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HYBRID CONTROL STRATEGY FOR SEISMIC PROTECTION OF BENCHMARK CABLE-STAYE BRIDGE

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ABSTRACT:

This paper presents a hybrid control strategy for seismic protection of a benchmark cable-stayed bridge, which is provided as a *testbed* structure for the development of strategies for the control of cable-stayed bridges. In this study, a hybrid control system is composed of a passive control system to reduce the earthquake-induced forces in the structure and an active control system to further reduce the bridge responses, especially deck displacements. Lead rubber bearings are used for the passive control part and an H_2/LQG control algorithm is adopted for the active control part. Numerical simulation results show that the performance of the proposed hybrid control strategy is superior to that of the passive control strategy and slightly better than that of the active control strategy. The proposed control method is also more reliable than the fully active control method due to the passive control part. Therefore, the proposed hybrid control strategy can effectively be used to seismically excited cable-stayed bridges.

1. Introduction

A cable-stayed bridge has become a popular type of bridges throughout the world because of its aesthetic shape, structural efficiency, and economical construction. However, such a structure might be vulnerable to strong earthquake excitations due to its large flexibility and low damping ratios. Structural control systems, such as passive, active, semiactive or a combination thereof, could provide an efficient means for seismic protection of cable-stayed bridges, but the control of such type of bridge is a new, unique and challenging problem because those structures are very large and flexible.

Under the coordination of the ASCE Committee on Structural Control, Dyke *et al.*^[1] developed the first generation of benchmark structural control problems for seismically excited cable-stayed bridges to investigate the effectiveness of various control strategies. In this study, a hybrid control strategy for the seismic protection of a cable-stayed bridge is investigated by using this ASCE first generation benchmark bridge model. The hybrid control strategy is composed of a passive control system to reduce the earthquake-induced forces in the structure and an active control system to further reduce the bridge responses, especially deck displacements, are employed. Following a summary of the benchmark problem statement, a seismic control system using the hybrid control strategy is proposed. Then, numerical simulation results are presented to demonstrate the effectiveness of the proposed control strategy.

2. Benchmark Problem Statement

For completeness, this section briefly summarizes the benchmark cable-stayed bridge problem, including discussion of the bridge model, ground excitations considered, and evaluation criteria, that was developed under the coordination of the ASCE Task Committee

on Structural Control Benchmarks. More details can be found in reference [1] and at the website <http://wusceel.cive.wustl.edu/quake/>.

This benchmark problem considers the cable-stayed bridge shown in Fig. 1, which is scheduled for completion 2003 in Cape Girardeau, Missouri, USA. In this benchmark study, only the cable-stayed portion of the bridge is considered, because the Illinois approach has a negligible effect on the dynamics of the cable-stayed portion of the bridge. Based on detailed drawings of the bridge, Dyke *et al.* developed and made available a three-dimensional linearized evaluation model that effectively represents the complex behavior of the full-scale benchmark bridge. The stiffness matrices used in this linear model are those of the structure determined through a nonlinear static analysis corresponding to the deformed state of the bridge with dead loads. Because this bridge is assumed to be attached to bedrock, the effect of the soil-structure interaction has been neglected. A one-dimensional ground acceleration is applied in the longitudinal direction for seismically excited cable-stayed bridges. The bridge model resulting from the finite element formulation, which is modeled by beam elements, cable elements, and rigid links, has a large number of degrees-of-freedom and high frequency dynamics.

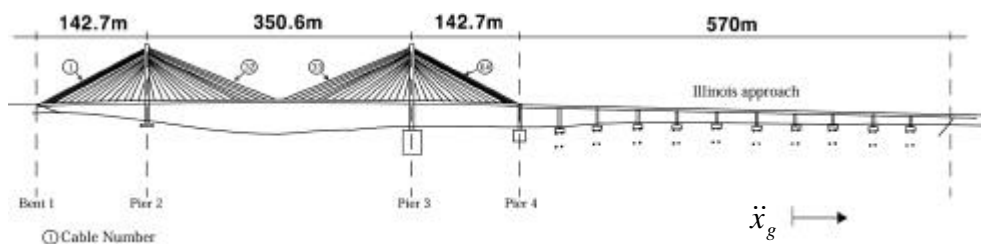


Fig. 1 Schematic of the Cape Girardeau Bridge^[1]

Application of static condensation to the full model of the bridge as a model reduction scheme resulted in a 419 DOF reduced-order model, designated the evaluation model. Each mode of this evaluation model has 3% of critical damping, which is consistent with assumptions made during the design of bridge. The first ten frequencies of the evaluation model for the uncontrolled system are 0.2899, 0.3699, 0.4683, 0.5158, 0.5812, 0.6490, 0.6687, 0.6970, 0.7102, and 0.7203 Hz. The deck-tower connections in this model are fixed (*i.e.*, the dynamically stiff shock transmission devices are present). On the other hand, another evaluation model should be formed in which the connection between the tower and the deck are disconnected to place control devices acting longitudinally. The first ten frequencies of this second model are 0.1618, 0.2666, 0.3723, 0.4545, 0.5015, 0.5650, 0.6187, 0.6486, 0.6965, and 0.7094 Hz, which are much lower than those of the nominal bridge model.

Three historical earthquake records are considered as ground excitations for numerical simulations of seismic protective systems installed in the bridge: i) 1940 El Centro NS; ii) 1985 Mexico City; and iii) 1999 Turkey Gebze NS. The Mexico City earthquake is selected because geological studies have indicated that the Cape Girardeau region is similar to Mexico City. The El Centro and Turkey Gebze earthquakes allow to test proposed control strategies on earthquakes with different characteristics. These three earthquakes are each at or below the design peak ground acceleration level of 0.36g for the bridge. Eighteen criteria have been defined to evaluate the capabilities of each proposed control strategy. The first six evaluation criteria are peak responses of the bridge to consider the ability of the controller. The second five evaluation criteria consider normed responses over the entire simulation time. The last seven evaluation criteria consider the requirements of each control system itself. More details can be found in the reference [1].

3. Seismic Control System using Hybrid Control Strategy

The current level of active control devices was not ready to immediately apply to disaster mitigation strategies for severe earthquakes^[6]. To enhance the safety of structures against severe earthquake, more advanced active control strategies with the principle of less energy and better performance should be urgently developed. On the other hand, the h

ybrid control system can operate well due to the passive control part even if the active control part may not work. Therefore, the hybrid system is more reliable and effective than the passive or active control method alone. In this section, a description of the proposed hybrid control strategy is provided. *Bouc-Wen* model^[3] is used to consider the nonlinear behavior of passive control devices, *i.e.*, lead rubber bearing (LRB) and an H_2/L QG control algorithm^{[7],[8]}, which was used for sample controller in the benchmark study, is employed for the active control part of hybrid control strategy.

3.1. Control Devices

3.1.1 Passive Control Devices

In the hybrid control strategy, passive devices have a great role for the effectiveness of the control method. In this study, conventional base isolation devices such as LRBs are used. The bearings fabricated using rubber and lead offer a simple method of passive control and are relatively easy and inexpensive to manufacture. The design of passive control device follows a general and recommended procedure^[2]. In the design procedure, the asymptotic stiffness ratio of the bearings at the bent and tower are assumed to be 1.0. The design shear force level for the yielding of lead plugs is taken to be 0.10M, where M is the part of the deck weight carried by bearings. As the results of the design procedure, a total of 24 LRBs are placed between the deck and pier/bent. Six LRBs are installed between the each deck and pier/bent. The properties of LRBs are shown in Table 1 and these LRBs are installed after removing the horizontal stiffness of beam element in pier 4.

Nonlinear model proposed by Wen^[3] is used to simulate the motion of nonlinear dynamics of the LRB. The restoring force of the model is composed of the linear and the nonlinear terms as

$$F_{LRB}(x_r, \dot{x}_r) = \mathbf{a}k_0x_r + (1 - \mathbf{a})k_0D_y y \quad (1)$$

where k_0 and \mathbf{a} are the linear stiffness and its contribution to restoring force, x_r and \dot{x}_r are relative displacement and relative velocity of nodes which LRBs are installed, respectively. And D_y and y are the yield displacement of LRB and the variable, respectively, satisfying the following equation:

$$\dot{y} = \frac{1}{D_y} \left(A_i \dot{x}_r - \mathbf{g} |\dot{x}_r| |y|^{n-1} |y| - \mathbf{b} \dot{x}_r |y|^n \right) \quad (2)$$

where A_i , \mathbf{g} , \mathbf{b} and n are the constant that affect the hysteretic behavior. The values of $A_i=n=1$ and $\mathbf{a}=\mathbf{b}=0.5$ are used to simulate the characteristic curve of the LRB in this study.

Table 1 The Properties of the LRB

Property	Value
Elastic stiffness, k_e (N/m)	3.571×10^7
Plastic stiffness, k_p (N/m)	3.139×10^6
Yield displacement of lead plugs, D_y (cm)	0.765
Design shear force level for the yielding of lead plugs, Q_i (kg)	2.540×10^4

3.1.2 Active Control Devices

In this study, a total of 24 hydraulic actuators (HAs), which are used in the benchmark problem, are employed^[1]. Eight between the deck and pier 2, eight between the deck and pier 3, four between the deck and bent 1, and four between the deck and pier 4. The actuators have a capacity of 1000 kN. Actuator dynamics are neglected and the actuator is considered to be ideal. The equations describing the forces produced by the actuators are

$$\mathbf{f} = \mathbf{K}_f \mathbf{u} = \mathbf{G}_{dev} \mathbf{D}_d \mathbf{u} = \begin{bmatrix} 2\mathbf{I}_{2 \times 2} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & 4\mathbf{I}_{4 \times 4} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & 2\mathbf{I}_{2 \times 2} \end{bmatrix} \mathbf{D}_d \mathbf{u} \quad (3)$$

$$\mathbf{y}_f = \mathbf{D}_d \mathbf{u} = D_d \mathbf{I}_{8 \times 8} \mathbf{u} \quad (4)$$

where \mathbf{f} is the force output of devices applied to the structure, \mathbf{y}_f is the force output of devices used for feedback in the control algorithm, $D_d=100\text{kN/V}$ is the device gain, and \mathbf{K}_f is a matrix that accounts for the gain of the relationship between the input voltage and the desired control force.

Five accelerometers and four displacement sensors are employed for feedback. Four accelerometers are located on top of the tower legs, and one is located on the deck at mid span. Two displacement sensors are positioned between the deck and pier 2 and two displacement sensors are located between the deck and pier 3. All sensor measurements are obtained in the longitudinal direction to the bridge and are assumed to be ideal, having a constant magnitude and phase^[1]. The sensors can be modeled as

$$\mathbf{y}_s = \mathbf{D}_s \mathbf{y}_m \dot{\mathbf{1}} + \quad (5)$$

where \mathbf{y}_s is a vector of the measured absolute accelerations and device displacements in Volts,

\mathbf{y}_m is the vector of measured continuous-time absolute accelerations and device displacement in physical units, and \mathbf{v} is the measurement noise, which has an *rms* value of 0.003 V. Sensor gain matrix \mathbf{D}_s is

$$\mathbf{D}_s = \begin{bmatrix} \mathbf{I}_{5 \times 5} G_a & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{4 \times 4} G_d \end{bmatrix} \quad (6)$$

where $G_a=0.714 \text{ V}/(\text{m}/\text{sec}^2)$ is the sensor gain for acceleration and $G_d=30 \text{ V}/\text{m}$ is the displacement sensor gain.

3.2 Control Design Model

A reduced order model of the system is developed for control design, which is formed from the evaluation model and has 30 states. This model obtained by forming a balanced realization of the system and condensing out the states with relatively small controllability and observability grammians^[4]. The resulting state space system is represented as follows

$$\dot{\mathbf{x}}_d = \mathbf{A}_d \mathbf{x}_d + \mathbf{B}_d \mathbf{u} + \mathbf{E}_d \ddot{\mathbf{x}}_g \quad (7)$$

$$\mathbf{z} = \mathbf{C}_d^z \mathbf{x}_d + \mathbf{D}_d^z \mathbf{u} + \mathbf{F}_d^z \ddot{\mathbf{x}}_g \quad (8)$$

$$\mathbf{y}_s = \mathbf{C}_d^y \mathbf{x}_d + \mathbf{D}_d^y \mathbf{u} + \mathbf{F}_d^y \ddot{\mathbf{x}}_g + \quad (9)$$

where \mathbf{x}_d is the design state vector, $\ddot{\mathbf{x}}_g$ is the ground acceleration, \mathbf{u} is the control command input, and \mathbf{z} is the regulated output vector including evaluation outputs (*i.e.*, shear force and moments in the tower, deck displacements, and cable tension forces, *etc.*).

3.3 Control Algorithm

In this study, an H_2 /LQG control design is adopted for the active control part. For this design, \dot{x}_g is taken to be a stationary white noise, and an infinite horizontal cost function is chosen as

$$J = \lim_{t \rightarrow \infty} \frac{1}{t} E \left[\int_0^t \{ \mathbf{z}^T \mathbf{Q} \mathbf{z} + \mathbf{u}^T \mathbf{R} \mathbf{u} \} dt \right] \quad (10)$$

where \mathbf{R} is an identity matrix of order 8, and \mathbf{Q} is the response weighting matrix. Further, the measurement noise is assumed to be identically distributed, statistically independent *Gaussian* white noise process, and the ratio of autospectral density function is 25.

In the optimal control such as LQG, obtaining the appropriate weighting parameters is very important to get well-performed controllers. In this study, the maximum response approach is used as follows: i) select the responses which could be considered as the important responses for the overall behaviors of the bridge as shown in Table 2; ii) perform the simulations in each parameter with varying the value of the parameter and determine the appropriate weighting parameters and combination; iii) perform the additional simulation in the combination of the weighting parameters selected in the previous step and finally select the appropriate values of each weighting parameters.

Table 2 The Selected Responses for Optimal Weighting Parameters

Responses	Corresponding weighting parameters
base shears at piers 2 and 3	q_{bs}
shears at deck level at piers 2 and 3	q_{sd}
overturning moments at base of piers 2 and 3	q_{om}
moments at deck level at piers 2 and 3	q_{md}
deck displacements at bent 1 and pier 4	q_{dd}
top displacements at towers 1 and 2	q_{td}

Consequently, the following combination and values of weighting parameters are obtained through the abovementioned approach for active and hybrid control systems:

$$\mathbf{Q}_{om_dd} = \begin{bmatrix} q_{om} \mathbf{I}_{4 \times 4} & \mathbf{0} \\ \mathbf{0} & q_{dd} \mathbf{I}_{4 \times 4} \end{bmatrix} \quad (11)$$

where q_{om} and q_{dd} are overturning moment-weighted and deck displacement-weighted parameters, respectively. Fig. 2 shows the selection of appropriate optimal weighting parameters by three-dimensional analysis.

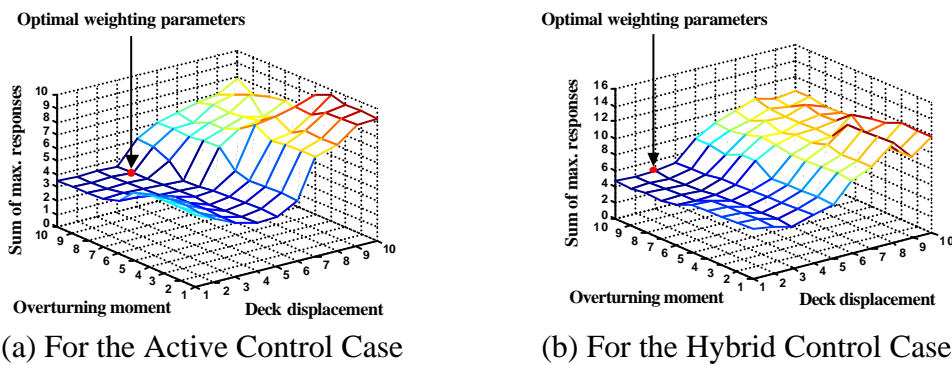


Fig. 2 Selection of Appropriate Optimal Weighting Parameters

The values of the appropriate optimal weighting parameters are $q_{om}=4 \times 10^{-9}$, $q_{dd}=1 \times 10^4$ for the active control system and $q_{om}=5 \times 10^{-9}$, $q_{dd}=1 \times 10^5$ for the hybrid control system.

4. Numerical Simulation Results

A set of numerical simulations is performed for the three historical earthquakes to verify the effectiveness of the hybrid control strategy. Simulation results of the hybrid control design are compared to those of a passive and an active control designs. Tables 3 to 5 show the values of eighteen evaluation criteria for each earthquake. While the controller presented Dyke *et al.*^[1] is not intended to be competitive control design, the associated performance indices are given in these tables for the readers' reference.

Table 3 Evaluation Criteria for the 1940 El Centro NS Earthquakes

Criterion	Dyke <i>et al.</i> ^[1]	Passive Control	Active Control	Hybrid Control
J ₁ - peak base shear	0.3868	0.3976	0.2713	0.2642
J ₂ - peak shear at deck level	1.0681	1.1846	0.7899	0.7227
J ₃ - peak overturning mom.	0.2944	0.3054	0.2545	0.2298
J ₄ - peak mom. at deck level	0.6252	0.6077	0.4597	0.3828
J ₅ - peak dev. of cable tension	0.1861	0.2077	0.1474	0.1462
J ₆ - peak deck displacement	1.2006	1.4250	1.0059	0.7461
J ₇ - normed base shear	0.2257	0.2304	0.2001	0.1978
J ₈ - normed shear at deck level	1.1778	1.0909	0.7164	0.6933
J ₉ - normed overturning mom.	0.2665	0.2473	0.2010	0.1882
J ₁₀ - normed mom. at deck level	0.8813	0.7128	0.5122	0.4953
J ₁₁ - normed dev. of cable tension	2.2968e-2	2.2327e-2	1.6176e-2	1.8215e-2
J ₁₂ - peak control force	1.5887e-3	1.3436e-3	1.9608e-3	LRB+HA: 2.6438e-3 LRB: 9.2908e-4 HA: 1.9608e-3
J ₁₃ - peak stroke	0.7883	0.9356	0.6604	0.4899
J ₁₄ - peak power	2.7011e-3	-	4.5696e-3	3.3152e-3
J ₁₅ - peak total power	4.2871e-4	-	7.2528e-4	7.1034e-4
J ₁₆ - no. of control devices	24	24	24	LRB+HA: 24+24
J ₁₇ - no. of sensors	9	-	9	9
J ₁₈ - no. of resources	30	-	30	30

Table 4 Evaluation Criteria for the 1985 Mexico City Earthquakes

Criterion	Dyke <i>et al.</i> ^[1]	Passive Control	Active Control	Hybrid Control
J ₁ - peak base shear	0.4582	0.5459	0.5071	0.4854
J ₂ - peak shear at deck level	1.3693	1.1097	0.9095	0.9270
J ₃ - peak overturning mom.	0.5836	0.6188	0.4485	0.4471
J ₄ - peak mom. at deck level	0.6140	0.4468	0.4154	0.3521
J ₅ - peak dev. of cable tension	7.7483e-2	4.8766e-2	4.4979e-2	4.6071e-2
J ₆ - peak deck displacement	2.3317	2.0197	1.6661	1.0797
J ₇ - normed base shear	0.3983	0.4211	0.3755	0.3723
J ₈ - normed shear at deck level	1.2118	0.9634	0.7700	0.7315
J ₉ - normed overturning mom.	0.4192	0.3989	0.3563	0.3336
J ₁₀ - normed mom. at deck level	1.1067	0.6536	0.6912	0.5246
J ₁₁ - normed dev. of cable tension	1.0276e-2	5.1805e-3	6.2671e-3	6.3367e-3
J ₁₂ - peak control force	5.7440e-4	7.7631e-4	1.2201e-3	LRB+HA: 1.0851e-3 LRB: 6.4325e-4 HA: 7.5552e-4
J ₁₃ - peak stroke	1.1742	1.0171	0.8390	0.5473
J ₁₄ - peak power	1.7523e-3	-	2.6195e-3	1.0937e-3
J ₁₅ - peak total power	2.3343e-4	-	3.4895e-4	1.9669e-4
J ₁₆ - no. of control devices	24	24	24	LRB+HA: 24+24
J ₁₇ - no. of sensors	9	-	9	9
J ₁₈ - no. of resources	30	-	30	30

As seen from the tables, the overall performances of the hybrid control system are superior to those of the passive control system and are slightly better than those of the active control system. The deck displacements of the structure with LRBs are larger than other control strategies. However, the increased deck displacements are still less than the allowable displacement (30 cm)^[5] and are decreased by additional active devices in the hybrid control

method. Tension in the stay cables remains within a recommended range of allowable values in the considered control strategies.

Table 5 Evaluation Criteria for the 1990 Gebze Earthquakes

Criterion	Dyke <i>et al.</i> ^[1]	Passive Control	Active Control	Hybrid Control
J ₁ - peak base shear	0.4540	0.4230	0.4139	0.3791
J ₂ - peak shear at deck level	1.3784	1.4616	1.1576	0.9360
J ₃ - peak overturning mom.	0.4434	0.5012	0.3419	0.2848
J ₄ - peak mom. at deck level	1.2246	1.2656	0.8792	0.6719
J ₅ - peak dev. of cable tension	0.1481	0.1598	9.0146e-2	9.5259e-2
J ₆ - peak deck displacement	3.5640	3.8289	1.8023	1.6629
J ₇ - normed base shear	0.3231	0.3340	0.2951	0.2773
J ₈ - normed shear at deck level	1.4271	1.5502	0.9510	0.9169
J ₉ - normed overturning mom.	0.4552	0.4815	0.3507	0.3240
J ₁₀ - normed mom. at deck level	1.4569	1.4429	0.7618	0.7799
J ₁₁ - normed dev. of cable tension	1.7052e-2	1.7129e-2	8.9002e-3	1.0398e-2
J ₁₂ - peak control force	1.7145e-3	2.1611e-3	1.9608e-3	LRB+HA: 2.4626e-3 LRB: 1.2246e-3 HA: 1.7824e-3
J ₁₃ - peak stroke	1.9540	2.0993	0.9886	0.9118
J ₁₄ - peak power	7.3689e-3	-	9.3311e-3	6.6678e-3
J ₁₅ - peak total power	6.9492e-4	-	8.7997e-4	8.4888e-4
J ₁₆ - no. of control devices	24	24	24	LRB+HA: 24+24
J ₁₇ - no. of sensors	9	-	9	9
J ₁₈ - no. of resources	30	-	30	30

In the case of the hybrid control system, all the structural responses (J₁ ~ J₄, J₆ ~ J₁₀) are decreased by 14% to 45% (under El Centro earthquake), 11% ~ 24% (under Mexico City earthquake), and 10% ~ 57% (under Gebze earthquake) compared to the passive control system.

The structural responses with the hybrid control system under El Centro earthquake are decreased by 1% ~ 26% compared to the active control system. All the structural responses except the peak shear at deck level (J₂) are decreased by 0.3% ~ 35% under Mexico City earthquake. J₂ is increased by 2%. In the case of Gebze earthquake, the structural responses are decreased by 4% ~ 24%, whereas the normed base moment is (J₉) increased by 2%.

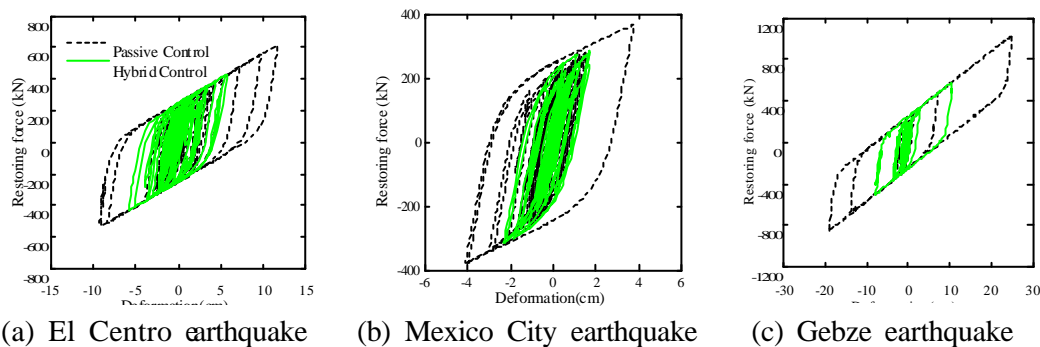


Fig. 3 Restoring Force of LRB at Pier 2

To demonstrate the feasibility of these controllers, peak values of the force, stroke, and velocity are provided for each earthquake in Table 6. The force, stroke, and velocity requirements presented Dyke *et al.*^[1] are 1000 kN, 0.2 m, and 1 m/sec. As seen from Table 6, all the three maximum responses satisfy the actuator requirements in the active and hybrid control cases and the values of the hybrid control system is smaller than the active control system.

Table 6 Actuator Requirements for Control Strategies

Earthquake	Max.	Dyke <i>et al.</i> ⁽⁷⁾	Active Control	Hybrid Control
1940 El Centro NS	Force(kN)	810.26	1000	1000
	Stroke(m)	0.1172	0.0982	0.0728
	Vel. (m/s)	0.6846	0.5499	0.5323
1985 Mexico City	Force(kN)	292.94	622.23	385.31
	Stroke(m)	0.0567	0.0405	0.0263
	Vel. (m/s)	0.3243	0.2374	0.2043
1990 Gebze NS	Force(kN)	874.41	1000	909.03
	Stroke(m)	0.2563	0.1297	0.1196
	Vel. (m/s)	0.5620	0.4157	0.4223

5. Conclusions

In this paper, a hybrid control strategy, which is composed of a passive control system to reduce the earthquake-induced forces in the structure and an active control system to further reduce the bridge responses, especially deck displacements, has been proposed by investigating the ASCE first generation benchmark control problem for seismic responses of cable-stayed bridges. The proposed control design uses conventional base isolation devices such as LRBs for the passive control part. The *Bouc-Wen* model is used to simulate the nonlinear behavior of these devices. An H_2/LQG control algorithm is adopted for the active control part. The numerical results show that all the structural responses with the proposed hybrid control strategy except the peak shear at deck level under Mexico City earthquake and the normed base moment under Gebze earthquake are decreased by 0.3% ~ 35% compared to the active control strategy. In the comparison to the passive control system, all the structural responses are decreased by 10% ~ 57% because of the additional active control devices in the hybrid control system. The hybrid control strategy is also more reliable than the active control method due to the passive control part. Therefore, the proposed hybrid control strategy can effectively be used to seismically excited cable-stayed bridges.

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