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MULTIOBJECTIVE OPTIMIZATION OF STRUCTURE USING MODIFIED ε - CONSTRAINT APPROACH

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Abstract. *The ε-constraint approach, a well-known multiobjective optimization technique is modified in efficient way. The algorithm of proposed approach is an ε-constraint approach, however the scheme treating secondary objective functions and the procedure generating initial design points differ from the conventional approach. As examples, the multiobjective optimal design of steel box girder bridge is presented. In the example, Pareto solutions are found using not only proposed approach but constraint approach.*

1 INTRODUCTION

Multiobjective optimization simultaneously optimizes all the objective functions considered within the design region defined by constraints. Usually because there are several competing objectives that have each optimal design value respectively, the results of multiobjective optimization cannot be further improved without impairing some of the objectives. The solution sets with this property are called the Pareto optimal solutions after Italian economist Pareto[1]. For three decades the Pareto concept was used in the engineering fields like operations research, control theory and structural design optimization.

Several approaches have been proposed to solve the multiobjective optimization problems: weighting method; ε-constraint approach; goal programming approach; game theory approach. Among these, the ε-constraint approach is known to be efficient in obtaining the Pareto optimal solutions. This approach was used by Cohn et al.[5] for the multiobjective optimization of prestressed concrete structures and by Carmichiael[6] for the multiobjective optimization of five bar planar truss. However, it is very difficult to select the initial design value inside the feasible region. To avoid this difficulty in practical work, optimization is usually conducted successively; the previous optimization result is used as the initial design value because this design value is in the feasible region anyway. Hence, the total solution time is increased linearly with the increased number of the Pareto solutions.

The main purpose of this paper is to obtain the Pareto optimal solutions in efficient way by improving the ε-constraint approach. The following sections of this paper deal with the proposed approach, and a numerical example is presented to demonstrate the validity and the applicability of the proposed approach.

2 PROPOSED APPROACH

When a lot of Pareto optimal solutions are required, much computational effort is necessary in the ε-constraint approach because the optimization must be performed successively. However, if initial values could be obtained independently, each Pareto optimal solution can be found independently by using parallel processing. To make this idea available, the proposed approach transforms objective functions like

$$\text{Minimize } f_p(X) \tag{1}$$

$$\text{subject to } f_i(X) \le f_i(X_0)\ i = 1,2,\cdots,m(\neq p) \tag{2}$$

$$g_j(X) \le 0\ j = 1,2,\cdots,J \tag{3}$$

$$h_n(X) = 0\ n = 1,2,\cdots,N \tag{4}$$

where f_p is a primary objective function and f_i 뒬are the secondary objective functions. That is, the upper bounds of the secondary objective functions are their initial function values. The initial vector X_0 in Eq. (2) is obtained by using the single-objective optimization results as in Eqs. (5) and (6).

$$X_0 = \sum_{i=1}^{m} c_i X_i^*, \quad \sum_{i=1}^{m} c_i = 1 \tag{5),(6}$$

where X_i^* is a design vector which minimize $f_i(X)$ and c_i is arbitrary coefficient. One can easily show that the initial vector X_0 is in the feasible design region, if the region defined by the constraint Eqs. (3) and (4) is convex. Therefore the initial vector X_0 can be directly used in optimization technique like the modified feasible direction method. One of the key-points of this paper is that the initial vector can be obtained efficiently with above manner.

The solution scheme of the proposed approach is as follows; An initial vector is first produced in Eqs. (5) and (6) with arbitrary c_i펄 Then the transformed optimization problem of Eqs. (1) to (4) is solved through the modified feasible direction method. Because the initial vector can be produced independently, the Pareto optimal solution can be also found independently. The efficiency of computational effort is high-lighted when this approach is combined with the parallel processing technique.

The overall comparison between the ε-constraint and the proposed approach is described in Figs. 1 and 2. The feasible design region is reduced by arbitrarily chosen ε value in the ε-constraint approach, and the Pareto solution is found with the initial vector being the Pareto solution found in the previous stage. The next Pareto solution can be found with a little bit increased ε value and the current step Pareto solution. While in the proposed approach, the initial vectors are calculated by using the convex combination of vectors which are obtained in the single-objective optimzations. Then each initial vector determines the upper limits of secondary objective functions, and the desired number of the Pareto solution is found in parallel.

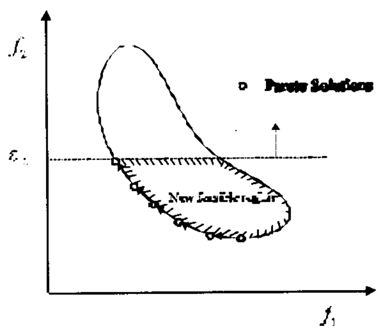

Fig. 1 ε-constraint approach

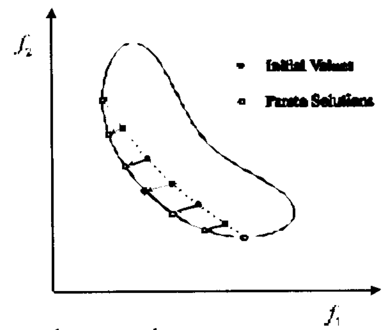

Fig. 2 The proposed approach

3 NUMERICAL EXAMPLE

The steel box girder bridge in Figure 3 is to be designed for the minimizations of both the cross sectional area and the maximum deflection of bridge. The bridge girder consists of three steel boxes and supports reinforced concrete slab on it. The bridge has four design lanes of 3.5m width each. All dead loads are included in the design of steel box. Live load by standard truck is described in Figure 4(c). The constraints include all requirements of the Korea Highway Bridge Design Code(1992). Design variables are the flange width B, web depth D, bottom flange thickness t_{bf}, upper flange thickness t_{uf} and web thickness t_w. Design loads are calculated through all the possible load cases and the influence lines over the span and width of bridge. The allowable stresses of steel are 1900kg/cm^2 in both compression(σ_{ca}) and tension(σ_{ta}), 1100kg/ cm^2 in shear(τ_a), and the Young펄 modulus of steel(E_s) is 2.1×10^6 kg/cm^2. Finally, multiobjectivc optimization problem can be formulated as

X_1^{*} =[B=200.0; D=223.1; t_{bf}=2.464; t_{uf}=2.207; t_w=1.716] in cm, and $f_1(X_1^{*}) = 1699.8$ cm^2. At this point maximum deflection of bridge is $f_2(X_1^{*}) = 5.849$cm. The minimization of the maximum deflection of bridge alone yields the optimal design vector X_2^{*} =[B=266.0; D=300.0; t_{bf}=3.0; t_{uf}=3.0; t_w=3.0] in cm, and $f_2(X_2^{*}) = 1.824$cm. At this point cross sectional area of a steel box is $f_1(X_2^{*}) = 3378.0$cm^2. Multiobjective optimization is done by both the ε-constraint and the proposed approach. The previous optimal design vectors are used for the ε-constraint approach in which ε is increased equally, and initial vectors are produced by Eqs. (5) and (6) with nine different c values for the proposed approach. The results of multiobjective optimization are given in Table I and II, and plotted in Fig. 7.

In this example, the maximum deflection of bridge is under the allowable limit suggested in KHBDC(span/500=8cm). So the objective for the cross sectional area can be mostly decreased according to the curve in Fig. 4. Both two approaches give nine Pareto optimal solutions well, but the proposed approach is more efficient in that it can be combined with parallel processing technique.

Table 1. Pareto solutions by the ε-constraint approach

$f_1(cm^2)$	$f_2(cm)$	$B(cm)$	$D(cm)$	$t_{bf}(cm)$	$t_{uf}(cm)$	$t_w(cm)$
2700.1	2.226	235.5	300.0	2.703	2.888	2.307
2413.8	2.629	206.3	300.0	2.390	2.600	2.307
2219.1	3.031	200.0	295.4	2.193	2.193	2.271
2096.4	3.434	200.0	281.4	2.196	2.196	2.164
1995.4	3.837	200.0	269.4	2.198	2.198	2.071
1910.8	4.239	200.1	258.8	2.201	2.201	1.990
1838.7	4.642	200.1	249.5	2.203	2.203	1.919
1776.4	5.045	200.1	241.2	2.204	2.204	1.855
1722.1	5.447	200.1	233.6	2.206	2.206	1.797

Table 2. Pareto solutions by the proposed approach

c	$f_1(cm^2)$	$f_2(cm)$	$B(cm)$	$D(cm)$	$t_{bf}(cm)$	$t_{uf}(cm)$	$t_w(cm)$
0.1	2891.1	2.021	255.6	300.0	2.896	3.000	2.307
0.2	2684.5	2.246	234.4	300.0	2.679	2.869	2.307
0.3	2494.1	2.503	216.6	300.0	2.467	2.656	2.307
0.4	2318.5	2.797	200.0	300.0	2.244	2.427	2.307
0.5	2185.0	3.136	200.1	291.5	2.195	2.195	2.242
0.6	2071.3	3.527	200.0	278.4	2.197	2.197	2.141
0.7	1963.2	3.982	200.0	265.4	2.199	2.199	2.041
0.8	1860.7	4.511	200.0	252.4	2.202	2.202	1.941
0.9	1763.9	5.120	200.0	239.5	2.204	2.204	1.842

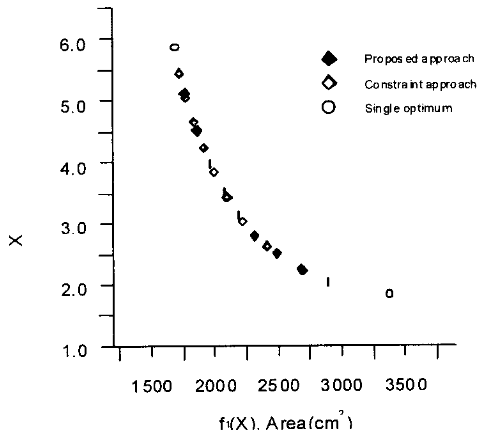

Fig. 4 Pareto optimal solutions of a steel box girder

CONCLUSIONS

A new multiobjective optimization technique is proposed. It is shown that a large number of Pareto optimal solutions can be obtained efficiently with the proposed approach. Because the proposed approach generates an initial vector independently of the Pareto solution found in the previous stage, it is possible to adopt the parallel processing technique in multiobjective optimization. If the parallel processing technique is used in finding the Pareto solutions, the total solution time can be dramatically decreased.

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