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Mode Localization in Multispan Beam with Massive and Stiff Couplers on Supports

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ABSTRACT

The influences of the coupler consisting of stiffness and mass between neighboring two spans on mode localization are studied theoretically, and the results are confirmed by numerical examples. The mass of the coupler makes a structure sensitive to mode localization especially in higher modes while the stiffness does in all modes. A new type of delocalization phenomenon is observed for the first time in some modes for which mode localization does not occur or is very weak although structural disturbances are severe. A spring-mass consisting of two substructures and a coupler connecting them is considered in the part of analytical study. As example structures for numerical analysis, simply supported continuous two-span beams with couplers having a rotational stiffness and a mass moment of inertia are considered.

INTRODUCTION

Many engineering structures are made of identical substructures connected by couplers, shaping into periodic structures. Periodically stiffened long plates, continuous girder bridges,

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multispan rahmen bridges, etc. are well known linear periodic structures. And cooling towers of a nuclear power plant, bladed rotors of a turbomachinery, etc. may be the cyclic periodic structures. However, in real structures, no substructures will be perfectly identical since no one can be free from manufacturing errors and damages. In the area of structural dynamics, a phenomenon that any structural disturbances make the specific part of a free vibrational mode have significantly larger deflection relative to the rest of it is referred to as mode localization.

In solid-state physics, the localization phenomenon of electron field in disordered solid was first observed by Anderson (1958) who shared the 1977 Nobel Prize in physics for his work. Hodges (1982) was the first to recognize that the wave localization may occur in the disordered periodic structures and it leads to mode localization. After his work, various structures have been considered and many methods have been proposed to discuss the characteristics of mode localization (Pierre *et al.* 1987, Bouzit and Pierre 1992, Lust *et al.* 1993, Kim and Lee 1998). It is well known that under a condition of weak coupling, the mode shapes undergo dramatic changes to become strongly localized when small disorder is introduced.

The present study is an attempt to prove that the mass, as well as the stiffness, of the coupler exerts important influences upon mode localization and weak coupling conditions. To accomplish this objective, a dynamic analysis of a spring-mass system is performed in the theoretical background section. In the numerical examples, mode localization of simply supported continuous two-span beam as a most simple periodic structure is analyzed under the various coupling conditions, and the results of the theoretical approach are confirmed.

THEORETICAL BACKGROUND

In this section the characteristics of mode localization of a simple spring-mass system consisting of two substructures and a coupler is discussed qualitatively. Figure 1 shows the structure considered.

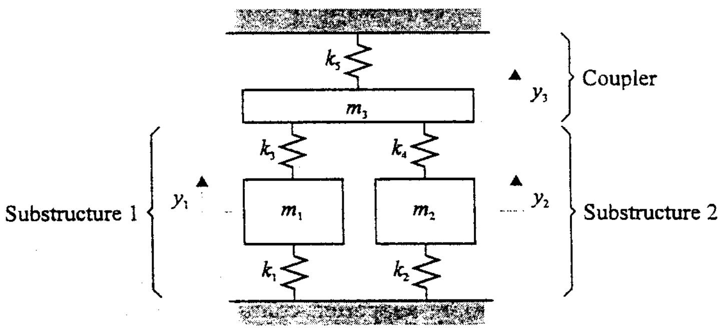


Figure 1. Simple structure constituted with two substructures and a coupler.

The eigenvalue problem for free vibration analysis of the structure may be written as

$$\begin{bmatrix} k_1 + k_3 - \lambda m_1 & 0 & -k_3 \\ 0 & k_2 + k_4 - \lambda m_2 & -k_4 \\ -k_3 & -k_4 & k_3 + k_4 + k_5 - \lambda m_3 \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \\ y_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \tag{1}$$

where λ denotes an eigenvalue or the square of circular natural frequency ω_n of the structure. A ratio of free vibration amplitude of the two substructures can be a measure of degree of mode localization. The displacement of the coupler y_3 in equation (1) is eliminated first to get an equation for the ratio, which becomes

$$(k_3 + k_4 + k_5 - \lambda m_3)(\lambda^{(1)} - \lambda^{(2)}) - \left(\frac{k_3^2}{m_1} - \frac{k_4^2}{m_2}\right) = k_3 k_4 \left(\frac{1}{m_1}\frac{y_2}{y_1} - \frac{1}{m_2}\frac{y_1}{y_2}\right). \tag{2}$$

where $\lambda^{(1)}$ and $\lambda^{(2)}$ are the eigenvalues of the substructures 1 and 2, respectively:

$$\lambda^{(1)} = (k_1 + k_3)/m_1, \qquad \lambda^{(2)} = (k_2 + k_4)/m_2. \tag{3, 4}$$

Multiplying each side of equation (2) by y_2/y_1, after simplification, results in

$$(r - s_1)(r - s_2) = \alpha r \tag{5}$$

where $r = y_2/y_1$ is a measure of mode localization and

$$s_1 = -\frac{k_3}{k_4}, \qquad s_2 = \frac{k_4}{k_3}\frac{m_1}{m_2}, \tag{6, 7}$$

and

$$\alpha = \frac{m_1}{k_3 k_4}(k_3 + k_4 + k_5 - \lambda m_3)(\lambda^{(1)} - \lambda^{(2)}). \tag{8}$$

Equation (5) is very useful in discussing the influences of the stiffness and the mass of the coupler on mode localization phenomenon. The left-hand side of equation (5) is a parabolic function having roots at s_1 and s_2, and the right-hand side is a line passing origin with slope α.

Assuming that the two substructures are identical, $\lambda^{(1)} = \lambda^{(2)}$, the ratios and the slope are $r_1 = -1$, $r_2 = 1$ and $\alpha = 0$, respectively. This indicates that the vibration amplitudes of the two substructures are equal to each other and the corresponding modes are not localized at all. Small disturbances making the substructures different to each other make the amplitude different, resulting in mode localization. Especially, when r_1 and r_2 are close to zero and infinity, respectively or reversely, corresponding modes are perfectly localized ones. Variations of r_1 and r_2 are closely related to the variation of α. If some disturbances produce significant variation in α, the degrees of mode localization drastically changed and so it can be said that the structure is very sensitive to mode localization.

A case of $m_3 = 0$ is discussed first and the structural disturbances are realized by the variation of difference of the eigenvalues of the two substructures, $1 >> |\lambda^{(1)} - \lambda^{(2)}| \neq 0$. It is obvious that when $k_5 / k_3 k_4 >> 1$, the small variations in the eigenvalues may lead to the significant change in α, resulting in drastic occurrence of mode localization. The condition of $k_5 / k_3 k_4 >> 1$ is equivalent to the weak coupling condition most popular precondition of mode localization.

Considering the term of λm_3, an additional pre-condition for the drastic occurrence of mode localization may be derived. As one can see in equation (8), if the condition of $\lambda m_3 >> k_3 + k_4 + k_5$ is satisfied, the small changes in the eigenvalues lead to the significant change in α. Therefore the large mass of the coupler and/or the large eigenvalue of the structure make the structure sensitive to mode localization and the coupling weak.

Considering the mass of the coupler, an interesting phenomenon of delocalization can be observed. When $k_3 + k_4 + k_5 - \lambda m_3 \cong 0$, corresponding mode is not sensitive to mode localization although the classical weak coupling condition, $k_5 / k_3 k_4 >> 1$, is satisfied. That is, if a natural frequency of the structure is close to that of the coupler, corresponding mode is a delocalized one for which mode localization does not occur or is very weak although structural disturbances are severe. The natural frequency of the coupler, ω_c, is the square root of the eigenvalue of the coupler given by $\lambda^{(3)} = (k_3 + k_4 + k_5) / m_3$. This delocalization phenomenon caused by the mass and stiffness of coupler is observed in this study for the first time.

NUMERICAL EXAMPLES

Here, influences of the stiffness and the mass of the couplers on mode localization of multispan beams are verified, and the results of the previous section are confirmed. As example structures, simply supported continuous two-span beams with couplers consisting of rotational stiffness and mass moment of inertia on supports are considered. Figure 3 shows the geometry of the example structure. The Young's modulus of the beam is $E = 207 GPa$, the mass density $\rho = 7750\ kg/m^3$, and the moment of inertia $I = 2.54 \times 10^{-10} m^4$. The span lengths are $l_1 = l_2 = 0.3\,m$ in the undisturbed perfect structures.

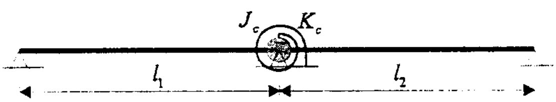


Figure 3. Simply supported continuous two-span beam with a coupler of the rotational stiffness and mass at the mid support.

The structural disorders or disturbances are realized by introducing the length variation into the last span in the example structure. Finite element method is used for dynamic analysis of the example structures. The rotational stiffness and the mass moment of inertia of the coupler are represented by the nondimensional quantities as $\bar{K}_c = K_c l / EI$ and $\bar{J}_c = 6 J_c / M l^2$ where K_c and J_c are the rotational stiffness and the mass moment of inertia of the couplers, respectively. EI is the flexural rigidity and l the span length of the perfect structures. Degrees of mode localization are computed and compared for various coupling conditions.

In this study, to facilitate discussion for mode localization of multispan beams, a measure of the degree of mode localization (DML) is defined here as

$$DML \equiv (m - m_c)/(m-1) \tag{9}$$

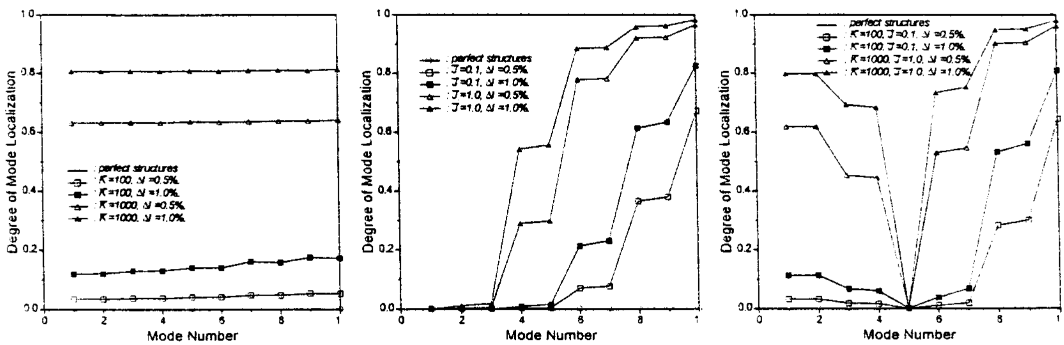
where m is the total number of spans and m_c is the number of spans in which vibrations are confined ($1 \le m_c \le m$). Here, m_c can be computed by

$$m_c = \left(\sum_{i=1}^{m} \bar{y}_i \right)^2 \left(\sum_{i=1}^{m} \bar{y}_i^2 \right)^{-1} \tag{10}$$

where $\bar{y}_i$ is the absolute value of the maximum amplitude associated with the i-th span. If the mode is extremely localized, then $DML = 1$, and if the mode is not localized at all, then $DML = 0$.

The influences of K_c on mode localization are studied first. Localization curves in Figure 4(a) show the typical localization behavior and influences of $\bar{K}_c$, on mode localization. Degrees of mode localization increase with increasing disturbance, and with increasing $\bar{K}_c$.

The influence of $\bar{J}_c$ differs from that of $\bar{K}_c$ and the localization curves in Figure 4(b) show it. Degrees of mode localization increase with increasing length disturbance in the second span, with increasing mode number, and with increasing the mass of the coupler. That is, as predicted in the section of theoretical background, the mass of coupler makes the coupling weak and the structure sensitive to mode localization, that influence are more pronounce in higher modes.



(a) Stiffness influence (b) Mass influence (c) Combined influences

Figure 4. Influence of the rotational stiffness and the mass moment of inertia of coupler.

The combined influences of the stiffness and the mass of the coupler on mode localization are also studied. Localization curves plotted in Figure 4(c) show the combined influences of $\bar{K}_c$ and $\bar{J}_c$ on mode localization. Degrees of mode localization decrease with increasing mode number until fifth mode, but after that mode they increase abruptly with increasing mode number. The fifth mode is delocalized one, and its frequency is close to the coupler's frequency, $\omega_c = 1780 Hz$. The delocalization phenomenon is more dramatic in cases of $\bar{K}_c = 1000$ and $\bar{J}_c = 1.0$. The behavior of mode localization is governed by $\bar{K}_c$ for lower modes but by $\bar{J}_c$ for higher modes under the condition that the modes are far from the localized ones. These results are consistent with the results of the previous section of theoretical background.

CONCLUDING REMARKS

In this work the influences of the stiffness and the mass of the coupler on mode localization have been studied by the theoretical approach and the numerical one. The consistent results have been drawn in both approaches. Some important conclusions drawn in the course of this work can be summarized as follows:

1) Degree of mode localization varies with the disturbances introduced into the structures.
2) The sensitivity to mode localization increases with increasing stiffness of couplers.
3) The sensitivity to mode localization increases with increasing mass of coupler and with increasing mode number.
4) The mass and stiffness of coupler causes a delocalization phenomenon for some modes for which mode localization does not occur or is very weak although structural disturbances are severe, The delocalization frequency is equal to that of a coupler.
5) The behavior of mode localization is governed by the stiffness in the lower modes but by the mass in the higher modes, and the delocalized modes are observed between them.

The first two results agree with those of previous researches in which the characteristics of mode localization and the influences of the stiffness of coupler are studied. The last three results are observed in this study for the first time. The results of this work may give very useful guide to design a structure insensitive to mode localization.

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