Mode Localization in Non-Periodic Structures

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Abstract

The mode localization phenomenon in non-periodic multispan beam is theoretically investigated. When localization occurs, the free vibration amplitude of a normal mode becomes confined to a local region of the structure. It is well known that the weakly coupled periodic structures are sensitive to certain types of periodicity-breaking disorder, resulting in the mode localization. The results of this study indicate that the mode localization occurs also in non-periodic structures and the degrees of mode localization of some modes are very sensitive to system parameters. Free vibration analysis of simply supported two-span beams of arbitrary span lengths is performed. Degrees of mode localization and their sensitivities to system parameters are appraised by considering the characteristic graph and the structural line defined in this study first.

Introduction

The natural frequencies and the mode shapes are the dynamic characteristics of the structural systems, which are functions of the geometric configuration and the material properties of the structures. The mode localization is a phenomenon that the magnitude of the specific part of the free vibrational mode is large relative to the rest of the mode.

Identifying the mode localization is very important to calculate the structural stability and to design the vibration controller. For some structures, the structural damages or the manufacturing errors may produce the undesirable mode localization. The dynamic characteristics may be changed extremely by the small changes in the system parameters when the mode localization occurs. This may be the reason why the performance of the controller decreases and why the stability changes. It is interesting that structures with localized modes is to possess passive motion confinement properties. If a structure is designed to have intentionally localized modes, the structure can be safer from the specific disturbances such as the adverse shock and vibration without supplementary control devices. It is, therefore, very important not only to calculate the natural frequencies and the mode shapes but to identify the degrees of localization and the localization sensitivities of the modes.

In the previous works[1-11] for mode localization, the disordered periodic structures are concerned. However, the mode localization phenomenon in the non-periodic structures has been passed over. In this paper, the mode localization phenomena are discussed both in the periodic structures and the non-periodic ones. The analysis of the free vibration and the mode localization in the simply supported two-span beam is theoretically investigated.

Free vibration of the two-span beam

Consider the two-span beam shown in Figure 1. The beam is simply supported at both ends, and is constrained to have zero deflection at $x = l_1$. Moreover, a torsional spring of K_R exerts a restoring moment at $x = l_1$. The system can be divide into two substructures.

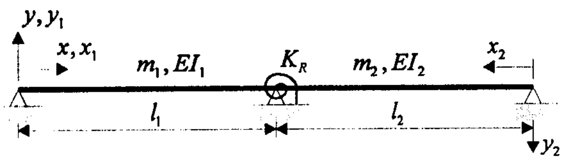

Figure 1. Simply supported two-span beam with rotational stiffness at mid support.

The torsional spring plays the role of a decoupler. As $K_R \to \infty$, the spans are fully decoupled from each other. For $K_R = 0$, the substructures are strongly coupled.

The eigenvalue problems for free bending vibrations of each substructure can be written as in equations (1) and (2),

$$EI_1 \frac{d^4 y_1}{d x_1^4} - \omega^2 m_1 y_1 = 0 \tag{1}$$

and

$$EI_2 \frac{d^4 y_2}{d x_2^4} - \omega^2 m_2 y_2 = 0 , \tag{2}$$

where EI_1 and EI_2 are flexural rigidity, m_1 and m_2 are masses per unit length of each substructure, respectively, ω is natural frequency of the system, and y_1 and y_2 are the transverse displacements of each substructure. The general solutions of equations (1) and (2) can be written as

$$y_1(x_1) = A_1 \sin \lambda_1 x_1 + B_1 \cos \lambda_1 x_1 + C_1 \sinh \lambda_1 x_1 + D_1 \cosh \lambda_1 x_1 \tag{3}$$

and

$$y_2(x_2) = A_2 \sin \lambda_2 x_2 + B_2 \cos \lambda_2 x_2 + C_2 \sinh \lambda_2 x_2 + D_2 \cosh \lambda_2 x_2 , \tag{4}$$

where

$$\lambda_1^4 = \omega^2 \frac{m_1}{EI_1}, \qquad \lambda_2^4 = \omega^2 \frac{m_2}{EI_2} . \tag{5, 6}$$

The boundary conditions are

$$y_1(0) = 0 , \qquad y_2(0) = 0 , \tag{7,8}$$

$$y_1(l_1) = 0 , \qquad y_2(l_2) = 0 , \tag{9, 10}$$

$$\frac{d^2 y_1(0)}{d x_1^2} = 0 , \qquad \frac{d^2 y_2(0)}{d x_2^2} = 0 \tag{11, 12}$$

$$\frac{d y_1(l_1)}{d x_1} = \frac{d y_2(l_2)}{d x_2} , \tag{13}$$

and

$$EI_1 \frac{d^2 y_1(l_1)}{d x_1^2} + EI_2 \frac{d^2 y_2(l_2)}{d x_2^2} = -K_R \frac{d y_2(l_2)}{d x_2} . \tag{14}$$

Applying those boundary conditions to equations (3) and (4) gives $B_1 = C_1 = B_2 = C_2 = 0$ and four algebraic equations for A_1, C_1, A_2 and C_2, which can be written in a matrix form as

$$\begin{bmatrix} \sin \lambda_1 l_1 & \sinh \lambda_1 l_1 & 0 & 0 \\ 0 & 0 & \sin \lambda_2 l_2 & \sinh \lambda_2 l_2 \\ \cos \lambda_1 l_1 & \cosh \lambda_1 l_1 & -\frac{\lambda_2}{\lambda_1} \cos \lambda_2 l_2 & -\frac{\lambda_2}{\lambda_1} \cosh \lambda_2 l_2 \\ -\sin \lambda_1 l_1 & \sinh \lambda_1 l_1 & -\frac{EI_2 \lambda_2^2}{EI_1 \lambda_1^2} \sin \lambda_2 l_2 + \frac{K_R \lambda_2}{EI_1 \lambda_1^2} \cos \lambda_2 l_2 & \frac{EI_2 \lambda_2^2}{EI_1 \lambda_1^2} \sinh \lambda_2 l_2 + \frac{K_R \lambda_2}{EI_1 \lambda_1^2} \cosh \lambda_2 l_2 \end{bmatrix} \begin{Bmatrix} A_1 \\ C_1 \\ A_2 \\ C_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{15}$$

Nontrivial solutions of equation (15) can be obtained if and only if the determinant of the its coefficient matrix vanishes. This gives an equation for the determination of natural frequencies, ω, which is called the frequency equation or characteristic equation:

$$K_R \left(\cos \beta_1 - \frac{\cosh \beta_1}{\sinh \beta_1} \sin \beta_1 \right) \left(\cos \beta_2 - \frac{\cosh \beta_2}{\sinh \beta_2} \sin \beta_2 \right) - 2 \frac{EI_2}{l_2} \beta_2 \sin \beta_2 \left(\cos \beta_1 - \frac{\cosh \beta_1}{\sinh \beta_1} \sin \beta_1 \right) - 2 \frac{EI_1}{l_1} \beta_1 \sin \beta_1 \left(\cos \beta_2 - \frac{\cosh \beta_2}{\sinh \beta_2} \sin \beta_2 \right) = 0 \tag{16}$$

where

$$\beta_1 = \lambda_1 l_1 = \omega^{\frac{1}{2}} l_1 \left(\frac{m_1}{EI_1} \right)^{\frac{1}{4}} \tag{17}$$

and

$$\beta_2 = \lambda_2 l_2 = \omega^{\frac{1}{2}} l_2 \left(\frac{m_2}{EI_2} \right)^{\frac{1}{4}} \qquad (18)$$

In equation (16), the only unknown is natural frequency, ω. However, it is convenient to use two variables, β_1 and β_2, for describing the characteristics of the system. From equations (17) and (18), we get

$$\beta_2 = \alpha \beta_1 . \qquad (19)$$

Equation (19) is an equation for line, where the slope α is defined as

$$\alpha \equiv \frac{\beta_2}{\beta_1} = \frac{l_2}{l_1} \left(\frac{m_2}{EI_2} \frac{EI_1}{m_1} \right)^{\frac{1}{4}} . \qquad (20)$$

In this study, the line and the slope were named structural line and structural slop respectively since they represent the geometry and material properties of the structure.

Mode localization of the two-span beam

The mode localization factor can be used for measure of the degree of mode localization of each mode. And using the characteristic graph, one can roughly forecast the effects of the changes in the system parameters on the mode localization.

The mode localization is the vibration energy confinement, so the degree of mode localization can be represented by logarithmic value of the ratio of the mean squared vibrational magnitude of the second substructure to that of the first substructure. Equation (21) is its approximated form.

$$\gamma \approx 2 \log \frac{|A_2|}{|A_1|} . \qquad (21)$$

where γ is the mode localization factor and it represents the degree of mode localization. If one of the following conditions is satisfied(not both), the absolute value of γ is to be large and the mode localization occurs.

$$\beta_1 \approx \left(m + \frac{1}{4} \right) \pi \text{ where } m = 1, 2, 3, \cdots \qquad (23)$$

or

$$\beta_2 \approx \left(n + \frac{1}{4} \right) \pi \text{ where } n = 1, 2, 3, \cdots . \qquad (24)$$

Now we can draw a characteristic graph.

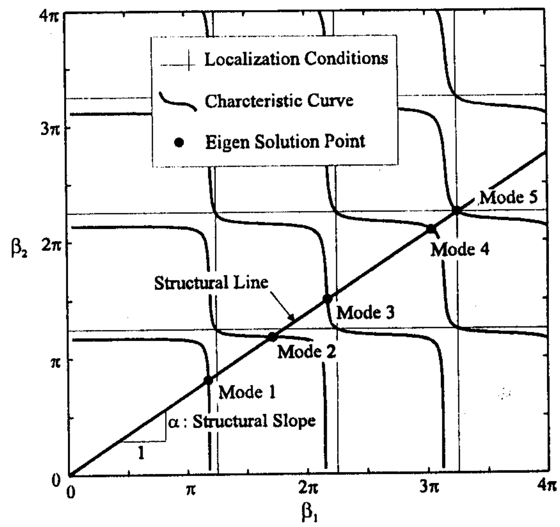

Figure 2. Characteristic graph of the two-span beam.

In Figure 2, the characteristic curves are solutions of the characteristic equation. The vertical and horizontal thin solid lines mean the localization conditions. The crossing points of the characteristic curves and the structural line indicate the eigen solutions of the system. The more an eigen solution is close to one of the localization conditions and is far from the other ones, the more the mode is strongly localized. However, if an eigen solution is close to the vertical and the horizontal localization condition lines concurrently or on the crossing point of them, the mode is not to be localized. What makes the best use of the characteristic graph is that one can roughly predict the mode localization phenomenon occurred by any disturbances in a two-span beam.

Examples

Periodic structures;

The mode localization phenomena in the ordered and the disordered periodic two-span beams are examined for two coupling conditions. In the disordered cases of this example the ratio of the span length of the second substructure to that of the first one is 0.95 while the ratio is unity in the ordered cases. The masses per unit length of each substructure are m_1=m_2=25.0 kg/m, the flexural rigidities $EI_1=EI_2=2.\times10^6$ Nm^2, and the span length $l_1=l_2=1.0$ m in ordered cases while $l_2=0.95$ m in disordered ones. The torsional spring constants are $K_R=0.00$ Nm and $K_R=2.\times10^8$ Nm in cases 1 and 2 respectively. The substructures of case 1 are strongly coupled with each other, and case 2 is weakly coupled system. The results of the ordered and the disordered cases are shown in Figure 3, 4, 5 and 6, and tables 1 and 2.

Non-periodic structures;

The normal modes of the non-periodic two-span beams are examined for three coupling conditions. In the cases of the non-periodic systems, the words of *ordered* and *disordered* do not have meanings any longer because the undisturbed initial structures have no regularity now. So in this study the structures of case 3-a and case 4-a are called by *initial* system, and the others are *disturbed* system. The material properties, mass per unit length and flexural rigidity, of these cases are equal to those of the systems discussed in the previous section. The results of the ordered and the disordered cases are discussed in pair, and they are shown in Figure 7, 8, 9 and 10, and tables 3 and 4.

Conclusions

The main findings are summarized as follows.

(1) The mode localization of an periodic beam occurs simultaneously in all modes.

(2) The mode localization of a non-periodic beam occurs in some(not all) modes.

(3) The mode localization in the higher modes is more sensitive to the system parameters than that in the lowest ones.

(4) The weak coupling makes the mode localization more sensitive in the lowest modes.

The present work can be applied to the study on the mode localization phenomenon in general structures and to the design of the structures which should be safe from any adverse loads such as shocks and vibrations.

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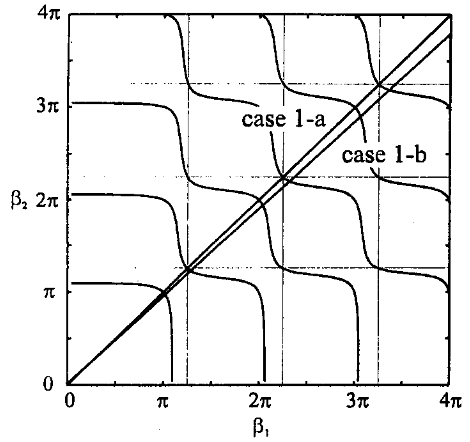
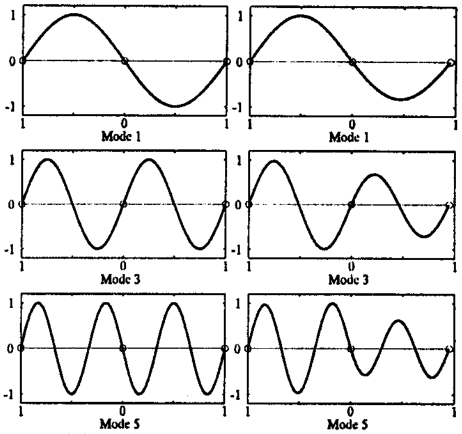


Figure 3. Characteristic graph of the case 1; case 1-a: α =1.0; case 1-b: α =0.95.



(a) Case 1-a (b) Case 1-b

Figure 4. Mode shapes of the case 1.

Table 1. Natural frequencies(ω) and degrees of mode localization(γ) : case 1.

Mode	Case 1-a		Case 1-b		Differences	
	ω ; Hz	γ	ω ; Hz	γ	δω (%)	δγ
1	444.28	0.000	465.46	-0.1384	4.77	-0.1384
2	694.06	0.000	733.86	0.1731	5.73	0.1731
3	1777.2	0.000	1854.7	-0.2695	4.36	-0.2695
4	2249.2	0.000	2387.3	0.3032	6.14	0.3032
5	3998.6	0.000	4158.4	-0.3880	4.00	-0.3880

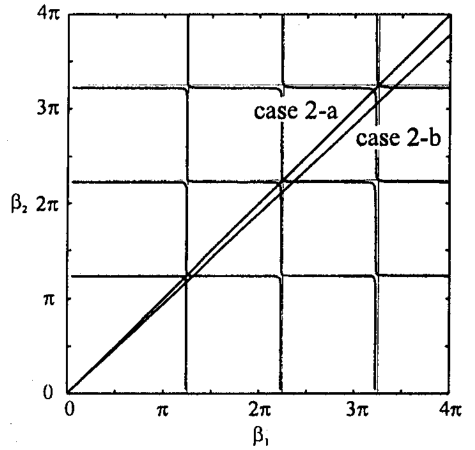
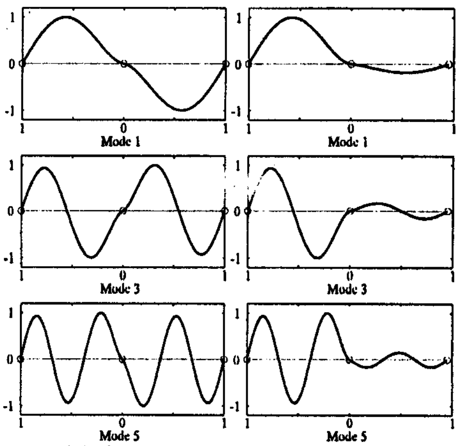


Figure 5. Characteristic graph of the case 2; case 2-a: α =1.0; case 2-b: α =0.95.



(a) Case 2-a (b) Case 2-b

Figure 6. Mode shapes of the case 2.

Table 2. *Natural frequencies(ω) and degrees of mode localization(γ): case 2.*

Mode	Case 3-a		Case 3-b		Differences	
	ω ; Hz	γ	ω ; Hz	γ	δω (%)	δγ
1	669.01	0.00	679.16	-1.487	1.52	-1.487
2	694.06	0.00	756.90	1.485	9.05	1.485
3	2172.6	0.00	2204.2	-1.521	1.45	-1.521
4	2249.2	0.00	2454.5	1.521	9.13	1.521
5	4541.7	0.00	4605.4	-1.546	1.40	-1.546

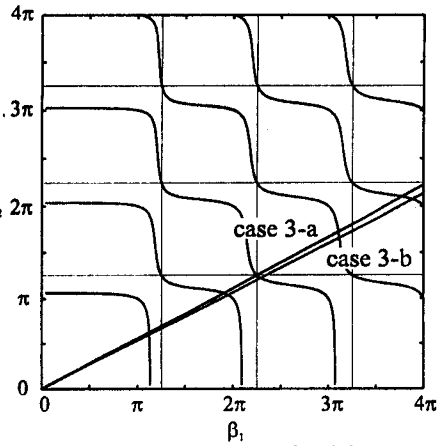
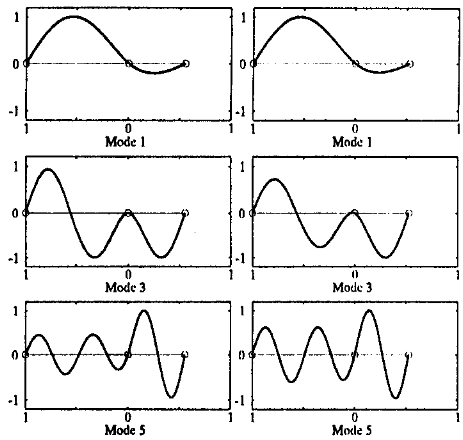


Figure 7. Characteristic graph of the case 3; case 3-a: α =0.5556; case 3-b: α =0.5278.



(a) Case 4-a (b) Case 4-b

Figure 8. Mode shapes of the case 3.

Table 3. Natural frequencies(ω) and degrees of mode localization(γ): case 3.

Mode	Case 4-a		Case 4-b		Differences	
	ω ; Hz	γ	ω ; Hz	γ	δω (%)	δγ
1	560.52	-0.7785	564.94	-0.7778	0.78	0.0007
2	1615.1	0.3878	1703.1	0.2031	5.45	-0.1847
3	2249.0	0.0000	2346.4	0.2273	4.33	0.2273
4	4289.1	-0.7223	4328.1	-0.7637	9.09	-0.0414
5	6331.6	0.6684	6771.0	0.3960	6.94	-0.2724

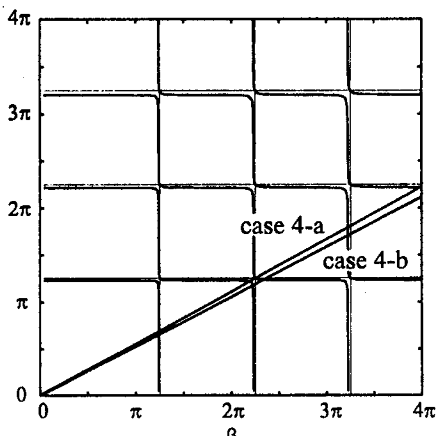
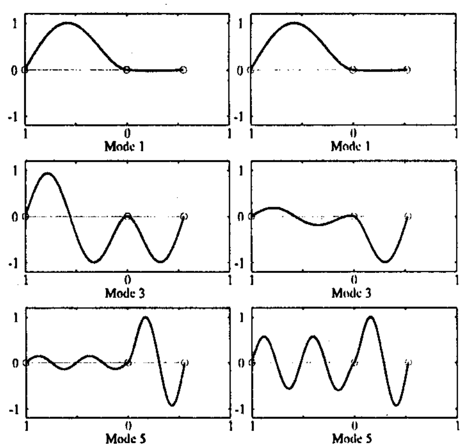


Figure 9. Characteristic graph of the case 4; case 4-a: α =0.5556; case 4-b: α =0.5278.



(a) Case 4-a (b) Case 4-b

Figure 10. Mode shapes of the case 4.

Table 4. Natural frequencies(ω) and degrees of mode localization(γ): case 4.

Mode	Case 6-a		Case 6-b		Differences	
	ω ; Hz	γ	ω ; Hz	γ	δω (%)	δγ
1	681.42	-2.889	681.48	-2.873	0.009	0.016
2	2142.8	0.511	2197.5	-0.925	2.55	-1.437
3	2249.0	-0.000	2426.1	1.428	7.87	1.428
4	4615.1	-2.136	4616.9	-2.153	0.039	-0.017
5	7026.7	1.617	7703.7	0.425	9.63	-1.192