JSME Centenial Grand Congressby the Japan Society of Mechanical Engineers
International Symposium on OPtimization and Innovative Design (OPID97)
July 28 th to July 30 th, 1997 at Tokyo International Forum
Edited by H. Yamakawa, M. Yoshimura, S. Morishita and M. Arakawa
Paper #146

MULTIOBJECTIVE OPTIMIZATION OF STRUCTURES USING MODIFIED ε - CONSTRAINT APPROACH

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The ε-constraint approach, a well-known multiobjective optimization technique is modified in efficient way. The algorithm of proposed approach is an ε -constraint approach, however the scheme treating secondary objective functions, and the procedure generating initial design points differ from the conventional approach. First, the starting points of multiobjective optimization is generated using the vectors obtained in each objective function optimization, then secondary objective functions are restricted to their initial function values. Second, primary objective function is optimized within the reduced design region using existing optimization technique such as feasible direction method, penalty method etc. As examples for demonstration and applicability of the proposed method, the multiobjective optimal designs of I-beam and steel box girder bridge are presented. In the examples, Pareto solutions are found using not only proposed approach but also constraint approach. Both approaches shows good agreement in Pareto curve, which means proposed approach to be valid in Pareto optimization.

INTRODUCTION

So far there have been many developments in the field of structural optimization. However it is not easy to apply the optimization techniques developed to the design of real structures. In real structural design there are many objectives (possibly conflicting) such as minimum cost, minimum deflection, maximum reliability, minimum dynamic response and so forth. So, it is necessary to consider simultaneously all types of objectives for the optimization of overall structural system. As an alternative approach to these practical problems, multiobjective optimization has been studied for decades and known to offer reasonable solutions.

Multiobjective optimization simultaneously optimizes all the objective functions considered within the design region defined by constraints. Usually because there are several competing objectives that have each optimal design value respectively, the results of multiobjective optimization cannot be further improved without impairing some of the objectives. The solution sets with this property are called the Pareto optimal solutions after Italian economist Pareto[1]. For three decades the Pareto concept was used in the engineering fields like operations research, control theory and structural design optimization.

Several approaches have been proposed to solve the multiobjective optimization problems: weighting method; ε-constraint approach; goal programming approach; game theory approach. The weighting method transforms the multiobjective function to a single-objective function through a set of relative weighting of the objective functions. The entire Pareto set can then be generated with the variation of the weights. However, because the characteristics often Pareto set are unknown, it is difficult to determine beforehand the variations of the weights. Both the game theory approach and the goal programming approach produce one optimal design which minimizes the newly defend criterion; supercriterion in the game theory; deviations from the set goals in the goal programming.

Among these, the c -constraint approach is known to be efficient in obtaining the Pareto optimal solutions. This approach was used by Cohn et al.[5] for the multiobjective optimization of prestressed concrete structures and by Carmichiael[6] for the multiobjective optimization of five bar planner truss. However, it is very difficult to select the initial design value inside the feasible region. To avoid this difficulty in practical work, optimization is usually conducted successively; the previous optimization result is used as the initial design value because this design value is in the feasible region anyway. Hence, the total solution time is increased linearly with the increased number of the Pareto solutions.

The main purpose of this paper is to obtain the Pareto optimal solutions in efficient way by improving the ε -constraint approach. When two or more objective functions exist, the most important objective function is adopted as the primary criterion and the other objective functions are transformed into the constraints by imposing upper or lower limits on them. The multiobjective optimization then can be treated as the single-objective optimization. If the feasible design region defined by the constraints is convex, the initial vectors are generated in the feasible region independently. So, the parallel processing can be used in the proposed multiobjective optimization technique. If the multiobjective optimization can be performed with the parallel processing technique, there is no solution time increase regardless often number often Pareto solutions.

The following sections of this paper deal with the ε-constraint approach and the proposed approach, and numerical examples are presented to demonstrate the validity and the applicability of the proposed approach.

MULTIOBJECTIVE OPTIMIZATION

The multiobjective optimization problem with more than two objective functions can be formulated as

$$\textit{Minimize} \quad F = [f_1(X), f_2(X), \cdots, f_m(X)] \tag{1}$$

$$\textit{subject to} \quad g_j(X) \le 0 \quad j = 1,2,\cdots,J \tag{2}$$

$$h_n(X) = 0 \quad n = 1,2,\cdots,N \tag{3}$$

where F is a vector of objective functions and f_i's are the objective functions to be minimized. Any optimization problem can be written as equations (1) to (3) since some objective functions to be maximized can be converted into objective functions to be minimized. Equations (2) and (3) represent inequality and equality conditions respectively. In general there is no single optimal solution that simultaneously minimizes all m objective functions. Instead, there is a set of solution, so called the Pareto optimal solutions as shown in Figure 1. The Pareto optimal solutions of the multiobjective optimization with two objective functions,

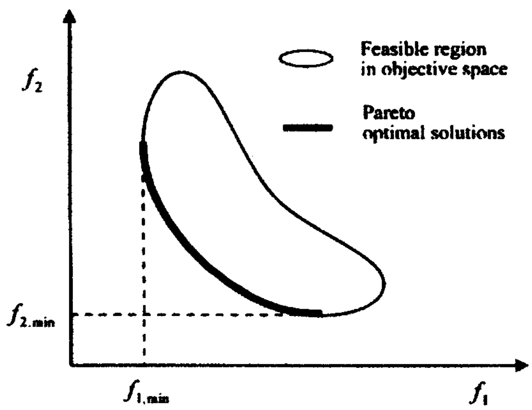


Figure 1. Pareto optimal solutions

f_1 and fa are on the bolded curve. If f_1 is to be increased, then f_2 must be decreased along the curve and vice versa.

This information may be very helpful in determining the final design. For example, one may decrease the most important objective function by increasing other less important objective functions. In addition. it helps engineer choose the levels of limits in case that some limits on the design values arena available in design code. Among several techniques, the 8 -constraint approach is known to be efficient in obtaining the Pareto optimal solutions. In the e -constraint approach, the multiobjective optimization problem showed in equations (1), (2) and (3) is transformed into a single-objective optimization problem as

$$\textit{Minimize} \quad f_p(X) \tag{4}$$

$$\textit{subject to} \quad f_i(X) \le \varepsilon_i \quad i = 1,2,\cdots,m(\ne p) \tag{5}$$

$$g_j(X) \le 0 \quad j = 1,2,\cdots,J \tag{6}$$

$$h_n(X) = 0 \quad n = 1,2,\cdots,N \;. \tag{7}$$

The strategy of this approach is very simple. Restricting (m-1) secondary objective functions with equation (5), single-objective optimization is conducted within the reduced design space. A Pareto optimum then will be found on the bolded curve as in Figure 1 . The next step optimization to find another Pareto optimum is done with a little bit increased upper limits and the current step Pareto optimum being the initial value, because the current Pareto optimum is in the feasible region. The adequate 8 values in equation (5) are bounded as

$$f_i(X_i^*) \le \varepsilon_i \le f_i(X_p^*) \quad i = 1,2,\cdots,m(\ne p) \tag{8}$$

where X_i* and X^*_p represent the optimal design vectors corresponding to the i-th and p-th objective functions respectively. Finally the desired number of the Pareto optimal solutions can be found in several repetitions of this process.

The inefficiency of this approach may be caused by the successive optimization. When a large number of the Pareto optimal solution are required in practical purpose, the total solution time is increased according to the number of the Pareto optimal solutions.

PROPOSED APPROACH

When a lot of Pareto optimal solutions are required, much computational effort is necessary in the ε-constraint approach because the optimization must be performed successively.

However, if initial values could be obtained independently, each Pareto optimal solution can be found independently by using parallel processing. To make this idea available, the proposed approach transforms equations (4) to (7) into equations (9) to (11) as follows.

$$\textit{Minimize} \quad f_p(X) \tag{9}$$

$$\textit{subject to} \quad f_i(X) \leq f_i(X_0) \quad i = 1,2,\cdots,m(\neq p) \tag{10}$$

$$g_j(X) \leq 0 \quad j = 1,2,\cdots,J \tag{11}$$

$$h_n(X) = 0 \quad n = 1,2,\cdots,N \tag{12}$$

where f_p is a primary objective function and f_i's are the secondary objective functions. That is, the upper bounds of the secondary objective functions are their initial function values. The initial vector X_0 in equation (10) is obtained by using the single-objective optimization results as in equations (13) and (14).

$$X_0 = \sum_{i=1}^{m} c_i X_i^* \tag{13}$$

$$\sum_{i=1}^{m} c_i = 1 \tag{14}$$

where X_i^* is a design vector which minimize $f_i(X)$ and c_i is arbitrary coefficient. One can easily show that the initial vector Xi is in the feasible design region, if the region defend by the constraint equations (11) and (12) is convex. Therefore the initial vector X_0 can be directly used in optimization technique like the modified feasible direction method. One of the key-points of this paper is that the initial vector can be obtained efficiently with above manner.

The solution scheme of the proposed approach is as follows; An initial vector is first produced in equations (13) and (14) with arbitrary c_i's. Then the transformed optimization problem of equations (9) to (12) is solved through the modified feasible direction method. Because the initial vector can be produced independently, the Pareto optimal solution can be also found independently. The efficiency of computational effort is high-lighted when this approach is combined with the parallel processing technique. The overall comparison between the ε-constraint and the proposed approach is described in Figure 2 and 3. The feasible design region is reduced by arbitrarily chosen ε value in the ε-constraint approach, and the Pareto solution is found with the initial vector being the Pareto solution found in the previous stage. The next Pareto solution can be found with a little bit increased 8 value and the current step Pareto solution. While in the proposed approach, the initial vectors are calculated by using the convex combination of vectors which are obtained in the single-objective optimzations. Then each initial vector determines the upper limits of secondary objective functions, and the desired number of the Pareto solution is found in parallel.

NUMERICAL EXAMPLES

Example 1 : I-beam design The I beam used in Reference 1 is adopted as the first example. It is to be designed for two objective functions; the cross sectional area and the midspan deflection of beam. The design variables are the web depth h, flange width b, web thickness t_w, and flange thickness t_f.

The constraints considered are strength constraints and geometric constraints only. The mathematical statement often optimum design problem can be written as

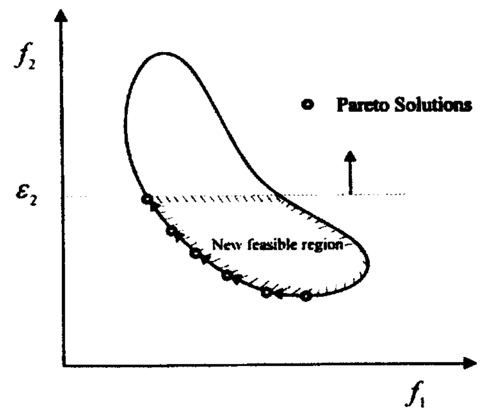


Figure 2. Strategy of the ε-constraint approach

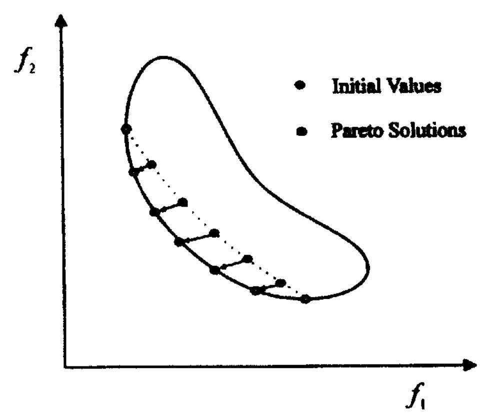


Figure 3. Strategy of the proposed approach

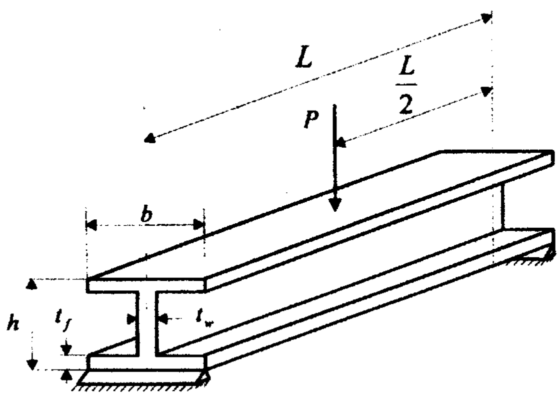


Figure 4. Simply supported I-beam

$$\textit{Minimize} \quad f_1(X) = 2bt_f + t_w(h - 2t_f) \qquad (15)$$

$$f_2(X) = PL^3 / 48EI \qquad (16)$$

$$\text{where } I = \frac{t_w(h-2t_f)^3 + 2bt_f[4t_f^2 + 3h(h-2t_f)]}{12} \qquad (17)$$

$$\textit{subject to} \quad \frac{Mc}{I} \le \sigma_a \qquad (18)$$

$$\frac{VQ}{It} \le \tau_a \qquad (19)$$

$$10 \le h \le 80, \quad 10 \le b \le 50 \qquad (20)$$

$$0.9 \le t_w \le 5, \quad 0.9 \le t_f \le 5 \qquad (21)$$

where P=600kN;L=200cm; E=204.08MPa; ,7., =1 86.4MPa; τ_a=107.8MPa, M and V represent the maximum moment and shear force respectively. Equations (18) and (19) represent the normal and shear stress constraints respectively. Equations (20) and (21) represent the geometric constraints to limit the design variables in centimeters.

The modified feasible direction method implemented in ADS was used to obtain the optimal design of each ob.jective function. The design vector which minimizes the objective function f_1 is X_1* =[h=73.21, b=13.44, t_w=0.9, t_f=0.9] in cm and $f_1(X_1^*)$ = 88.45 cm^2. At this point midspan deflection is $f_2(X_1^*)$ = 0.0849cm. The minimization of midspan deflection yields the optimal design vector X_2*=[h=80, b=50, t_w=5, t_f=5] in cm and f_2 (X_2^*)= 0.0059cm. At this point cross sectional area is $f_1(X_2^*)$ = 850.0cm^2. The set of Pareto optima is obtained by both the ε-constraint and the proposed approach. The previous Pareto point is used for the ε-constraint approach in which ε is increased equally, and equation (22) is used to obtain initial vector for the proposed approach. The results of the ε-constraint and the proposed approach are given in Table I and II, and plotted in Figure 5. Figure 5 shows that the proposed approach can give the Pareto optimal solutions on the curve.

$$X_0 = cX_1^* + (1-c)X_2^*, \qquad c \in [0,1] \qquad (22)$$

Example 2 : Steel box girder bridge design

The steel box girder bridge in Figure 6 is to be designed for the minimizations of both the cross sectional area and the maximum deflection of bridge. The bridge girder consists of three steel boxes and supports reinforced

Table I. Pareto optimal solutions of I-beam by the ε -constraint approach

$f_1(cm^2)$	$f_2(cm)$	$h(cm)$	$b(cm)$	$t_w(cm)$	$t_f(cm)$
290.60	0.0138	80.0	37.27	0.9	3.005
199.46	0.0217	80.0	27.92	0.9	2.359
157.78	0.0296	80.0	22.74	0.9	1.964
133.49	0.0375	80.0	23.01	0.9	1.390
118.27	0.0454	80.0	19.67	0.9	1.233
107.74	0.0533	80.0	15.51	0.9	1.223
99.96	0.0612	80.0	11.92	0.9	1.269
93.87	0.0691	80.0	10.42	0.9	1.148
89.00	0.0769	79.9	10.00	0.9	0.935

Table II. Pareto optimal solutions of I-beam by the proposed approach

c	$f_1(cm^2)$	$f_2(cm)$	$h(cm)$	$b(cm)$	$t_w(cm)$	$t_f(cm)$
0.1	560.40	0.00694	80.0	50.0	0.9	4.974
0.2	472.15	0.00812	80.0	50.0	0.9	4.078
0.3	395.87	0.00971	80.0	50.0	0.9	3.298
0.4	331.60	0.0118	80.0	45.17	0.9	3.000
0.5	275.02	0.0147	80.0	36.96	0.9	2.815
0.6	224.19	0.0187	80.0	32.00	0.9	2.447
0.7	180.00	0.0247	80.0	27.21	0.9	2.053
0.8	142.29	0.0342	80.0	22.13	0.9	1.656
0.9	110.74	0.0508	80.0	16.59	0.9	1.234

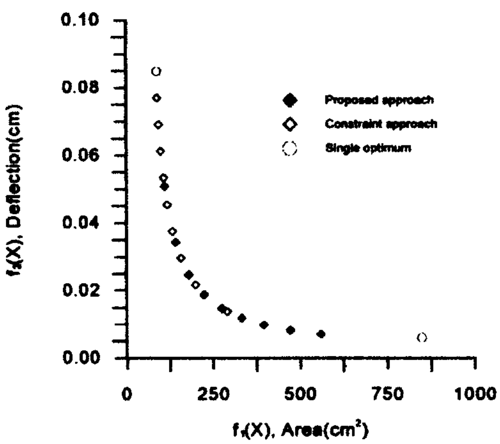


Figure 5. Pareto optimal solutions of I-beam

concrete slab on it. The bridge has four design lanes of 3.5m width each. All dead loads are included in the design of steel box. Live load by standard truck is described in Figure 6. The constraints include all requirements of the Korea Highway Bridge Design Code(1992). Design variables are the flange width B, web depth D, bottom flange thickness t_{bf}, upper flange thickness t_{uf} and web thickness t_w. Design loads are calculated through all the possible load cases and the influence lines over the span and width of bridge. The allowable stresses of steel are 1900kg/cm^2 in both compression(σ_{ca}) and tension (σ_{ta}), 1100kg/ cm^2 in shear (τ_a), and the Young's modulus of steel (E_s) is 2.1×10^6 kg/cm^2. Finally, multiobjectivc optimization problem can be formulated as

$$\text{Minimize} \quad f_1(X) = B(t_{bf} + t_{uf}) + 2Dt_w \tag{23}$$

$$f_2(X) = \Delta_{L+D}(X) \tag{24}$$

$$\text{subject to} \quad \frac{My_t}{I} - \sigma_{ta} \le 0 \tag{25}$$

$$\frac{My_c}{I} - \sigma_{ca} \le 0 \tag{26}$$

$$\frac{VQ}{It} - \tau_a \le 0 \tag{27}$$

$$\left[\frac{\sigma_m}{\sigma_a}\right]^2 + \left[\frac{\tau_m}{\tau_a}\right]^2 - 1.2 \le 0 \tag{28}$$

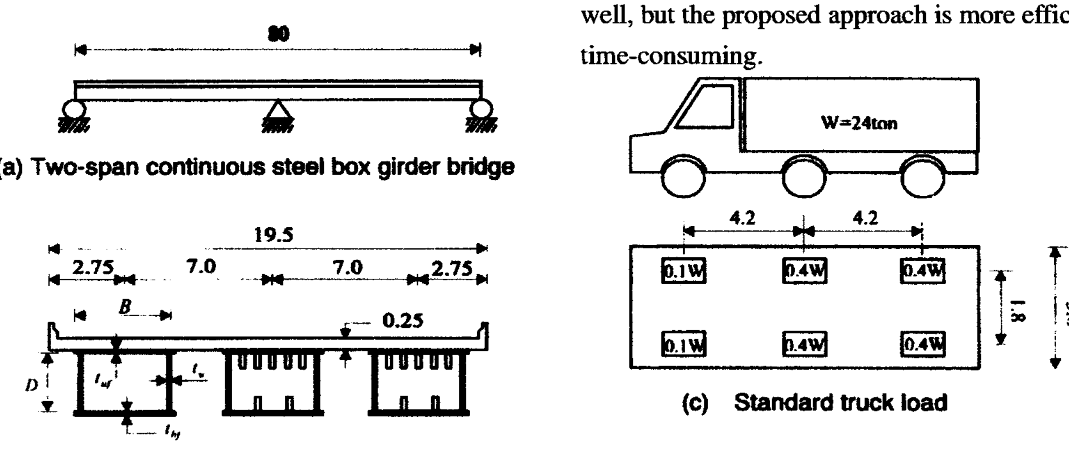
$$\frac{1}{80}(B - t_w - 20) - t_{uf} \le 0 \tag{30}$$

$$\frac{D}{130} - t_w \le 0 \tag{31}$$

where $f = 0.65(\phi/n)^2 + 0.13(\phi/n) + 1.0$ in which ϕ is the stress distribution factor, and n is the number of channel divided by ribs. The objective functions of equations (23) and (24) represent the cross sectional area of a steel box and the maximum deflection of bridge respectively. The constraints of equations (25) to (27) are to assure that the stresses in steel do not exceed their allowable limits. Equation (28) represents the stress combination constraint required in KHBDC. Equations (29) to (3 1) represent minimum thickness constraints of web and flanges. The geometric constraints for constructibility, machinability and repair are considered in the optimization; the lower limits of B and D are 200 and 180cm respectively, and those of t_{bf}, t_{uf} and t_w are1.0 cm's; the upper limits of both B and D are 300cm's, and those of t_{bf}, t_{uf} and t_w are 3 cm's.

The minimization of the cross sectional area of a steel box yields the optimal design vector X_1^* =[B=200.0; D=223.1; t_{bf}=2.464; t_{uf}= 207; t_w=1.716] in cm, and $f_1(X_1^*)$ = 1699.8 cm^2. At this point maximum deflection of bridge is $f_2(X_1^*)$ = 5.849cm. The minimization of the maximum deflection of bridge alone yields the optimal design vector X_2^*=[B=266.0; D=300.0; t_{bf}=3.0; t_{uf}=3.0; t_w=3.0] in cm, and $f_2(X_2^*)$ = 1.824cm. At this point cross sectional area of a steel box is $f_1(X_2^*)$ = 3378.0cm$_2$. Multiobjective optimization is done by both the ε-constraint and the proposed approach. The previous optimal design vectors are used for the ε-constraint approach in which ε is increased equally, and initial vectors are produced by equation (22) with nine different c values for the proposed approach. The results of multiobjective optimization are given in Table III and IV, and plotted in Figure 7.

In this example, the maximum deflection of bridge is under the allowable limit suggested in KHBDC(span/500=8cm). So the objective for the cross sectional area can be mostly decreased according to the curve in Figure 7. Both two approaches give nine Pareto optimal solutions well, but the proposed approach is more efficient and less time-consuming.



(a) Two-span continuous steel box girder bridge

(b) Cross section of bridge

(c) Standard truck load

Figure 6. Bridge Configurations and Truck Load [dimension in meter]

Table III. Pareto optimal solutions of the steel box girder bridge by the ε -constraint approach

$f_1(cm^2)$	$f_2(cm)$	$B(cm)$	$D(cm)$	$t_{bf}(cm)$	$t_{wf}(cm)$	$t_w(cm)$
2700.1	2.226	235.5	300.0	2.703	2.888	2.307
2413.8	2.629	206.3	300.0	2.390	2.600	2.307
2219.1	3.031	200.0	295.4	2.193	2.193	2.271
2096.4	3.434	200.0	281.4	2.196	2.196	2.164
1995.4	3.837	200.0	269.4	2.198	2.198	2.071
1910.8	4.239	200.1	258.8	2.201	2.201	1.990
1838.7	4.642	200.1	249.5	2.203	2.203	1.919
1776.4	5.045	200.1	241.2	2.204	2.204	1.855
1722.1	5.447	200.1	233.6	2.206	2.206	1.797

Table IV. Pareto optimal solutions of the steel box girder bridge by the proposed approach

c	$f_1(cm^2)$	$f_2(cm)$	$B(cm)$	$D(cm)$	$t_{bf}(cm)$	$t_{wf}(cm)$	$t_w(cm)$
0.1	2891.1	2.021	255.6	300.0	2.896	3.000	2.307
0.2	2684.5	2.246	234.4	300.0	2.679	2.869	2.307
0.3	2494.1	2.503	216.6	300.0	2.467	2.656	2.307
0.4	2318.5	2.797	200.0	300.0	2.244	2.427	2.307
0.5	2185.0	3.136	200.1	291.5	2.195	2.195	2.242
0.6	2071.3	3.527	200.0	278.4	2.197	2.197	2.141
0.7	1963.2	3.982	200.0	265.4	2.199	2.199	2.041
0.8	1860.7	4.511	200.0	252.4	2.202	2.202	1.941
0.9	1763.9	5.120	200.0	239.5	2.204	2.204	1.842

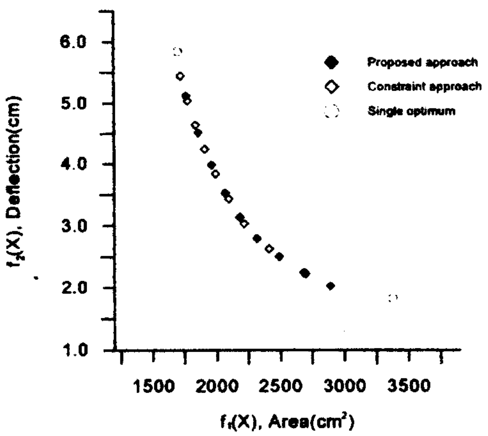


Figure 7. Pareto optimal solutions of a steel box girder

CONCLUSION

A new multiobjective optimization technique is proposed. It is shown that a large number of Pareto optimal solutions can be obtained efficiently with the proposed approach. Because the proposed approach generates an initial vector independently of the Pareto solution found in the previous stage, it is possible to adopt the parallel processing technique in multiobjective optimization. If the parallel processing technique is used in finding the Pareto solutions, the total solution time can be dramatically decreased.

Examples of I-beam and steel box girder bridge design show how the objective functions are sensitive to each other. Designer can choose the final design with this information. Especially, one can decrease important objective function by sacrificing less important objective functions in the choice level; the objective for maximum deflection is quite below the requirement of the Korea Highway Bridge Design Code in the steel box girder design example, so the objective for cross-sectional area can be minimized to its true minimum point while the objective for deflection is a little bit increased.

ACKNOWLEDGEMENT

The supports by the Korea Industrial Co. LTD. and the Korea Science and Engineering Foundation are gratefully acknowledged.

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