IMPLEMENTATION OF MODAL CONTROL FOR SEISMICALLY EXCITED STRUCTURES USING MR DAMPER

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ABSTRACT

This paper proposes an implementation of modal control for seismically excited structures using magnetorheological (MR) dampers. Modal control reshapes the motion of a structure by merely controlling a few selected vibration modes. Hence, a modal control scheme is more convenient to design the controller than other control algorithms. Although modal control has been investigated for the several decades, its potential for semi-active control, especially for the MR damper, has not been exploited. Thus, in order to study the effectiveness for MR damper system, a modal control scheme is implemented to seismically excited structures. A Kalman filter is included in a control scheme to estimate modal states from measurements by sensors. Moreover, a low-pass filter is applied to eliminate the spillover problem. The numerical results indicate that the motion of the structure is effectively suppressed by merely controlling a few lowest modes, although resulting responses varied greatly depending on the choice of measurements available and weightings.

INTRODUCTION

Magnetorheological (MR) dampers are one of semi-active control devices, which use MR fluids to provide controllable damping forces. A number of control algorithms have been adopted for semi-active systems including the MR damper. Jansen and Dyke (2000) discussed recently proposed semi-active control algorithms including the decentralized bang-bang controller (MaClamroch and Gavin 1995), the controller based on Lyapunov stability theory (Brogan 1991; Leitmann 1994), the clipped-optimal controller (Sack et al. 1994; Dyke 1996), the modulated homogeneous friction controller (Inaudi 1997), and the maximum energy dissipation algorithm. They, also, formulated these algorithms for use with MR dampers and evaluated and compared the performance of each algorithm.

Modal control represents one control class, in which the motion of a structure is reshaped by merely controlling some selected vibration modes. Modal control is especially desirable for the vibration control of civil engineering structure may involve hundred or even thousand degrees of freedom, its vibration is usually dominated by the first few modes. Therefore, the motion of the structure can be effectively suppressed by merely controlling these few modes. A modal control scheme, which uses modal state estimation, is desirable (Meirovitch 1990).

The purpose of this study is to implement modal control for seismically excited structures that use MR dampers and to compare the performance of the proposed method with that of other control algorithms previously studied. A modal control scheme with a Kalman filter and a low-pass filter is applied. A Kalman filter is included in a control scheme to estimate modal states from measurements by sensors. Three cases of the structural measurement are considered by a Kalman filter to verify the effect of each measurement; displacement, velocity, and acceleration, respectively. Moreover, a low-pass filter is applied to eliminate the spillover problem.

MODAL CONTROL SCHEME FOR MR DAMPERS

In this section, a modal control scheme with a Kalman filter and a low-pass filter is implemented to seismically excited structure. After the implementation of modal control scheme, numerical simulation is presented in a subsequent section for comparisons between control algorithms.

Modal Control

Consider a seismically excited structure controlled with m MR dampers. Assuming that the forces provided by the control devices are adequate to keep the response of the primary structure from exiting the linear region, the equations of motion can be written

$$\boldsymbol{M}\ddot{\boldsymbol{x}}(t)+\boldsymbol{C}\dot{\boldsymbol{x}}(t)+\boldsymbol{K}\boldsymbol{x}(t)=\boldsymbol{\Lambda}\boldsymbol{f}(t)-\boldsymbol{M\Gamma}\ddot{x}_g \qquad (1)$$

where $\boldsymbol{M}$, $\boldsymbol{C}$ and $\boldsymbol{K}$ are the $n\times n$ mass, damping, and stiffness matrices, respectively; $\boldsymbol{x}$ is the n-dimensional vector of the relative displacements of the floors of the structure; $\boldsymbol{f} = [f_1, f_2, \ldots, f_m]^T$ is the vector of measured control forces generated by m MR dampers; x_g is ground acceleration; Γ is the column vector of ones; and Λ is the matrix determined by the placement of MR dampers in the structure. The displacement can be expressed as the linear combination

$$\boldsymbol{x}(t) = \sum_{r=1}^{n} \phi_r \eta_r(t) = \boldsymbol{\Phi}\boldsymbol{\eta}, \; r = 1, 2, \ldots, n \tag{2}$$

where $\eta_r(t)$ is the r th modal displacement; ϕ_r is the r th eigenvector; $\boldsymbol{\Phi}$ is a eigenvector set; and $\boldsymbol{\eta}$ is a modal displacement vector. In modal control, only a limited number of lower modes are controlled. Hence, l controlled modes can be selected with $l < n$ and the displacement may be partitioned into controlled and uncontrolled parts as $\boldsymbol{x}(t) = \boldsymbol{x}_C(t) + \boldsymbol{x}_R(t)$, where $\boldsymbol{x}_C$ and $\boldsymbol{x}_R$ represent the controlled and uncontrolled displacement vector, respectively. We refer to the uncontrolled modes as residual. The eigenvectors are orthogonal and can be normalized so as to satisfy the orthonormality conditions. Inserting Eq. (2) into Eq. (1), multiplying by $\boldsymbol{\phi}_r^T$ and considering orthogonal condition between eigenvectors, we obtain

$$\ddot{\eta}_r + 2\zeta_r \omega_r \dot{\eta}_r + \omega_r^2 \eta_r = \boldsymbol{\phi}_r^T \Lambda f - \boldsymbol{\phi}_r^T \Gamma \ddot{x}_g, \quad r = 1, 2, \ldots, l \tag{3}$$

where ζ_r are modal damping ratios, ω_r is a natural frequency. Then, Equation (3) can be rewritten in state-space form such as

$$\dot{\boldsymbol{w}}_C(t) = \boldsymbol{A}_C \boldsymbol{w}_C(t) + \boldsymbol{B}_C\, f(t) + \boldsymbol{E}_C \ddot{x}_g, \quad \boldsymbol{y}_C(t) = \boldsymbol{C}_C \boldsymbol{w}_C(t) \tag{4}$$

where $\boldsymbol{w}_c$ is a $2l$-dimensional modal state vector by the controlled modes and

$$\boldsymbol{A}_C = \begin{bmatrix} 0 & \boldsymbol{I}_C \\ -\boldsymbol{\Omega}_C^2 & -\boldsymbol{\Delta}_C \end{bmatrix}, \quad \boldsymbol{B}_C = \begin{bmatrix} 0 \\ \boldsymbol{B}'_C \end{bmatrix}, \quad \boldsymbol{E}_C = \begin{bmatrix} 0 \\ \boldsymbol{E}'_C \end{bmatrix} \tag{5}$$

are the $2l\times2l$, $2l\times m$ matrixes and a $2l\times1$ vector, respectively, and Δ_c is the diagonal matrix listing $2\omega_r\zeta_r$; Ω_c^2 is the diagonal matrix listing $\omega_1^2, \ldots, \omega_n^2$; $B_c' = \Phi^T \Lambda$; and $E_c' = \Phi^T \Gamma$. For feedback control, the control vector is related to the modal state vector according to

$$\mathbf{f}(t) = -\mathbf{K}_C \mathbf{w}_C(t) \tag{6}$$

where K_C is an $m\times2l$ control gain matrix. Because the force generated in the i th MR damper depends on the responses of the structural system, the MR damper cannot always produce the desired optimal control force f_{Ci}. Thus, the strategy of a clipped-optimal control(Dyke et al. 1996) is used.

Design of Optimal Controller

Referring to the discussions in above section, control gain matrix K_C should be decided. Although a variety of approaches may be used to design the optimal controller, H2/LQG (Linear Quadratic

Gaussian) methods are advocated because of their successful application in previous studies (Dyke et al. 1996). For the controller design, x_g is taken to be a stationary white noise, and an infinite horizon performance index is chosen that weights the modal states by controlled modes such as

$$J = \lim_{\tau \to \infty} \frac{1}{\tau} E\left[\int_0^{\tau} (\boldsymbol{w}_C^T \boldsymbol{Q} \boldsymbol{w}_C + \boldsymbol{u}^T \boldsymbol{R} \boldsymbol{u}) dt \right] \tag{7}$$

where $\boldsymbol{R}$ is a 2×2 identity matrix because the numerical example has two MR dampers, and $\boldsymbol{Q}$ is a $2l \times 2l$ diagonal matrix. It should be noted that the size of $\boldsymbol{Q}$ is reduced from $2n \times 2n$ to $2l \times 2l$ because the limited lower modes are controlled. Therefore, it can be said that it is more convenient to design the smaller weighting matrix of modal control. For example, when the lowest one mode is controlled for calculating the modal control action, $\boldsymbol{Q}$ is a 2×2 diagonal matrix.

Modal State Estimation

An observer for modal state estimation should be provided, since real sensors may not estimate the full modal states directly or the system may be expensive to prepare the sensors for the full states. To estimate the modal state vector $\boldsymbol{w}_C(t)$ from the measured output $\boldsymbol{y}(t)$, we consider a Kalman-Bucy filter as an observer(Meirovitch, 1990). Not only, in this paper, the state feedback including velocities or displacements is considered, but also the acceleration feedback is implemented for the modal state estimation using a Kalman-Buch filter. In any case, we can write a modal observer in the form

$$\dot{\hat{\boldsymbol{w}}}_C(t) = \boldsymbol{A}_C \hat{\boldsymbol{w}}_C(t) + \boldsymbol{B}_C \boldsymbol{f}(t) + \boldsymbol{E}_C \ddot{x}_g + \boldsymbol{L}[\boldsymbol{y}(t) - \boldsymbol{C}_C \hat{\boldsymbol{w}}_C(t) - \boldsymbol{D}_C \boldsymbol{f}(t)] \tag{8}$$

where $\hat{\boldsymbol{w}}_c(t)$ is the estimated controlled modal state and $\boldsymbol{L}$ is the optimally chosen observer gain matrix by solving a matrix Riccati equation, which assumes that the noise intensities associated with earthquake and sensors are known. $\boldsymbol{C}_c$ is changeable according to the signals which are used for the feedback and $\boldsymbol{D}_c$ is generally zero except the acceleration feedback. For modal state estimation from the displacements, $\boldsymbol{C}_c = [\boldsymbol{\Phi}_c\ 0\ ]$. For control with the velocity feedback, $\boldsymbol{C}_c = [\ 0\ \boldsymbol{\Phi}_c\ ]$. For control with the acceleration feedback,

$$\mathrm{C_C} = [\boldsymbol{M}^{-1}\boldsymbol{K}\ \boldsymbol{M}^{-1}\boldsymbol{C}] \begin{bmatrix} \boldsymbol{\Phi}_C & 0 \\ 0 & \boldsymbol{\Phi}_C \end{bmatrix} \quad \textit{and} \quad \mathrm{D_C} = \boldsymbol{M}^{-1}\boldsymbol{\Lambda} \tag{9}$$

Upon obtaining the estimated controlled modal state from Eq. (8), we compute the feedback control forces

$$\mathrm{f}(\mathrm{t}) = -\mathbf{KC}\, \hat{\boldsymbol{w}}_C(t) \tag{10}$$

Until now, the uncontrolled modes are ignored. In reality, however, the sensor signals will include contributions from all the modes, so that the output vector is corrected to

$$y = C\,w(t) = C_C w_C(t) + C_R w_R(t) \qquad (11)$$

To examine the effect of the control forces on the uncontrolled modes, residual modes can be written

$$\dot{w}_R(t) = A_R w_R(t) + B_R f(t) + E_R \ddot{x}_g \qquad (12)$$

where w_R is a residual state vector by uncontrolled modes. Substituting Eq. (10) into Eq. (4) and considering Eq. (12), we obtain

$$\dot{w}_C(t) = A_C w_C(t) - B_C K_C \hat{w}_C(t) + E_C \ddot{x}_g, \quad \dot{w}_R(t) = A_R w_R(t) - B_R K_C \hat{w}_C(t) + E_R \ddot{x}_g \qquad (13)$$

Moreover, substituting Eqs. (10) and (11) into Eq. (8), we can write the observer equation in the form

$$\dot{\hat{w}}_C(t) = (A_C - B_C K_C)\hat{w}_C(t) + LC_C(w_C(t) - \hat{w}_C(t)) + LC_R w_R(t) + E_C \ddot{x}_g \qquad (14)$$

The error vector is defined such as $e_C(t) = \hat{w}_C(t) - w_C(t)$. Then the Equations can be written in the matrix form

$$\begin{bmatrix} \dot{w}_C(t) \\ \dot{w}_R(t) \\ \dot{e}_C(t) \end{bmatrix} = \begin{bmatrix} A_C - B_C K_C & 0 & -B_C K_C \\ -B_R K_C & A_R & -B_R K_C \\ 0 & LC_R & A_C - LC_C \end{bmatrix} \begin{bmatrix} w_C(t) \\ w_R(t) \\ e_C(t) \end{bmatrix} + \begin{bmatrix} E_C \\ E_R \\ E_C \end{bmatrix} \ddot{x}_g \qquad (15)$$

Note that the term $-B_R K_C$ in Eq. (15) is responsible for the excitation of the residual modes by the control forces and is known as control spilloveralas. If C_R is zeros, which means the sensor signal only include controlled modes, the term $-B_R K_C$ has no effect on the eigenvalues of the closed-loop system. Hence, we conclude that control spillover cannot destabilize the system, although it can cause some degradation in the system performance. Normally, however, the above system can not satisfy the separate principle because the term LC_R affects eigenvalues of the controlled system by the observer. This effect is known as observation spillover and can produce instability in the residual modes. However, a small amount of damping inherent in the structure is often sufficient to overcome the observation spillover effect. At any rate, observation spillover can be eliminated if the sensor signals are prefiltered so as to screen out the contribution of the uncontrolled modes (Meirovitch, 1990)

Numerical Example

To evaluate the proposed modal control scheme for use with the MR damper, a numerical example is considered in which a model of a six-story building is controlled with four MR dampers (Fig. 1). This numerical example is the same with that of Jansen and Dyke (2000) and is adopted for direct comparisons between the proposed modal control scheme and other control algorithms. In simulation, the model of the structure is subjected to the NS component of the 1940 El Centro earthquake. Because the building system considered is a scaled model, the amplitude of the earthquake was scaled to ten percent of the full-scale earthquake. The various control algorithms were evaluated using a set of evaluation criteria based on those used in the second generation linear control problem for buildings (Spencer and Sain 1997) such as

$$J_1 = \max_{t,i}\left(\frac{|x_i(t)|}{x^{\max}}\right),\quad J_2 = \max_{t,i}\left(\frac{|d_i(t)/h_i|}{d_n^{max}}\right),\quad J_3 = \max_{t,i}\left(\frac{|\ddot{x}_{ai}(t)|}{\ddot{x}_a^{\max}}\right),\quad J_4 = \max_{t,i}\left(\frac{|f_i(t)|}{W}\right)$$

where $x_i(t)$ is the relative displacement of the ith floor over the entire response, x^{max} denotes the uncontrolled maximum displacement. h_i is the height of each floor (30cm), $d_i(t)$ is the interstory drift of the above ground floors over the response history, $d_n^{\max}$ denotes the normalized peak interstory drift in the uncontrolled response, $\ddot{x}_{ai}(t)$ is the absolute accelerations of the ith floor, $\ddot{x}_a^{\max}$ is the peak uncontrolled floor acceleration, and W is the total weight of the structure (1335 N). The corresponding uncontrolled responses are as follows: $x^{\max}$ = 1.313 cm, $d_n^{\max}$ = 0.00981 cm, $\ddot{x}_a^{\max}$ (t) = 146.95 cm/sec^2.

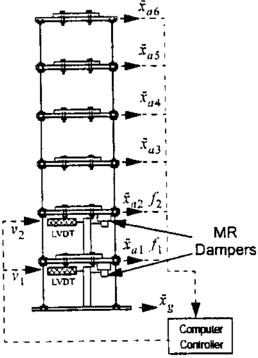

FIGURE 1
SCHEMATIC DIAGRAM
(Jansen and Dyke 2000)

The resulting evaluation criteria are presented in Table 1 for the control algorithms previously studied (Jansen and Dyke, 2000). The numbers in parentheses indicate the percent reduction as compared to the best passive case. To compare the performance of the semiactive system to that of comparable passive systems, two cases are considered in which MR dampers are used in a passive mode by maintaining a constant voltage to the devices. The results of passive-off (0 V) and passive-on (5 V) configurations are included.

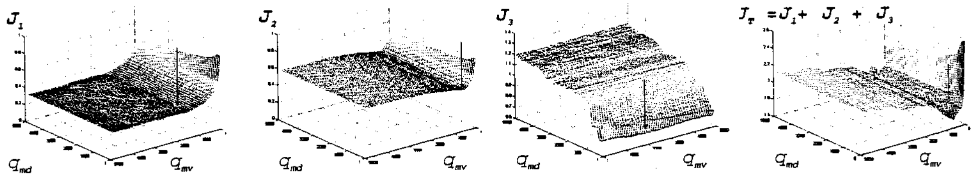

FIGURE 2 VARIATIONS OF EVALUATION CRITERIA WITH WEIGHTING PARAMETERS FOR THE ACCELERATION FEEDBACK

For modal control, three cases of the structural measurements are considered; displacements, velocities and accelerations. Using each structural measurement, a Kalman filter estimates the modal states. Fig. 2 represents the variations of each evaluation criteria for increasing weighting parameters in a 3-dimensional plot. J_T is the summation of evaluation criteria, J_1, J_2 and J_3. We can find the weighting for reduction of overall structural responses from the variations of J_T, whereas we can find the weighting for reduction of related responses from J_1, J_2 and J_3, Designer can decide which to use according to control objectives. By using the controller (H2/LQG) with designed weighting matrices from Fig. 2, we can get the results in Table 2. Similarly, for the displacement and velocity feedback cases, Table 2 summarize the results for each minimum evaluation criteria of the designed weighting matrices.

TABLE 1*

NORMALIZED CONTROLLED MAXIMUM RESPONSES
DUE TO THE SCALED EL CENTRO EARTHQUAKE

Control strategy	J_1	J_2	J_3	J_4
Passive-off	0.862	0.801	0.904	0.00292
Passive-on	0.506	0.696	1.41	0.0178
Lyapunov controller A	0.686(+35)	0.788(+13)	0.756(−16)	0.0178
Lyapunov controller A	0.326(−35)	0.548(−21)	1.39(+53)	0.0178
Decentralized bang-bang	0.449(−11)	0.791(+13)	1.00(+11)	0.0178
Maximum energy dissipation	0.548(+8)	0.620(−11)	1.06(+17)	0.0121
Clipped-optimal A	0.631(+24)	0.640(−8)	0.636(−29)	0.01095
Clipped-optimal B	0.405(−20)	0.547(−21)	1.25(+38)	0.0178
Modified homogeneous friction	0.421(−17)	0.599(−20)	1.06(+17)	0.0178

(* Jansen and Dyke 2000)

TABLE 2

NORMALIZED CONTROLLED MAXIMUM RESPONSES OF THE VARIOUS FEEDBACK DUE TO THE SCALED EL CENTRO EARTHQUAKE

Control strategy	Weighting parameters	J_1	J_2	J_3	J_4
Modal control $\mathbf{A}_{J1}$	qmd=400, qmv=1500	0.310(-39)	0.529(-24)	1.07(+18)	0.0178
Modal control $\mathbf{A}_{J2}$	qmd=1, qmv=500	0.398(-21)	0.485(-30)	0.870(-4)	0.0178
Modal control $\mathbf{A}_{J3}$	qmd=2200, qmv=100	0.549(+8)	0.618(-11)	0.697(-23)	0.0176
Modal control $\mathbf{A}_{JT}$	qmd=500, qmv=600	0.380(-25)	0.488(-30)	0.823(-9)	0.0178
Modal control $\mathbf{D}_{J1}$	qmd=100, qmv=4900	0.403(-20)	0.560(-20)	0.765(-15)	0.0177
Modal control $\mathbf{D}_{J2}$	qmd=100, qmv=4900	0.403(-20)	0.560(-20)	0.769(-15)	0.0178
Modal control $\mathbf{D}_{J3}$	qmd=200, qmv=4900	0.702(+39)	0.728(+5)	0.671(-26)	0.0178
Modal control $\mathbf{D}_{JT}$	qmd=3300, qmv=4700	0.408(-19)	0.566(-19)	0.721(-20)	0.0178
Modal control $\mathbf{V}_{J1}$	qmd=700, qmv=800	0.327(-35)	0.554(-20)	1.06(+17)	0.0178
Modal control $\mathbf{V}_{J2}$	qmd=1, qmv=400	0.383(-24)	0.487(-30)	0.874(-3)	0.0177
Modal control $\mathbf{V}_{J3}$	qmd=1300, qmv=100	0.541(+7)	0.611(-12)	0.632(-30)	0.0178
Modal control $\mathbf{V}_{JT}$	qmd=600, qmv=500	0.354(-30)	0.502(-28)	0.825(-9)	0.0176

CONCLUSIONS

In this paper, modal control was implemented to seismically excited structures using MR dampers. To this end, a modal control scheme was applied together with a Kalman filter and a low-pass filter. A Kalman filter considered three cases of the structural measurement to estimate modal states: displacement, velocity, and acceleration, respectively. Moreover, a low-pass filter was used to eliminate spillover problem.

Modal control reshapes the motion of a structure by merely controlling a few selected vibration modes. Hence, in designing phase of controller, the size of weighting matrix Q was reduced because the lowest one or two modes were controlled. Therefore, it is more convenient to design the smaller weighting matrix of modal control. This is one of the important benefits of the proposed modal control scheme. The numerical results show that the motion of the structure was effectively suppressed by merely controlling a few lowest modes, although resulting responses varied greatly depending on the choice of measurements available and weightings. The modal controller **A** and **V** achieved significant reductions in the responses. The modal controller $\mathbf{A}_{J1}$, $\mathbf{A}_{J2}$ and $\mathbf{V}_{J3}$ achieve reductions (39%, 30%, 30%) in evaluation criteria J_1, J_2 and J_3, respectively, resulting in the lowest values of all cases considered here. The modal controller AJT and VJT fail to achieve any lowest value of evaluation criteria, but have competitive performance in all evaluation criteria. Based on these results, the proposed modal control scheme is found to be suited for use with MR dampers in a multi-input control system. Further studies are underway to examine the influence of the number of controlled modes on the control performance.

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