GEOMETRIC NONLINEAR ANALYSIS OF UNDERGROUND LAMINATED COMPOSITE PIPES

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ABSTRACT

An analytical study was conducted using the Galerkin technique to determine the behaviour of thin fibre-reinforced and laminated composite pipes under soil pressure. Geometric nonlinearity and material linearity have been assumed. It is assumed that vertical and lateral soil pressure are proportional to the depth and lateral displacement of the pipe respectively. It is also assumed that radial shear stress is negligible because the ratio of thickness to the radius of pipe is very small. The above results are verified by the finite element analysis.

INTRODUCTION

Filament wound composite pipes have been used for small diameter water pipes, large diameter sewage pipes, grain storage tanks, rocket propellent containers, flow lines, salt water handlings, petrochemical plant facilities, structural frames of large structures, pressure vessels for military and aerospace applications, and many others. Compared with metal pipes, composite pipes have the following advantages.

Corrosion resistance - Maintenance free
Lightweight - Economical to install
Excellent flow properties
Paraffin build-up resistance
Scale build-up resistance
Nonconductive
Low thermal conductivity

In case of line pipe construction, cost of transporting pipes can be significant : it could be as much as or more than the cost of pipes, depending on the length of the pipeline. This transportation effort could become the major concern of the project if the size of the pipes is large. Because of its lightweight nature, composite pipes can be extensively used for pipeline construction. When pipe is buried, the surrouding soil and the pipe interact structurally, and the forces acting on the pipe are functions of the medium that surrouds it as well as the stiffness of the pipe itself. The contrubution of the surrounding soil in resisting external loads can be very important and can provide considerable saving in pipe material. The conventional recommendations for design and construction of underground pips are based on the extensive work of Marston and Spangler[1]. However, the use of this method for flexible pipes may cause high safety factors[2]. Earlier works on this subject by others include these of Meyerhof[3], Timmers[4], Brockenbrough[5], Valentine[6], et al. Costes[7] suggests to use arching theory to reduce the soil load directly above the pipe. Richards and Agrawal[8] applied Barjansky's tunnel solution by transforming the pipe section to an equivalent ring of soil. The loading on the cross section of the buried pipes is assumed to come from two sources : (A) The weight of backfill soil and any top loading present. The magnitude of this vertical forces, q_v, is the weight of backfill soil modified by the effect of shear stress due to settlement of backfill, plus the pressure due to any line load present, usually calculated by Boussinesq solution. (B) The force of the soil against the pipe sides which tends to prevent deformations caused by the (A) type force (see Figure 2). The assumption generally made here is that this force, q_v , is proportional to the horizontal pipe deflection, δ_h , and can be expressed as $K\delta_h$ where K is called as subgrade reaction coeffecient or modulus of the foundation. Defining the value of K correctly is rather complex. Rubin[9], using inextensional cylindrical shell theory, and assuming that the vertical deformation is approximately equal to the negative of the horizonatal deformation, obtained the value of K as the function of rigidity and diameter of the pipe. However, according to Valentine's experiment[6], the magnitude of the horizontal deformation is between 0 to 80 percent of the vertical deformation. Furthermore, it has to be noted that K value is related to soil properties. Molin[10] adds the horizontal earth pressure at rest to the Spangler's concept. This may be acceptable if the surrounding media is clay. However, it is general practice to put granular material around the pipes and to compact it. If the pipe deflects to "near" maximum under above condition, when the horizontal thrust from soil becomes significant, the $K\delta_h$ value can be assumed to be "close" enough to the passive earth pressure of the soil. Szyszkowski and Glockner[12] reported result of nonlinear analysis of buried aluminum tubes. The model used is similar to that of Spangler with $K\delta_h$ value as passive earth pressure. In this paper, underground pipes made of laminated composites are considered. Because of the flexible nature of thin composite walls, the problem is geometrically nonlinear. Assuming the pipe section under consideration is sufficiently "far" from end or bent, the problem is considered as that of plane strain. Equilibrium equations are obtained from the deformed shape and are solved by Galerkin's method. It is assumed that the vertical load on the pipe is proportional to the depth of the backfill and horizontal load as proportional to the horizontal deformation. Any modification of loading can be made easily depending on actual soil condition and live loads, if necessary. Given constant value of such surrounding condition, a composite pipe has different stiffness depending on number of layers and fiber directions and so on. This results in dirfferent amount of deflection and different soil-structure interaction. This paper presents a method of nonlinear analysis of underground composite pipes and effect of variable factors, such as fiber orientations and different values of subgrade reaction coefficients, on soil-structure interaction.

ANALYSIS METHOD

1. Displacement Relation

Equations of strain and displacement with respect to orthogonal curvilinear coordinates are as follows [13][14].

$$
\begin{aligned}
\varepsilon_x &= \frac{1}{\alpha}\Big[u_x + \frac{\alpha_y v}{\beta} + \frac{\alpha_z w}{\gamma} + \frac{1}{2\alpha}\Big(u_x + \frac{\alpha_y v}{\beta} + \frac{\alpha_z w}{\gamma}\Big)^2 + \frac{1}{2\alpha}\Big(v_x - \frac{\alpha_y u}{\beta}\Big)^2 + \frac{1}{2\alpha}\Big(w_x - \frac{\alpha_z u}{\gamma}\Big)^2\Big] \\
\varepsilon_y &= \frac{1}{\beta}\Big[v_y + \frac{\beta_z w}{\gamma} + \frac{\beta_x u}{\alpha} + \frac{1}{2\beta}\Big(v_y + \frac{\beta_y w}{\gamma} + \frac{\beta_x u}{\alpha}\Big)^2 + \frac{1}{2\beta}\Big(w_y - \frac{\beta_z v}{\gamma}\Big)^2 + \frac{1}{2\beta}\Big(u_y - \frac{\beta_x v}{\alpha}\Big)^2\Big] \qquad (1) \\
\varepsilon_z &= \frac{1}{\gamma}\Big[w_z + \frac{\gamma_x u}{\alpha} + \frac{\gamma_y v}{\beta} + \frac{1}{2\gamma}\Big(w_z + \frac{\gamma_y u}{\alpha} + \frac{\gamma_y v}{\beta}\Big)^2 + \frac{1}{2\gamma}\Big(u_z - \frac{\gamma_x w}{\alpha}\Big)^2 + \frac{1}{2\gamma}\Big(v_z - \frac{\gamma_y w}{\beta}\Big)^2\Big]
\end{aligned}
$$

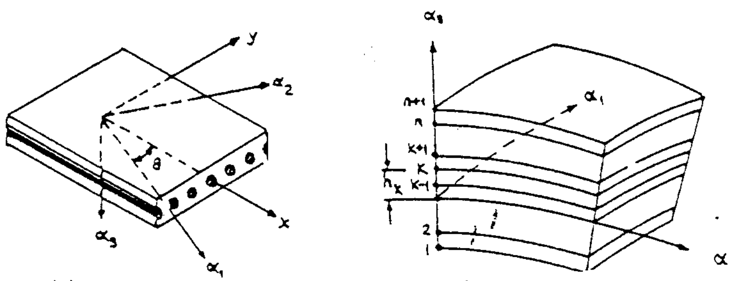
And where u,v,w are displacements with repect to axes and α, β, γ are Lame coefficients. When we ignore all square terms except it about w and transfer these to cylinderical coordinate, we can obtain the following relation between circumferential strain and displacements.

$$\varepsilon_\phi = \frac{1}{R}\Big[\Big(\frac{dv}{d\phi} - w\Big) - \frac{1}{2R}\Big(\frac{dw}{d\phi}\Big)^2\Big] \qquad (2)$$

where v, w are tangential and radial displacements. The positive of w is defined to be toward the center. According to inextention deformation, the displacements due to extention of the center line of a cylinder section are very samll in comparision with the displacements due to bending and usually can be neglected [15]

$$v = \int \left[w + \frac{1}{2R}\left(\frac{dw}{d\phi}\right)^2 \right] d\phi \tag{3}$$

2. Composite-pipe stiffness



(a) (b)

Fig. 1 Element of composite pipe (a) lamina (b) laminate

Consider an element of the composite pipe as shown in Fig. 1. It is assumed the thickness of the pipe is small relative to the radius and that deformation to the transverse direction varies linearly. The constitutive equtions for moment are as follows [16].

$$\begin{Bmatrix} M_{11} \\ M_{22} \\ M_{12} \end{Bmatrix} = [B] \begin{Bmatrix} \varepsilon^{\circ}_{11} \\ \varepsilon^{\circ}_{22} \\ \varepsilon^{\circ}_{12} \end{Bmatrix} + [D] \begin{Bmatrix} \kappa_{11} \\ \kappa_{22} \\ \kappa_{12} \end{Bmatrix} \quad \text{where} \quad \begin{aligned} B_{ij} &= \frac{1}{2}\sum_{k=1}^{n} (\overline{Q}_{ij})_k (h_{k+1}^2 - h_k^2) \\ D_{ij} &= \frac{1}{3}\sum_{k=1}^{n} (\overline{Q}_{ij})_k (h_{k-1}^3 - h_k^3) \end{aligned} \tag{4}$$

$$\begin{aligned} \overline{Q}_{11} &= U_1 + U_2\cos 2\theta + U_3\cos 4\theta \\ \overline{Q}_{22} &= U_1 - U_2\cos 2\theta + U_3\cos 4\theta \\ \overline{Q}_{12} &= U_4 - U_3\cos 4\theta = \overline{Q}_{21} \\ \overline{Q}_{33} &= U_5 - U_3\cos 4\theta \\ \overline{Q}_{13} &= -\frac{1}{2}U_2\sin 2\theta - U_3\sin 4\theta = \overline{Q}_{31} \\ \overline{Q}_{23} &= -\frac{1}{2}U_2\sin 2\theta + U_3\sin 4\theta = \overline{Q}_{32} \end{aligned} \qquad \begin{aligned} U_1 &= \frac{1}{8}(3Q_{11} + 3Q_{22} + 2Q_{12} + 4Q_{33}) \\ U_2 &= \frac{1}{2}(Q_{11} - Q_{22}) \\ U_3 &= \frac{1}{8}(Q_{11} + Q_{22} - 2Q_{12} - 4Q_{33}) \\ U_4 &= \frac{1}{8}(Q_{11} + Q_{22} + 6Q_{12} - 4Q_{33}) \\ U_5 &= \frac{1}{8}(Q_{11} + Q_{22} - 2Q_{12} + 4Q_{33}) \end{aligned} \qquad \begin{aligned} Q_{11} &= \frac{E_{xx}}{1 - \mu_{xy}\mu_{yx}} \\ Q_{22} &= \frac{E_{yy}}{1 - \mu_{xy}\mu_{yx}} \\ Q_{12} &= \frac{\mu_{yx}E_{yx}}{1 - \mu_{xy}\mu_{yx}} \\ Q_{21} &= \frac{\mu_{yx}E_{yy}}{1 - \mu_{xy}\mu_{yx}} \\ Q_{33} &= G_{xy} \end{aligned}$$

If the pipe section is made such that it is exectly symmetric about its middle surface, all components of the bending streching coupling matrix, [B], are vanished.

$$\begin{aligned} M_{11} &= D_{11}k_{11} + D_{12}k_{22} \\ M_{22} &= D_{12}k_{11} + D_{22}k_{22} \\ M_{12} &= D_{33}k_{12} \end{aligned} \tag{5}$$

3. Equilibrium Equation

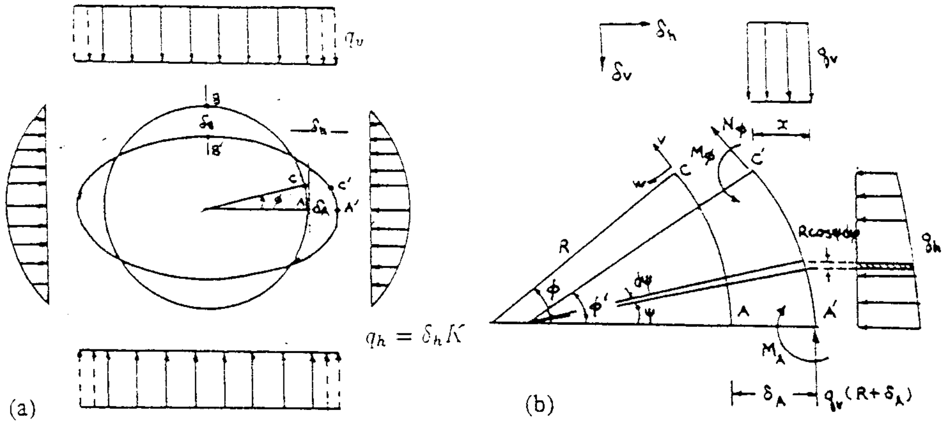


Fig. 2 Definition of loading and sign convention for conduit

$$
\begin{aligned}
\delta_h &= -v\sin\phi - w\cos\phi \\
\delta_v &= -v\cos\phi + w\sin\phi \\
x &= (R + \delta_A) - R\cos\phi - \delta_h \\
&= R(1-\cos\phi) + \delta_A + v\sin\phi + w\cos\phi
\end{aligned}
\tag{6}
$$

From Fig. 2 (b), equilibrium equation on deformed shape is as follows.

$$\Sigma M_c = 0$$

$$-M_\phi + M_A + q_v x\frac{x}{2} - q_v(R+\delta_A)x + \int_0^\phi \delta_h K(Rd\varphi\cos\phi)[R(\sin\phi - \sin\varphi)] = 0 \tag{7}$$

Substituting Eq. (6) into Eq. (7)

$$-M_\phi + M_A - \frac{q_v}{2}[R(1-\cos\varphi) + \delta_A + v\sin\varphi + w\cos\phi][R(1+\cos\varphi) + \delta_A - v\sin\phi - w\cos\phi]$$

$$+\int_0^\phi \delta_h KR^2(\sin\varphi - \sin\phi)]\cos\varphi d\varphi = 0 \tag{8}$$

Since the problem is assumed to be that of plane strain, the following relation can be obtained.

$$M_\phi = Dk_\phi \qquad \text{where} \quad \kappa_\phi = \frac{1}{R^2}\left(\frac{dv}{dw} - \frac{d^2w}{d\phi^2}\right) \tag{9}$$

$$D = D_{22} = \frac{1}{3}\sum_{k=1}^{n}(\overline{Q}_{22})_k(h_{k-1}^3 - h_k^3)$$

Since Eq. (8) is the differential Equation about w, Galerkin's method was used to obtain the solution.

$$\overline{w} = \frac{w}{R} = A_1\cos2\phi + A_2\cos4\phi \tag{10}$$

$$\int_0^{\frac{\pi}{2}} X\cos2n\phi = 0 \qquad n = 1,2 \tag{11}$$

And where X is left the side of Eq. (8). Substituting Eq. (3)(8)(9)(10) into (11), we can obtain the following nonlinear simultaneous equations.

$$\bar{q}[(1-A_1)^2-\frac{5}{4}A_2-\frac{3}{16}(\frac{A_1^2}{3}-6A_1A_2-\frac{5}{2}A_2^2)]=-12A_1(1+\frac{38.3}{210}\frac{\beta}{\pi})-A_2\frac{96.4}{360}\beta$$

$$\bar{q}[\frac{A_1}{8}-2A_2+\frac{3}{32}A_1^2+\frac{3}{8}A_1A_2]=15A_2(1+\frac{11.9}{1920}\frac{\beta}{\pi})+\frac{12.6}{385}A_1\frac{\beta}{\pi} \qquad (12)$$

where $\bar{q}=\frac{q_vR^3}{D}$; $\beta=\frac{KR^4}{D}$

Talyor's series is used to solve above equations to obtain the continuous displacement function expressed in trigonometic forms.

NUMERICAL EXAMPLE

A composite pipe made with Giass/Epoxy which has the following material property and geometry

$E_1=0.55\times10^6$ Kg/cm^2	R = 12.7 cm
$E_2=0.18\times10^6$ Kg/cm^2	4 layer with same thickness
$G_{12}=0.091\times10^6$ Kg/cm^2	total thickness = 0.203 cm
$\nu=0.25$	soil unit weight (γ) = 1995 Kg/m^3

is analysed. As results, Fig. 3 shows an effect of soil against pipe and nonlinearity of the displacement according to depth. Fig. 4 displays the verification of this buried composite pipe analysis method by FEM.

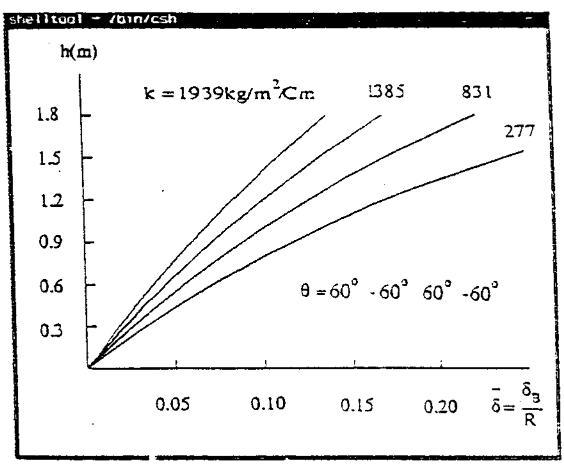


Fig. 3 Displacement according to depth and soil

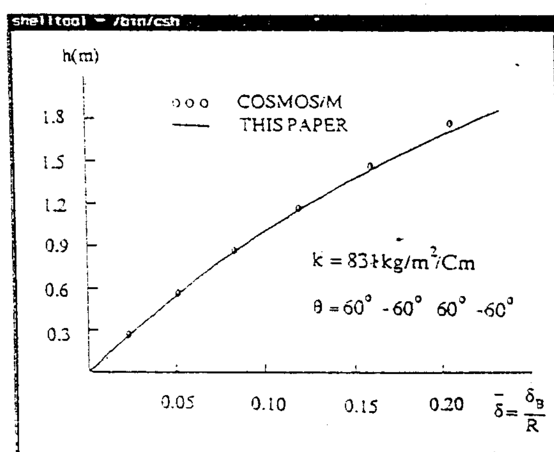


Fig. 4 Comparision with FEM

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STIFFNESS OF THIN-WALL OPEN-SECTION BARS MADE OF STRUCTURAL UNORTHOTROPIC REINFORCED PLASTIC

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ABSTRACT

The present paper advances the technique for determining elastogeometrical characteristics of an elementary strip (ES) in a thin-wall straight bar, which allows to consider the peculiarities of its wall deformation in the cross-section plane along the whole contour as well as in its seperate segments.

INTRODUCTION

For construction purposes a reinforced plastic laminate (RPL) structure which is unorthotropic in the bar axes, in some cases may prove more rational, e.g., for bars loaded asymmetrically to their longitudinal axis. Limited application of bars with structurally unorthotropic structure is explained by the absence of engineering theory for the calculations of their stress-strain states and elastogeometrical characteristics

RESULTS

The present paper analyzes the ESM an open-section thin-wall straight bar with a RPL cylindrical surface which is structurally unorthotropic but is symmetrical in relation to its middle surface structure (Fig.1). Structure and stress-strain states of RPL in each point of the ES are analyzed in the right-hand orthogonal coordinate system 1,2,3,4,5,6 beginning on the middle surface of the RPL and having the orientation of axes as shown in Fig.1. For general analysis of ES we introduce another orthogonal coordinate system X,Y, Z. The present research has been mainly aimed at determining the stress-strain states and elastogeometrical characteristics of thin-wall straight bars, considering their structure as well as the elastic characteristics of RPL, and defining the deformation law for ES. This law must find the correlation between longitudinal force N, flexural moments M_y, M_z, shear forces S_{yx}, S_{zx} and torsional moment m, acting in the cross-section of a thin-wall straight bar, and the corresponding deformations ε, K_y, K_z, γ_{yx}, γ_{zx} and θ', which are defined as follows:

$$\varepsilon=\mu_{o,x}; \quad K_y=-W_{o,x,x}; \quad K_z=-V_{o,x,x}; \quad \theta'=\theta_{ox}; \quad \gamma_{yx}=V_{o,x}; \quad \gamma_{zx}=W_{o,x} \qquad (1)$$

where μ_o, V_o, W_o are the displacements of the initial point of the coordinate system of the ES in directions X,Y,Z, respectively: θ_o-rotation of the given cross-section in counterclockwise direction around axis X. Referring to RPL we apply to the classic theory of laminated materials which allows to consider the middle surface instead of a real thin-wall straight bar. Disregard of local effects is possible owing to the following conditions with regard to curvature and strains of RPL:

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