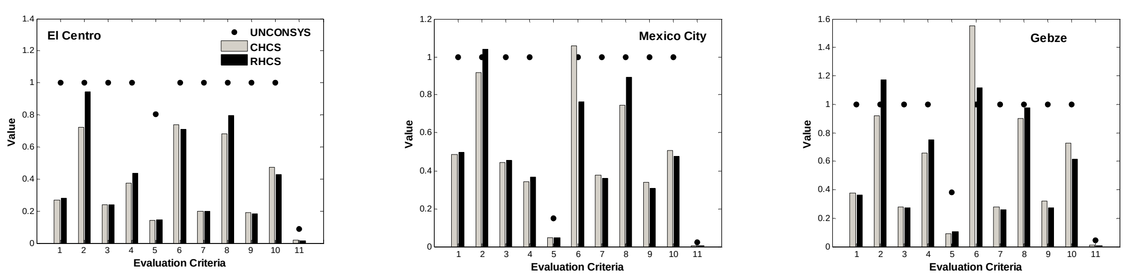
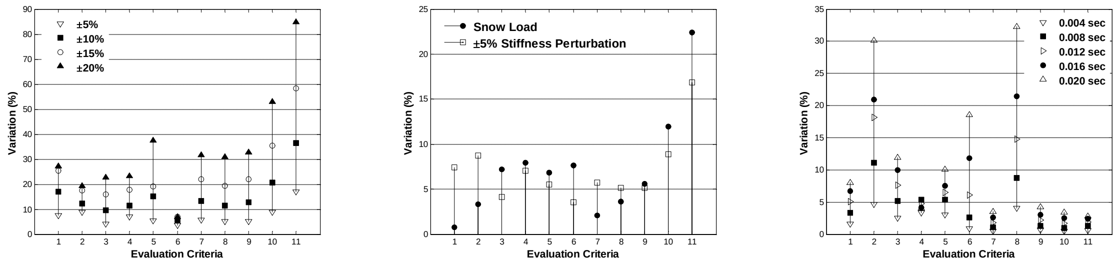


Fig. 4. Evaluation criteria under each earthquake

Controller robustness The evaluation model with the proposed control system produces desirable results based on the performance criteria set by Dyke *et al.* (2003). However, the dynamics of the real bridge may not be expected to be identical to the numerical model, so even if the proposed robust hybrid control system shows good performance in the evaluation model, it will not necessarily mean that it will yield good performance in the as-built bridge, too. Therefore, the robustness of proposed control system is investigated in order to verify the applicability of the control system with respect to various uncertainties.



(a) Stiffness perturbation (b) Snow load (c) Time delay

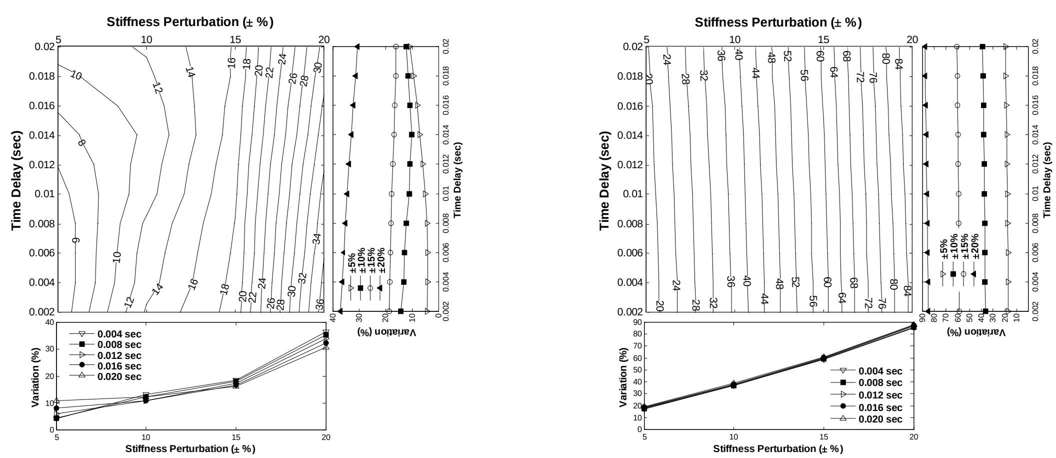
Fig. 5. Robustness of the proposed control system with respect to various perturbations

Fig. 5 shows the maximum variation of each evaluation criteria related to the structural responses (J_1 ~ J_{11}) for all three earthquakes (El Centro, Mexico City, and Gebze earthquakes) with respect to various perturbations. In this study, it is assumed that the control system fails to obtain the robustness if the maximum variation of evaluation criteria is greater than 100% and/or additional constraints are violated. As seen in Fig. 5(a), the variation of each evaluation criteria increases monotonously as the stiffness perturbation increases. Furthermore, the variation of cable tension deviation (J_5 and J_{11}) is larger than the others and deck displacement (J_6) is relatively insensitive to the stiffness perturbation.

There are many unexpected additional mass on the bridge deck, such as snows, vehicle, rains, and so on. Therefore, the effects of snow load on the model considered in phase II benchmark control problem (Caicedo *et al.* 2003) are used in this study. The variation of evaluation criteria for additional snow load in the deck are similar to that for ±5% stiffness perturbation as shown in Fig. 5(b) and the normed cable tension deviation (J_{11}) shows the largest variation among the 11 evaluation criteria similar to stiffness perturbation.

Time delay of actuator is introduced by many reasons, such as dynamics, A/D input and D/A output conversions, interaction between bridge and control devices, and so on. When there is time delay in the actuator, the variation of shears at deck level and deck displacements (J_2, J_8, and J_6) is larger than the others as seen in Fig 5(c), because the actuators are installed between deck and tower connections.

The normed devidation of cable tension (J_{11}) is increased from 0.007 to 0.015 when stiffness perturbation is -20% and there is additional snow load in the deck simultaneously under Gebze earthquake, so the variation of J_{11} is greater than 100%. Therefore, the control system may impacts negative effects on the bridge even though the cable tension satisfies the allowable range



(a) Maximum cable tension deviation (b) Normed cable tension deviation

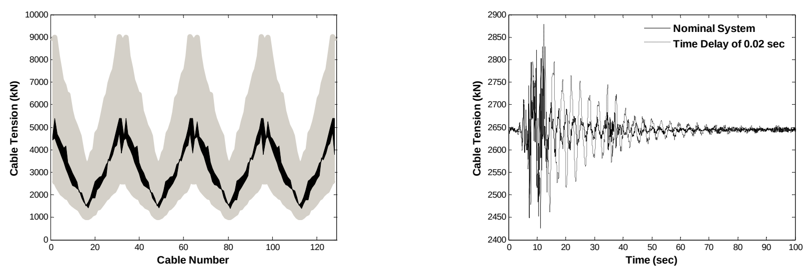
Fig. 6. Robustness of the proposed control system with respect to stiffness perturbation and time delay

Fig. 6 shows the maximum variation of cable tension deviation (J_5 and J_{11}) for all three earthquakes with stiffness perturbation of bridge and time delay in the actuator simultaneously. As seen in Fig. 6(a), the variation of cable tension deviation is much affected by the stiffness perturbation than the time delay. The effect of time delay is presented when the stiffness perturbation is small (±5%), however the effect of time delay decreases as the stiffness perturbation increases. The variation of normed cable tension deviation (J_{11}) is not sensitive to time delay when there are stiffness perturbations as seen in Fig. 6(b).

When there are time delay in the command signal and additional snow load in the deck, the variation of evaluation criteria larger than that of without snow load when time delay is small, however as the time delay increases the difference of variation between with and without snow load decreases.

When there are stiffness perturbation, snow load, and time delay simultaneously, the variation of normed cable tension deviation (J_{11}) under Gebze earthquake is greater than 100% with the -20% stiffness perturbation, even though the cable tension is satified the allowable range as seen in Fig. 7 and the structural responses die out after seismic events.

During the controller design, three earthquakes (El Centro, Mexico City, and Gebze earthquakes) are used. Therefore, it is required to investigate the robustness of the controller to other earthquakes which are not used in the design procedure. The NS component of Hachinohe earthquake (1968) and TCU084 component of Chi-Chi earthquake (1999) are considered as additional earthquake. Hachinohe earthquake has the peak ground acceleration (PGA) of 0.23g which is lower than the design PGA of Bill Emerson Memorial Bridge whereas the PGA of Chi-Chi earthquake is 0.42g which is greater than the design PGA.



(a) Deviation of tension of all cables (b) Deviation of tension of cable no. 10

Fig. 7. Deviation of cable tension under Gebze earthquake

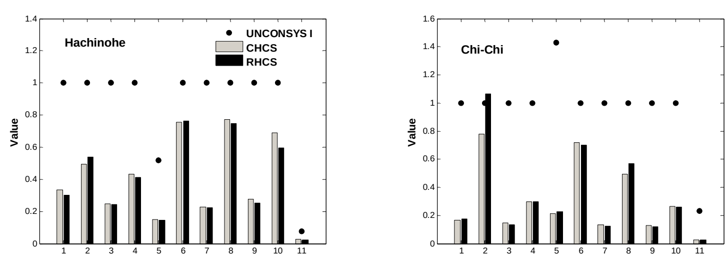
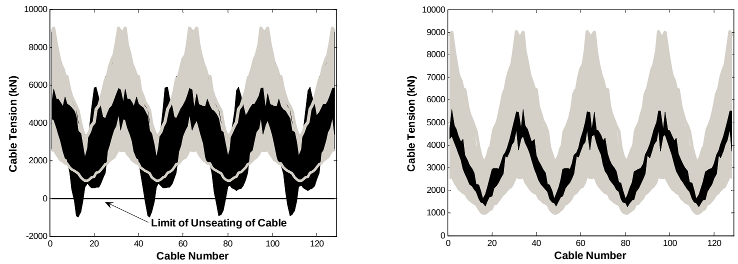


Fig. 8. Evaluation criteria under Hachinohe and Chi-Chi earthquakes

The evaluation criteria of the proposed control system are much smaller than those of uncontrolled system except deck shear (J_2) under Chi-Chi earthquake and shows similar control performances with the conventional hybrid control system as shown in Fig. 8. Furthermore, the cable tension of the proposed control system satisfies the allowable range whereas that of uncontrolled system falls below the lower bound and falls above the upper bound as shown in Fig. 9. There are unseating in the some stay cables for the uncontrolled system under Chi-Chi earthquake which may results in the failure of bridge.



(a) Uncontrolled system (b) Proposed control system

Fig. 9. Deviation of cable tension under Chi-Chi earthquake

CONCLUSIONS

The responses of bridge are much reduce in the proposed control system compared to uncontrolled case, and the proposed control system show similar control performance with the performance-oriented hybrid system under earthquakes considered in this study. Furthermore, the proposed control system does not show the instability under the perturbations considered in this study except -20% stiffness perturbation with additional snow load without loss of control performances. The responses of bridge with control system are much affected by the perturbation of stiffness of bridge than other perturbations, and the deviation of cable tension is much sensitive than other responses. The proposed control system shows desirable results even though the PGA of input excitation is exceed the design PGA whereas there are unseating of cables in the uncontrolled system.

Therefore, the hybrid system controlled by a μ-synthesis method could be proposed as an improved control strategy for a seismically excited cable-stayed bridge containing man uncertainties.

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