ROBUST HYBRID CONTROL OF A CABLE-STAYED BRIDGE

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ABSTRACT : This paper presents a robust hybrid control system for seismic response control of a cable-stayed bridge. Because multiple control devices are operating, a hybrid control system could alleviate some of the restrictions and limitations that exist when each system is acting alone. Lead rubber bearings are used as passive control devices to reduce the earthquake-induced forces in the bridge and hydraulic actuators are used as active control devices to further reduce the bridge responses, especially deck displacement. Three kinds of control algorithm are used to improve the controller robustness. A LQG control algorithm is adopted as a primary controller in the first hybrid control system. In addition, a secondary on-off type control scheme according to the responses of lead rubber bearings is combined with the primary controller. The second hybrid control system uses an H_2 control algorithm with various frequency weighting filters. A Kanai-Tajimi spectrum filter type is considered to capture the characteristics of input earthquakes. Furthermore, a high-pass filter type is adopted to prevent the spillover effects caused by the difference error of the desiged and evaluation models especially in the high frequency region. Finally, a low-pass filter type is used for regulated outputs to use the active control force effectively. The last hybrid control system uses an H_∞ control algorithm instead of H_2 control algorithm in the sencond hybrid control system. Numerical simulation results show that control performances of three hybrid control systems are similar to those of the hybrid control system with LQG algorithm. Furthermore, it is verified that three hybrid control systems are more robust than the hybrid control system with LQG algorithm and there are no signs of instability in the overall system whereas the hybrid control system with LQG algorithm shows instabilities in the ±5% stiffness perturbed system. This robustness of hybrid control systems comes from passive control deives that is robust inherently and robust control algorithms for the active control devices.

KEYWORDS : Robust hybrid control system, lead rubber bearing, hydraulic actuator, LQG algorithm, H_2 algorithm, H_∞ algorithm, benchmark cable-stayed bridge.

1. INTRODUCTION

A hybrid control system is typically defined as one that employs a combination of passive and active devices. This system could alleviate some of restrictions and limitations that exist when each system is acting alone, because multiple control devices are operating. Thus, higher level of performance may be achievable and the resulitng hybrid control system may be more relaible than a fully active system, because the passive components still offer some degree of protection in the case of power failure. However, the robustness of hybrid control system could be decreased by the active control devices. Therefore, control algorithms that gaurantee the controller robustness should be considered to improve the overall system robustness of the hybrid control system.

In this study, a hybrid control system combining lead rubber bearings (LRBs) and hydraulic actuators (HAs) is used for seismic response control of a cable-stayed bridge. LRBs are used as passive control devices to reduce the earthquake-induced forces in the bridge and HAs are used as active control devices to further reduce the bridge responses, especially deck displacement (i.e., deformation of LRBs). Three kinds of control algorithm, i.e., a LQG algorithm with on-off type control scheme and H_2, H_∞ control algorithm with frequency weighting filters, are used to improve the controller robustness. The results of the proposed hybrid control system are compared with those of conventional hybrid control system (i.e., hybrid control system with LQG control algorithm) to verify the performance and robustness of a robust hybrid control system.

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2. HYBRID CONTROL SYSTEM

2.1. Control Device and Sensors

The LRB of Park et al. (2003) are used as passive control devices in this study. This LRB is designed by recommended procedure proposed by Ali and Abdel-Ghaffar (1995). For the active control devices, HAs, which are used in the benchmark problem (Dyke et al. 2003), are used. The actuator is assumed to have a capacity of 1000 kN without its dynamics.

For feedback in the control algorithm, five accelerometers and four displacement sensors are used. All sensor measurements are obtained in the longitudinal direction to the bridge and the sensor dynamics are neglected. More details about HAs and sensors can be found in Dyke et al. (2003).

2.2. Control Algorithms

A hybrid control system is more robust than the fully active control system because of the passive control part that is robust inherently. However, the robustness of overall system is decreased by the active control part. Therefore, three control algorithms are used for HAs of the active control part in the hybrid control system to improve the controller robustness. A LQG control algorithm is adopted as a primary controller in the first hybrid control system. In addition, a secondary on-off type control scheme according to the responses of LRBs is combined with the primary controller (RHCS I). The block diagram of RHCS I is shown in **Figure 1**.

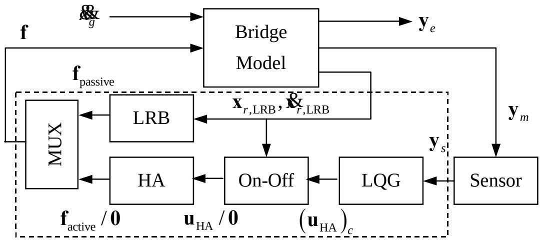
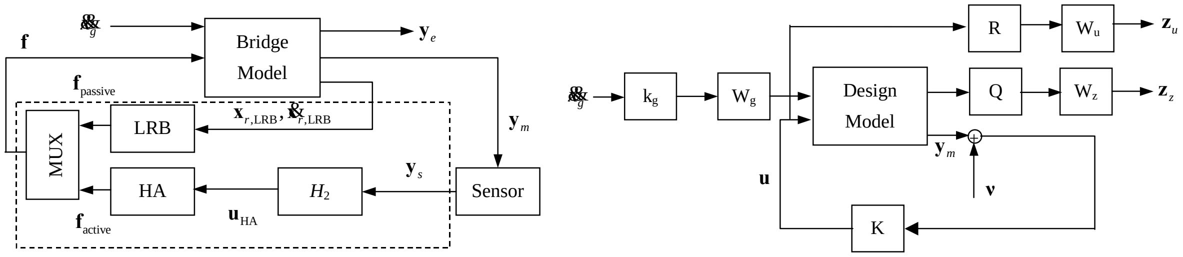


Figure 1. Block diagram of RHCS I

In the block diagram, $\mathbf{x}_{r,\mathrm{LRB}}$ and $\dot{\mathbf{x}}_{r,\mathrm{LRB}}$ are the relative displacement and velocity of nodes which LRBs installed, respectively. And $\mathbf{y}_e$, $\mathbf{y}_m$ and $\mathbf{y}_s$ are evaluation outputs, measured outputs and outputs from the sensor including noise, respectively. The acting conditions of HAs are determined as follows by extensive sensitivity analyses.

$$u_{\mathrm{HA},i} = \begin{cases} \left(u_{\mathrm{HA},i}\right)_c, & \left(x_{r,LRB}\right)_i \geq 0.005\mathrm{m} \text{ or } \left(\dot{x}_{r,LRB}\right)_i \geq 0.03\mathrm{m/s}^2 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

The second hybrid control system uses an H_2 control algorithm with various frequency weighting filters (RHCS II). **Figure 2** shows the block diagram of RHCS II.



(a) Hybrid control with H_2 controller (b) H_2 controller with various filters

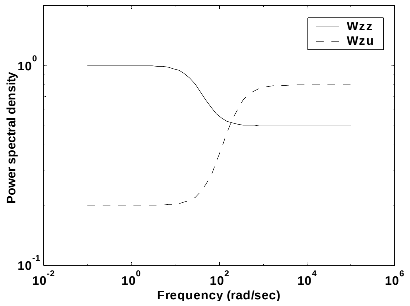
Figure 2. Block Diagram of RHCS II

In the LQG algorithm, the earthquake excitation is taken to be a stationary white noise. However, the energy of earthquakes is concentrated in low frequency domain in general as shown in **Figure 3(a)**. Therefore, a Kanai-Tajimi spectrum filter type with $\omega_g = 17$ rad/sec, $\xi_g = 0.3$, $S_0 = 2.5044$ is considered to capture the characteristics of input earthquakes.

Generally, a controller is designed based on the reduced-order model (i.e., design model) formulated from the full-order model (i.e., evaluation model). The quality of the design model depends on how closely its response matched those of the evaluation model. In this study, the reduction process of the less controllable and observable states resulted in large differences at high frequencies. Therefore, a high-pass filter type is adopted to prevent the spillover effects caused by the differences of the designed and evaluation models. Finally, a low-pass filter type is used for regulated outputs to use the active control force effectively. These control force and regulated output weights are shown in **Figure 3(b)**.

The third hybrid control system uses an H_∞ control algorithm instead of the H_2 control algorithm in RHCS II (RHCS III).

In the considered control algorithms, obtaining the appropriate weighting parameters is very important to get well-performed controllers. In this study, the *maximum response approach* (Park et al. 2003) is used.



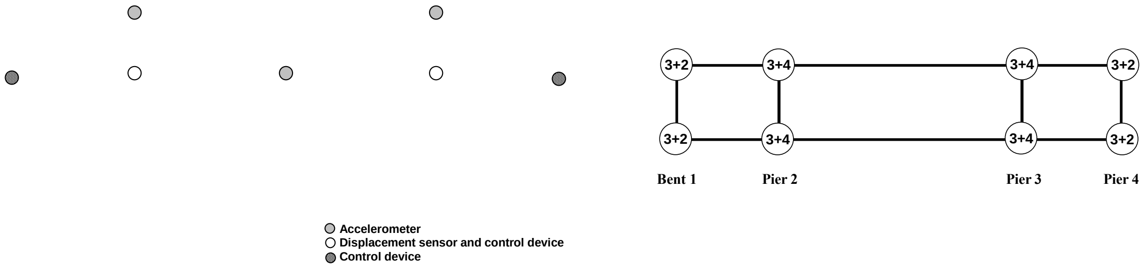
(a) Earthquakes and Kanai-Tajimi filter (b) Control force and regulated output filters

Figure 3. Power Spectral Density of Various Filters

3. NUMERICAL EXAMPLES

3.1. Bridge Model

The bridge model considered in this study is that of a benchmark control problem (Dyke et al. 2003) which is provided as a testbed structure for the development of strategies for the control of cable-stayed bridges. This benchmark bridge is composed of two towers, 128 cables, and 2 additional piers as shown in **Figure 4(a)**. **Figure 4(b)** shows the configuration of control devices.



(a) Location of control devices and sensors (b) Control device configuration (LRBs+HAs)

Figure 4. Schematic of the Bill Emerson Memorial Bridge and Location of Control Devices and Sensors

A three-dimensional linearized evaluation model is used to effectively represent the complex behavior of the full-scale benchmark bridge. However, the stiffness matrices used in this linear model are those of the structure determined through a nonlinear static analysis corresponding to the deformed state of the bridge with dead load (Wilson, and Gravelle 1991). Each mode of this evaluation model has 3% of critical damping, which is consistent with assumptions made during the design of bridge.

3.2. Analysis Results

3.2.1. Control Performances

Table 1 shows the maximum values of eighteen evaluation criteria (Dyke et al. 2003) for all three earthquakes. In **Table 1**, conventional hybrid control system (CHCS) proposed by Park et al. (2003) is also considered to compare the control performance.

Table 1. Maximum Evaluation Criteria for All Three Earthquakes

Criterion	CHCS (Park et al. 2003)	RHCS I	RHCS II	RHCS III
J_1- peak base shear	0.4841	0.4810	0.5319	0.4808
J_2- peak shear at deck level	0.9476	0.9508	0.9607	0.9579
J_3- peak overturning mom.	0.4444	0.4426	0.5057	0.4380
J_4- peak mom. at deck level	0.6750	0.6739	0.6441	0.5750
J_5- peak dev. of cable tension	0.1468	0.1469	0.1252	0.1473
J_6- peak deck displacement	1.6702	1.6787	1.0652	1.1923
J_7- normed base shear	0.3744	0.3749	0.3929	0.3514
J_8- normed shear at deck level	0.9261	0.9352	0.7868	0.9301
J_9- normed overturning mom.	0.3345	0.3389	0.3590	0.3132
J_{10}- normed mom. at deck level	0.7806	0.8055	0.5404	0.7458
J_{11}- normed dev. of cable tension	1.819e-2	1.769e-2	1.275e-2	1.782e-2
J_{12}- peak control force	2.643e-3	2.642e-3	2.924e-3	2.901e-3
J_{13}- peak stroke	0.9157	0.9204	0.5695	0.6537
J_{14}- peak power	9.099e-3	6.692e-3	9.477e-3	9.479e-3
J_{15}- peak total power	1.158e-3	8.519e-4	8.937e-4	8.939e-4
J_{16}- no. of control devices	48	48	48	48
J_{17}- no. of sensors	9	9	9	9
J_{18}- no. of resources	30	30	48	48

As shown in **Table 1**, the control performances of the proposed hybrid control system (RHCS I, RHCS II and RHCS III) are similar to those of CHCS. The peak deck displacement (J_6) of RHCS II and RHCS III is decreased by 36% and 29%, respectively compare to that of CHCS. And the peak stroke of control devices (J_{13}) is decreased by 38% and 27% compared to that of CHCS. These reductions in RHCS II and RHCS III are because of the low-pass filter in the regulated output. However, the number of resources (J_{18}) is increased from 30 to 48 because of filters.

To demonstrate the feasibility of controller, peak values of the force, stroke, and velocity are provided for each earthquake in **Table 2**. The force, stroke, and velocity requirements presented by Dyke et al. (2003) are 1000 kN, 0.2 m, 1 m/sec. As seen from the **Table 2**, all the maximum values satisfy the actuator requirements in the considered control systems.

Table 2. Actuator Requirements for Control Systems

Earthquake	Max. values	CHCS (Park et al. 2003)	RHCS I	RHCS II	RHCS III
El Centro	Force (kN)	1000	1000	1000	1000
	Stroke (m)	0.0735	0.0734	0.0771	0.0576
	Vel. (m/sec)	0.5332	0.5327	0.5568	0.4455
Mexico City	Force (kN)	398	362	370	639
	Stroke (m)	0.0262	0.0273	0.0259	0.0204
	Vel. (m/sec)	0.2096	0.2000	0.1707	0.1687

Gebze	Force (kN)	920	923	1000	1000
	Stroke (m)	0.1201	0.1207	0.0747	0.0857
	Vel. (m/sec)	0.4219	0.4157	0.4157	0.4426

3.2.2. Controller robustness

The evaluation model with the robust hybrid control systems produces desirable results based on the performance criteria set by Dyke et al. (2003). It is expected that the same controller might also have good performance when it will be connected to the real bridge. However, the dynamics of the real bridge may not be expected to be identical to the evaluation model. Furthermore, the reduced-order model that is used in designing the controller has large differences at high frequencies compare to the evaluation model. These differences may introduce the spillover effects in the overall system. Therefore, the robustness of the proposed hybrid control systems is investigated with respect to the uncertainties of the stiffness parameters. The stiffness matrix is perturbed by a small amount, and the resulting bridge model is simulated with the controller designed for the nominal system.

The maximum variation of evaluation criteria with the ±5% stiffness perturbed system for all three earthquakes are summarized in **Table 3**.

Table 3. Max. Variations of Evaluation Criteria for All Three Earthquakes (%, ±5% Perturbation)

Criterion	**CHCS (Park et al. 2003)**	**RHCS I**	**RHCS II**	**RHCS III**
J_1- peak base shear	9.75	10.34	9.20	7.69
J_2- peak shear at deck level	16.62	16.26	4.42	14.34
J_3- peak overturning mom.	16.38	15.97	4.93	5.01
J_4- peak mom. at deck level	4.46	5.37	6.21	8.91
J_5- peak dev. of cable tension	13.08	14.22	13.96	15.68
J_6- peak deck displacement	7.51	4.06	1.48	3.52
J_7- normed base shear	50.00	6.54	6.12	7.02
J_8- normed shear at deck level	139.17	7.94	4.93	10.68
J_9- normed overturning mom.	39.94	5.98	5.54	10.36
J_{10}- normed mom. at deck level	42.15	10.37	7.56	21.82
J_{11}- normed dev. of cable tension	41.32	18.65	13.78	30.31

The maximum variations of normed responses of CHCS are large (e.g., J_8=139.17%) and these indicate that the CHCS does not work in the stiffness matrix perturbation (i.e., may not work in the real bridge). In the case of robust hybrid control systems, major changes in response do not occur in the perturbed system. For ±5% stiffness perturbation, RHCS II is more robust that other systems.

Table 4. Max. Variations of Evaluation Criteria for All Three Earthquakes (%, ±20% Perturbation)

Criterion	**RHCS II**	**RHCS III**
J_1- peak base shear	36.51	27.33
J_2- peak shear at deck level	22.93	38.66
J_3- peak overturning mom.	33.08	30.86
J_4- peak mom. at deck level	34.48	40.75
J_5- peak dev. of cable tension	50.07	31.97
J_6- peak deck displacement	5.02	18.86
J_7- normed base shear	31.78	29.98
J_8- normed shear at deck level	39.33	35.21
J_9- normed overturning mom.	29.70	32.17
J_{10}- normed mom. at deck level	45.34	33.66
J_{11}- normed dev. of cable tension	72.35	47.83

Table 4 shows the robust analysis results for the ±20% stiffness perturbation. In this Table, the results of CHCS and RHCS I are not presented because these systems fail to ensure the controller robustness. As shown in **Table**

4, RHCS III is more robust that RHCS II. From the **Table 3 and 4**, RHCS II shows good robustness for a small amount of perturbation (±5%) whereas RHCS III shows good robustness for a large amount of perturbation (±20%).

4. CONCLUSIONS

In this paper, a hybrid control system, which is composed of lead rubber bearings to reduce the earthquake-induced forces in the bridge and hydraulic actuators to further reduce the bridge responses, especially deck displacement, has been proposed by investigating the benchmark control problem for seismic responses of cable-stayed bridges. The LQG algorithm with on-off type control scheme, H_2 and H_∞ control algorithm with frequency weighting filters are used for hydraulic actuators to improve the overall robustness. The numerical simulation results show that robust hybrid control systems have excellent robustness for stiffness perturbation without loss of control performances. Therefore, the proposed robust hybrid control systems could effectively be used to seismically excited cable-stayed bridges.

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6. REFERENCES

Park, K. S., Jung, H. J., Park, J. G., and Lee, I. W., (2003), “Integrated Passive-Active System for Seismic Protection of a Cable-Stayed Bridge”, *Journal of Earthquake Engineering*, Vol. 7, No. 3, 1-19.

Ali, H. M., and Abdel-Ghaffar, A. M., (1995), “Seismic Passive Control of Cable-Stayed Bridges”, *Shock and Vibration*, Vol. 2, No. 4, 259-272.

Dyke, S. J., Turan, G., Caicedo, J. M., Bergman, L. A., and Hague, S., (2003), “Phase I Benchmark Control Problem for Seismic Response of Cable-Stayed Bridges”, *Journal of Structural Engineering,* ASCE, Vol. 129, No. 7, 857-872.

Wilson, J., and Gravelle, W., (1991), “Modeling of a Cable-Stayed Bridge for Dynamic Analysis”, *Earthquake Engineering and Structural Dynamics*, Vol. 20, 707-721.