

## **A SURVEY ON STATE FEEDBACK CONTROL FOR SEISMIC RESPONSE REDUCTION BY ACTIVE MASS DAMPER**

K. H. Shim, D. H. Kim, K. S. Park and I. W. Lee

*Department of Civil Engineering, KAIST, Taejeon 305-701, Korea*

*[khshim@shimcontrol.com](mailto:khshim@shimcontrol.com), [jtkim@kaist.ac.kr](mailto:jtkim@kaist.ac.kr), [kyusik@kaist.ac.kr](mailto:kyusik@kaist.ac.kr), [iwlee@kaist.ac.kr](mailto:iwlee@kaist.ac.kr)*

### **Abstract**

For aseismic control by the active mass damper, usually the constant optimal gain is used. If general gain by pole placement is used, it is expected that the speed and pattern of the response are controlled arbitrary. Nevertheless, it is unusual to find such papers adopting the sub-optimal gain by pole assignment instead of using the optimal gain by linear quadratic regulator design. 'Why not' is investigated in this paper.

### **Introduction**

For feedback control, one measures the state variables as the output, finds the optimal gain by solving Riccati equation derived from the linear system and quadratic cost, and applies the feedback gain into the input for actuating. In this control process, usually one gets constant solution by solving the algebraic Riccati equation, and uses it for the real time feedback control.

Recently, the active mass damper (AMD) has been put to practical use. The AMD reduces the seismic response by compensating the momentum generated by the earthquake. It consists of motor, hydraulic actuator, and mass ball. The mass ball moves toward the opposite direction of the movement of structure to make up for the seismic momentum.

Spencer Jr. *et al.* made an experimental model of the three-story building with installation of the AMD on the top of the structure, obtained useful parameters and developed a simulation program for comparison with other control strategies. Chase and Smith resolved the actuator saturation problem by formulating the Lyapunov function and implemented it for the real building in Tokyo, Japan. Kim showed that, when CMAC used, training time is dramatically reduced in the neural networks for the seismic response control with the AMD. Wu and Soong studied an implementation of the Bang-Bang optimal control into the structural control in order to reduce the peak-response that conventional LQR design cannot decrease.

The optimal control is used to guarantee the minimum energy consumption of the linear system with appropriately selected weighting matrices in the cost functional. The optimal gain is obtained from solving the Riccati equation. But, it takes too much time to solve the differential Riccati equation. Therefore, the constant solution of the algebraic Riccati

equation is used for the real time operation in the control process. A drawback of using the optimal control is that the system response cannot be controlled arbitrary. Sub-optimal control is substitute for the optimal control by prescribing the poles; hence, the speed and pattern of the system response are changed according to the design purpose.

If general gain by pole placement is used, it is expected that the magnitude of the seismic response will be reduced more than that of the optimal gain. In this case, more electric power is consumed to generate the lager control force by the AMD. Comparing the cost of electric energy to the performance of AMD, the latter is much more important. An appropriately selected constant gain is utilized for all the control process within the limit of operation range of the AMD. So, it seems attractive that the pole assignment is adopted to design the feedback controller of the AMD. However, no such papers have been found so far.

Why the sub-optimal control by pole assignment is not used? This question is answered in this paper..

### AMD Model

Dynamic equation of a three-story miniature building with an AMD is given as,

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = -M[1]\ddot{x}_g(t) + Lu(t) \quad (1)$$

where  $L = [0 \ 0 \ 1 \ -1]^T$

Its state space model is obtained as

$$\dot{X} = AX + Bu + H\ddot{x}_g, \quad \dot{X} = [x \ \dot{x}]^T \quad (2)$$

$$B = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0.01 \ -0.06]^T \quad H = [0 \ 0 \ 0 \ 0 \ -1 \ -1 \ -1 \ -1]^T$$

where

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \quad A_{11} = 0, \quad A_{12} = I, \quad A_{21} = -M^{-1}K, \quad A_{22} = -M^{-1}C$$

$$A_{21} = \begin{bmatrix} -2250 & 1125 & 0 & 0 \\ 1125 & -2250 & 1125 & 0 \\ 0 & 1125 & -1143.55 & 18.55 \\ 0 & 0 & 206.10 & -206.10 \end{bmatrix} \quad A_{22} = \begin{bmatrix} -0.46 & -0.02 & 0.14 & 0 \\ -0.02 & -0.32 & 0.12 & 0 \\ 0.14 & 0.12 & -0.56 & 0.22 \\ 0 & 0 & 2.48 & -2.48 \end{bmatrix}$$

The state vector X is denoted as

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix} = \begin{bmatrix} \text{Dispalcement of the first story, m} \\ \text{Dispalcement of the second story, m} \\ \text{Dispalcement of the third story, m} \\ \text{Dispalcement of the AMD, m} \\ \text{Velocity of the first story, m/sec} \\ \text{Velocity of the second story, m/sec} \\ \text{Velocity of the third story, m/sec} \\ \text{Velocity of the AMD, m/sec} \end{bmatrix}$$

The S00E component of *El Centro* earthquake is used as an excitation input.

### State Feedback Control

A standard linear system and quadratic cost for the steady state case are given by

$$\dot{X}(t) = AX(t) + Bu(t), \quad J = \int_0^{\infty} \{X^T(t)QX(t) + u^T(t)Ru(t)\} dt \quad (3)$$

$$u(t)_{opt} = -GX(t) = -R^{-1}B^TKX(t) \quad (4)$$

$$0 = KA + A^TK + Q - KBR^{-1}B^TK \quad (5)$$

If the gain  $G$  is assigned arbitrary, the converging pattern of the response is easily controlled. The sub-optimal gains are given as  $3 * G_{opt}$ ,  $0.5 * G_{opt}$  respectively.

### Simulation Results

For Figure 2, the first and the second show the displacement and velocity of the story three of the closed-loop zero input response. The third and the fourth show the displacement and velocity of the story three of the closed-loop seismic input response. For Figure 3, the first and the second display the displacement and velocity of the AMD of the closed-loop zero input response. The third and the fourth display the displacement and velocity of the AMD of the closed-loop seismic input response.

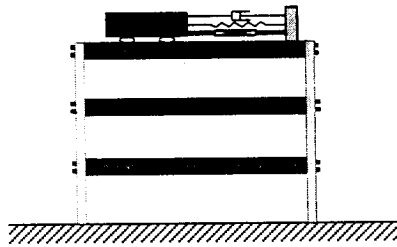
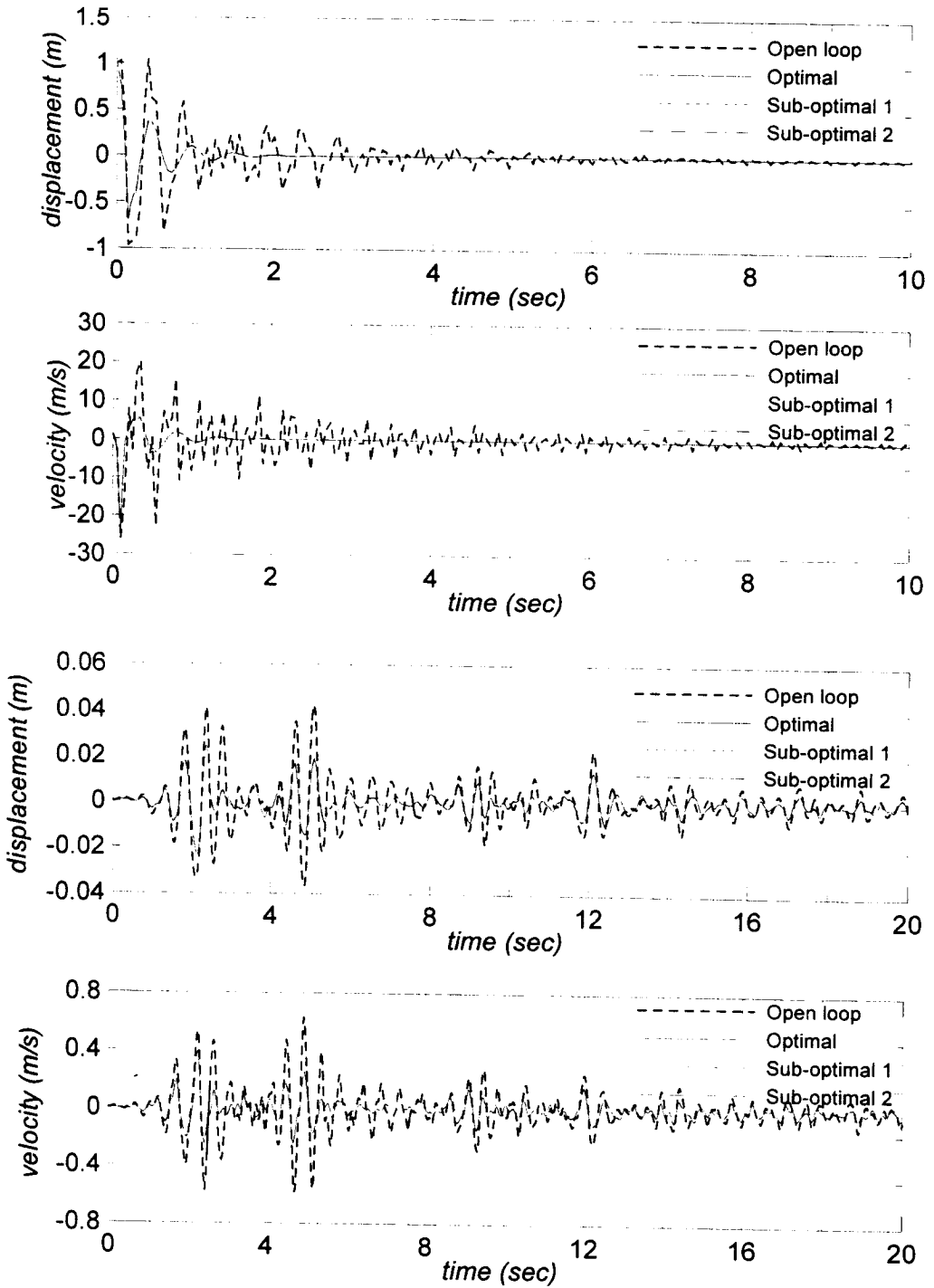


Figure 1: Structure with AMD



**Figure 2: Response of  $x_3, \dot{x}_3$**

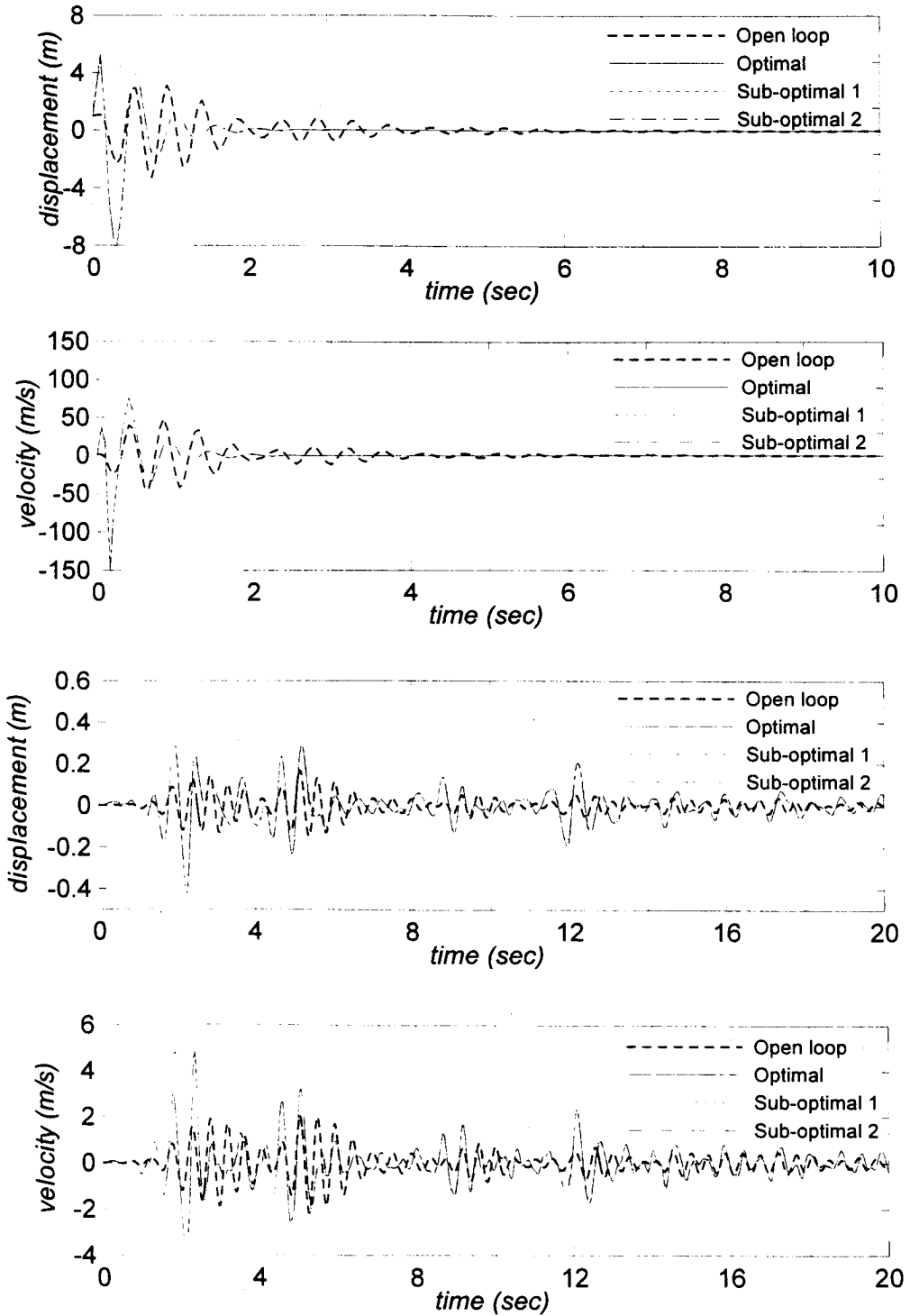


Figure 3: Response of  $x_4, \dot{x}_4$

## Discussion

The closed-loop state feedback design improves the displacement and velocity of the story three compared with that of open-loop case. However, there are shown no difference between the optimal and the sub-optimal design. Why can be explained as: The selected AMD system showed saturation characteristics both in the zero input state response and in the excitation input response of the closed-loop design.

The weighting Q's of the displacement and the velocity is given as a ratio of 100 to 1. The weighting Q's of the story three and the AMD both in displacement and velocity are adjusted a ratio of 100 to 1. And the weighting R is given as  $10^{-8}$ . Here, the values of Q and R are assigned quite reasonably considering the size of the building and the capacity of the AMD.

## Conclusions

In this paper, it is questioned that why only the optimal control is used for the LQR design to reduce the seismic response, and tried using the sub-optimal control by the pole assignment. When the weighting Q and R are given as above so as to produce reasonable control force, it is unavoidable that the saturation occurs in the response of the closed-loop feedback control. In this case, therefore, small gain is sufficient enough to reduce the seismic response. Further work should be focused on identifying the reason of such saturation.

## References

- Brogan, W.L. (1991), "Modern Control Theory", *Prentice-Hall*, Englewood Cliffs, New Jersey
- Chase J.G. and Smith H.A. (1996), "Robust  $H_{\infty}$  Control Considering Actuator Saturation. I. Theory", *Journal of Engineering Mechanics*, 122(10), 976-983
- Chase J.G., Smith H.A. and T. Suzuki (1996), "Robust  $H_{\infty}$  Control Considering Actuator Saturation. II. Theory", *Journal of Engineering Mechanics*, 122(10), 984-993
- Housner, G.W. *et al.* (1997), "Structural Control: Past, Present, and Future", *Journal of Engineering Mechanics*, 123(9), Special Issue
- Kim, D.H. (2000), "Vibration Control of Structures using Neural Networks", *Ph.D. Dissertation*, Korea Advanced Institute of Science and Technology
- Soong, T.T. (1990), "Active Structural Control: Theory and Application", *Longman Scientific and Technical*, Essex, England
- Spencer, B.F. Jr., Dyke S.J., and Deoskar, H.S. (1998), "Benchmark Problems in Structural Control: Part I- Active Mass Driver System", *Earthquake Engineering and Structural Dynamics*, 27, 1127-1139
- Wu, Z. and T.T. Soong (1996), "Modified Bang-Bang Control Law for Structural Control Implementation", *Journal of Engineering Mechanics*, 122(8), 771-777