# Three-dimensional Plate Analyses of Wind-loaded Structures

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## Abstract

Simplified beam analyses are commonly used to obtain the responses of wind-loaded structures, and sometimes three-dimensional plate analyses are adopted to more exactly calculate the responses of structures under wind loads. However, there is no systematic comparison between simplified beam analyses and three-dimensional plate analyses. This paper systematically compares simplified beam analyses with three-dimensional plate analyses to show disadvantages of conventional simplified beam analyses by considering a plate structure under wind loads. Bending moments and principal stresses from beam analyses are similar to those from plate analyses. However, torsional moments from beam analyses are quite different from those from plate analyses. It is possible to obtain member forces which vary along width directions from plate analyses. On the other hand, member forces along width directions cannot be exactly calculated from beam analyses because beam analyses give constant distributions of member forces along width directions. Therefore, torsional moments and width-directional member forces of wind-loaded structures must be calculated from three-dimensional plate analyses instead of simplified beam analyses.

## 1. Introduction

Large-scaled structures such as high-rise buildings, tall slender towers and long-span bridges are often sensitive to wind loads due to their flexibility. Accordingly, analyses of wind-loaded structures have become more important in structural designs.

Simplified beam analyses using lift coefficients, drag coefficients and moment coefficients are commonly used to obtain the responses of wind-loaded structures. In the field of wind engineering, many researches on the analyses of wind-loaded structures have still been performed by using beam modeling [1–3]. Three-dimensional plate analyses using pressure coefficients are adopted to more exactly calculate the responses of structures under wind loads [4]. It is commonly recognized that three-dimensional plate analyses give more exact results than simplified beam analyses.

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However, comparison of simplified beam analyses and three-dimensional plate analyses has not been investigated systematically. In addition, disadvantages of conventional simplified beam analyses have not been addressed clearly. This paper systematically compares conventional simplified beam analyses with exact three-dimensional plate analyses by considering a plate structure under wind loads, and shows disadvantages of conventional simplified beam analyses of wind-loaded structures.

## 2. Beam analyses of wind-loaded structures

The dynamic equation of motion of wind-loaded structures can be expressed as follows:

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K]\{u\}=\{f\} \tag{1}$$

where $[M]$, $[C]$ and $[K]$ represent mass, damping and stiffness matrices, respectively, and $\{u\}$ is displacement vector and $\{f\}$ is force vector due to wind loads. The procedure of beam analyses for wind-loaded structures is as follows. First, a structure is modeled with finite beam elements. Second, lift coefficients ($C_L$), drag coefficients ($C_D$) and moment coefficients ($C_M$) are evaluated according to the shapes of beam sections through experimental or analytical methods. Then, distributed forces are given by

$$L=(1/2)C_L rV^2B\,,\ \ D=(1/2)C_D rV^2B\,,\ \ M=(1/2)C_M rV^2B^2 \tag{2}$$

where $L$, $D$ and $M$ are lift force, drag force and moment per unit length. $r$ is wind density, $V$ wind speed and $B$ the width of beam section [5]. Finally, the responses can be obtained by solving Eq. (1) with $\{f\}$ which is composed of $L$, $D$ and $M$.

## 3. Plate analyses of wind-loaded structures

The procedure of plate analyses for wind-loaded structures is summarized as follows. First, a structure is modeled with triangular finite plate elements. Next, a pressure coefficient at each element is evaluated through experimental or analytical methods. Then a wind pressure at each element can be given by

$$p=(1/2)C_p rV^2 \tag{3}$$

where $p$ is the wind pressure and $C_p$ is the pressure coefficient. By multiplying $p$ by the area of plate element, the magnitude of a wind load acting on each element can be calculated. The direction of the wind load agrees with that of normal vector of a plate element. Therefore, the wind load can be calculated by

$$\vec{f_w}=(1/2)C_p rV^2A\vec{n} \tag{4}$$

where $\vec{f_w}$ is the wind load, $A$ is the area of plate element and $\vec{n}$ is the normal vector of plate element. By distributing such wind load to three nodal points of plate element equally, wind load vector can be composed as in

$$\{f\}_w = (1/6)C_p rV^2 A[n_x\, n_y\, n_z\, n_x\, n_y\, n_z\, n_x\, n_y\, n_z]^T \tag{5}$$

where $\{f\}_w$ is wind load vector and $n_x$, $n_y$ and $n_z$ are components of $\vec{n}$. Finally, the responses of wind-loaded structures can be calculated by using Eq. (1) with $\{f\}$ which is composed of $\{f\}_w$.

## 4. Numerical example

In order to compare beam analyses with plate analyses, a numerical example is introduced. The geometric and material properties of an example structure, which is a plate with seven hinge supports at both edges, are shown in Fig. 1. Each hinge has a rotational restraint about x-axis so that a torsional moment can be induced. The properties of an example wind are shown in Fig. 2. The velocity history of the example wind is generated artificially. In the generation of the example wind, Kaimal spectrum [5] is used as a reference spectrum. In this study, analyses are performed for the angle of attack $a$ = 15, 30, 45, 60, 75 and 90 degree. Pressure coefficients for plate analyses are analytically calculated from the formulas in Abernathy's paper [6]. $C_L$, $C_D$ and $C_M$ for beam analyses can be calculated respectively by integrating $C_p$ as follows:

$$C_L = \left(\int_0^B C_p\, dy / B\right)\cos a\ , C_D = \left(\int_0^B C_p\, dy / B\right)\sin a\ , C_M = \int_0^B C_p (B/2 - y)\, dy / B^2 \tag{6}$$

Fig. 3 represents $C_p$ along the width direction of the example structure. $C_L$, $C_D$ and $C_M$ of the example structure, which are calculated from Eq. (6), are shown in Table 1. In beam analyses, by using distributed loads ($L$, $D$ and $M$) calculated form Eqs. (2), Eq. (1) is solved with the beam model shown in Fig. 4. In plate analyses, wind load vectors $\{f\}_w$ are calculated from Eq. (5) with $C_p$ shown in Fig. 3. Eq. (1) is solved with the plate model shown in Fig. 5.

Fig. 6 represents bending moments at the center line of the example structure with varying $a$. Table 2 compares maximum bending moments from beam analyses with those from plate analyses. Differences of the results from two analyses are less than 5.8 %. Fig. 7 shows torsional moments at the edge line of the example structure with varying $a$. Table 3 compares maximum torsional moments from beam analyses with those from plate analyses. Differences of the results from two analyses have the range of 19.3 ~ 100 %. Fig. 8 represents principal stresses at the center line of the example structure with varying $a$. Table 4 compares maximum principal stresses from beam analyses with those from plate analyses. Differences of the results from two analyses are less than 7.2 %. As can be seen from the above results, torsional moments from beam analyses are quite different from those from plate analyses.

Fig. 9 and Table 5 represent bending moments along the center line of the example structure at the time of maximum point. Fig. 10 and Table 6 show torsional moments along the edge line of the example structure at the time of maximum point. These results verify that member forces which vary along width directions can be obtained from plate analyses, whereas those cannot be exactly calculated from beam analyses due to the limitation of the analyses that constant member forces are obtained along width directions.

## 5. Conclusions

Systematic comparison of simplified beam analyses and three-dimensional plate analyses is presented to verify disadvantages of conventional simplified beam analyses of structures under wind loads. The results from the numerical example can be summarized as follows:

(1) Bending moments and principal stresses from beam analyses are similar to those from plate analyses. On the other hand, torsional moments from beam analyses are quite different from those from plate analyses.

(2) Member forces which vary along width directions can be obtained from plate analyses, whereas those cannot be exactly calculated from beam analyses due to the limitation of the analyses that constant member forces are obtained along width directions.

Therefore, three-dimensional plate analyses are required to exactly calculate torsional moments and width-directional member forces of structures under wind loads, instead of simplified beam analyses.

## Acknowledgements

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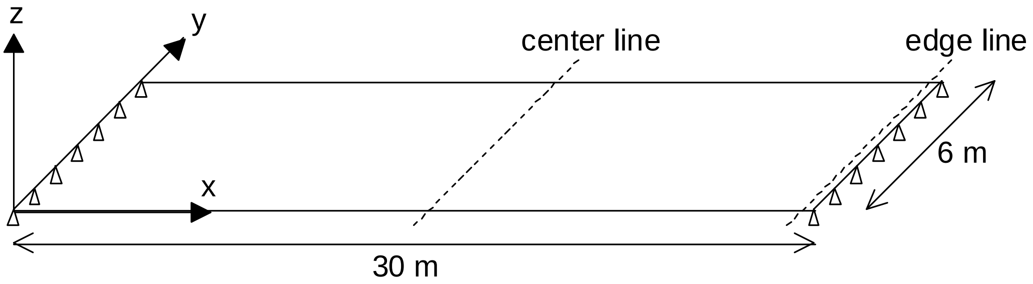

Fig. 1. The properties of an example structure. Young's modulus is $2.5\times10^{5}$ kgf/cm$^{2}$, mass density $2.446\times10^{-6}$ kgf-sec$^{2}$/cm$^{4}$, thickness 60 cm, Poisson ratio 0.17, damping ratio 5 %

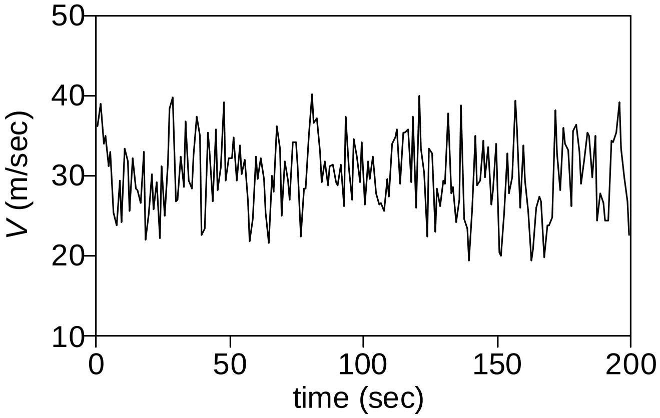

Fig. 2. The velocity history of an example wind

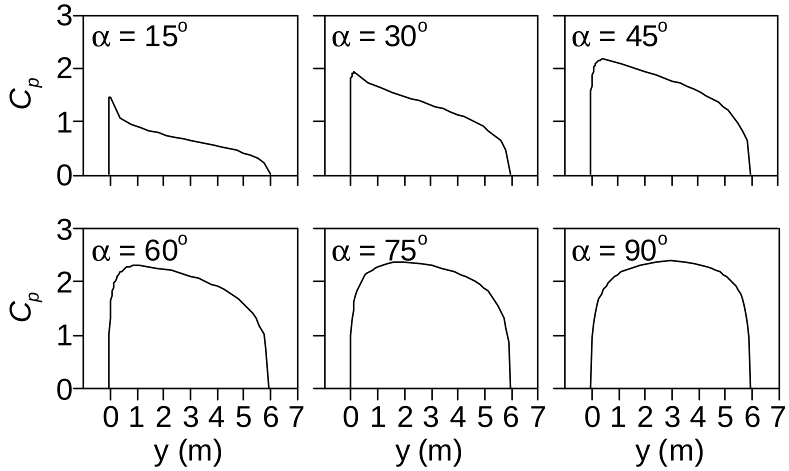

Fig. 3. Pressure coefficients along the width direction of the example structure

Table 1
Lift coefficients, drag coefficients and moment coefficients of the example structure

| $a$ (degree) | $C_L$ | $C_D$ | $C_M$ |
|---|---|---|---|
| 15 | 0.605 | 0.162 | 0.057 |
| 30 | 1.055 | 0.609 | 0.086 |
| 45 | 1.139 | 1.139 | 0.090 |
| 60 | 0.937 | 1.622 | 0.074 |
| 75 | 0.520 | 1.941 | 0.041 |
| 90 | 0.000 | 2.066 | 0.000 |

Fig. 4. Finite beam element model of the example structure for beam analyses

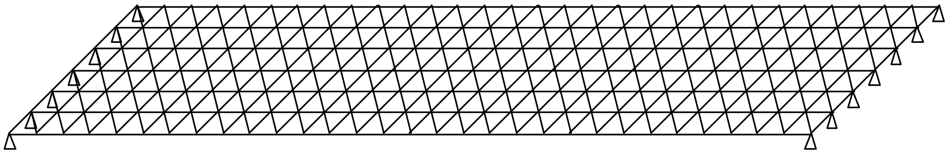

Fig. 5. Finite plate element model of the example structure for plate analyses

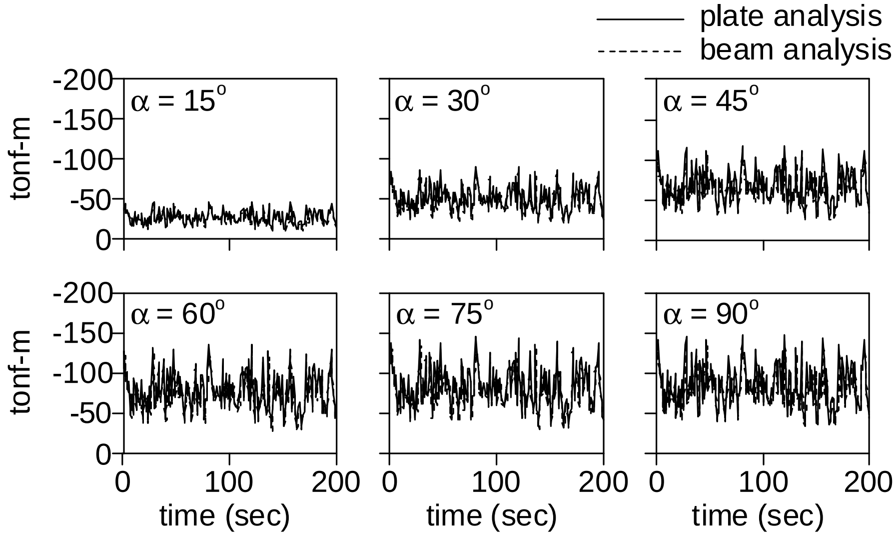

Fig. 6. Bending moments at the center line of the example structure

Table 2
Maximum bending moments at the center line of the example structure
(unit = tonf-m)

| *a* (degree) | plate analysis | beam analysis | difference (%) |
|---|---|---|---|
| 15 | -45.32 | -42.72 | 5.75 |
| 30 | -87.59 | -83.11 | 5.12 |
| 45 | -115.35 | -109.91 | 4.72 |
| 60 | -133.53 | -127.82 | 4.28 |
| 75 | -142.80 | -137.08 | 4.00 |
| 90 | -146.65 | -140.94 | 3.89 |

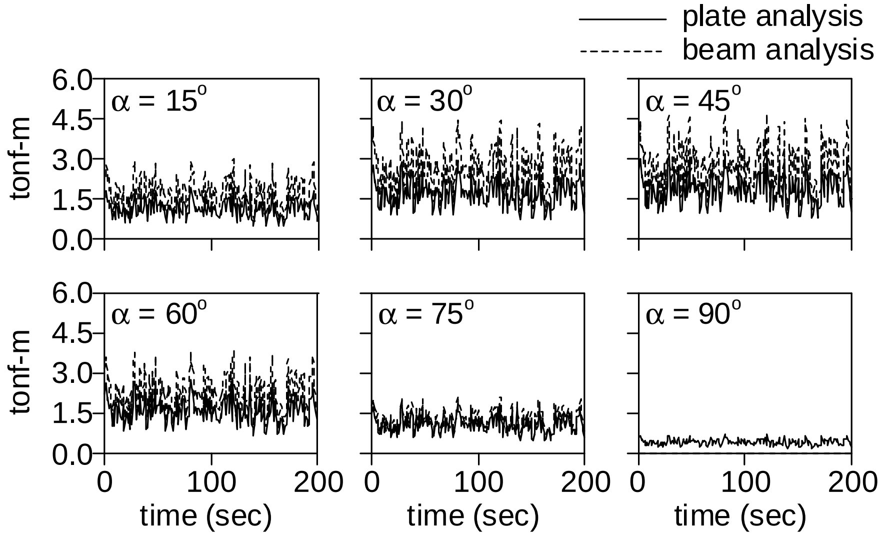

Fig. 7. Torsional moments at the edge line of the example structure

Table 3
Maximum torsional moments at the edge line of the example structure
(unit = tonf-m)

| *a* (degree) | plate analysis | beam analysis | difference (%) |
|---|---|---|---|
| 15 | 1.86 | 2.94 | 57.90 |
| 30 | 2.88 | 4.45 | 54.39 |
| 45 | 3.12 | 4.69 | 50.23 |
| 60 | 2.69 | 3.83 | 42.20 |
| 75 | 1.78 | 2.13 | 19.29 |
| 90 | 0.63 | 0.00 | 100.00 |

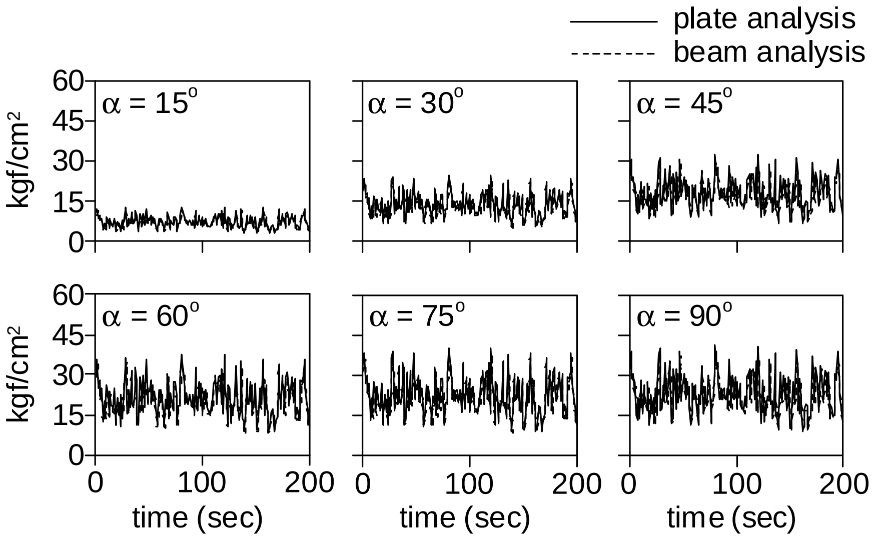

Fig. 8. Principal stresses at the center line of the example structure

Table 4
Maximum principal stresses at the center line of the example structure
(unit = kgf/cm$^2$)

| *a* (degree) | plate analysis | beam analysis | difference (%) |
|---|---|---|---|
| 15 | 12.78 | 11.87 | 7.16 |
| 30 | 24.65 | 23.09 | 6.34 |
| 45 | 32.41 | 30.53 | 5.80 |
| 60 | 37.45 | 35.51 | 5.19 |
| 75 | 39.97 | 38.08 | 4.74 |
| 90 | 40.97 | 39.15 | 4.43 |

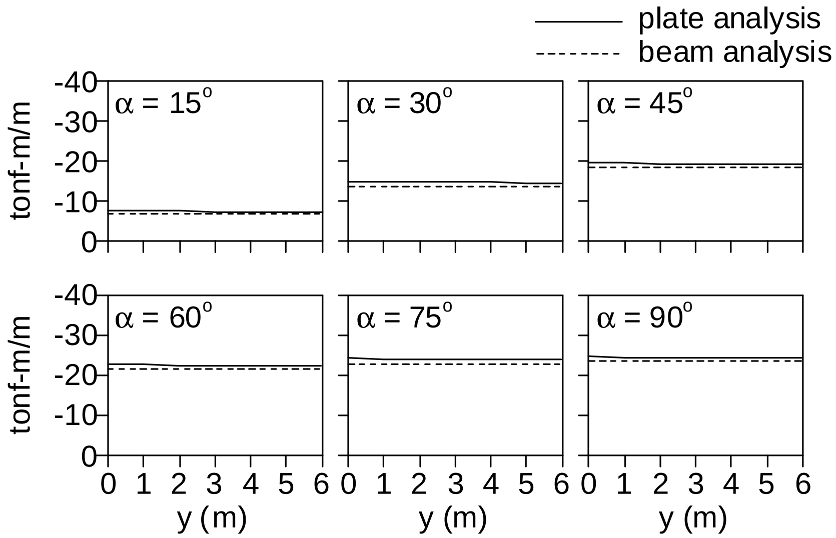

Fig. 9. Bending moments along the center line of the example structure (time = 81 sec)

Table 5
Bending moments along the center line of the example structure
(unit = tonf-m/m, time = 81 sec)

| y (m) | *a* = 15° | | *a* = 30° | | *a* = 45° | | *a* = 60° | | *a* = 75° | | *a* = 90° | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | plate | beam | plate | beam | plate | beam | plate | beam | plate | beam | plate | beam |
| 0 | -7.67 | -7.12 | -14.79 | -13.85 | -19.45 | -18.32 | -22.47 | -21.30 | -23.98 | -22.85 | -24.58 | -23.49 |
| 1 | -7.62 | -7.12 | -14.70 | -13.85 | -19.34 | -18.32 | -22.35 | -21.30 | -23.87 | -22.85 | -24.48 | -23.49 |
| 2 | -7.58 | -7.12 | -14.63 | -13.85 | -19.26 | -18.32 | -22.28 | -21.30 | -23.81 | -22.85 | -24.43 | -23.49 |
| 3 | -7.54 | -7.12 | -14.58 | -13.85 | -19.20 | -18.32 | -22.23 | -21.30 | -23.77 | -22.85 | -24.41 | -23.49 |
| 4 | -7.52 | -7.12 | -14.54 | -13.85 | -19.15 | -18.32 | -22.19 | -21.30 | -23.75 | -22.85 | -24.41 | -23.49 |
| 5 | -7.50 | -7.12 | -14.51 | -13.85 | -19.13 | -18.32 | -22.18 | -21.30 | -23.75 | -22.85 | -24.43 | -23.49 |
| 6 | -7.47 | -7.12 | -14.47 | -13.85 | -19.08 | -18.32 | -22.12 | -21.30 | -23.70 | -22.85 | -24.39 | -23.49 |

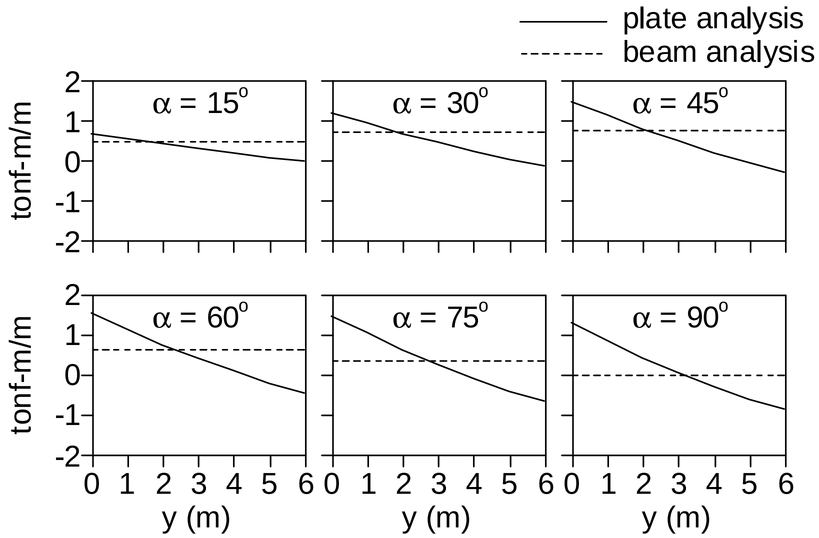

Fig. 10. Torsional moments along the edge line of the example structure (time = 81 sec)

Table 6
Torsional moments along the edge line of the example structure (unit = tonf-m/m, time = 81 sec)

| y (m) | *a* = 15° | | *a* = 30° | | *a* = 45° | | *a* = 60° | | *a* = 75° | | *a* = 90° | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | plate | beam | plate | beam | plate | beam | plate | beam | plate | beam | plate | beam |
| 0 | 0.68 | 0.49 | 1.20 | 0.74 | 1.46 | 0.78 | 1.53 | 0.64 | 1.45 | 0.35 | 1.29 | 0.00 |
| 1 | 0.55 | 0.49 | 0.94 | 0.74 | 1.12 | 0.78 | 1.14 | 0.64 | 1.04 | 0.35 | 0.86 | 0.00 |
| 2 | 0.42 | 0.49 | 0.68 | 0.74 | 0.78 | 0.78 | 0.75 | 0.64 | 0.62 | 0.35 | 0.43 | 0.00 |
| 3 | 0.30 | 0.49 | 0.46 | 0.74 | 0.49 | 0.78 | 0.41 | 0.64 | 0.25 | 0.35 | 0.06 | 0.00 |
| 4 | 0.18 | 0.49 | 0.23 | 0.74 | 0.20 | 0.78 | 0.07 | 0.64 | -0.10 | 0.35 | -0.30 | 0.00 |
| 5 | 0.08 | 0.49 | 0.03 | 0.74 | -0.06 | 0.78 | -0.22 | 0.64 | -0.41 | 0.35 | -0.62 | 0.00 |
| 6 | -0.01 | 0.49 | -0.13 | 0.74 | -0.27 | 0.78 | -0.46 | 0.64 | -0.67 | 0.35 | -0.88 | 0.00 |