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MODIFIED MODAL METHODS FOR CALCULATING EIGENPAIR SENSITIVITY OF ASYMMETRIC SYSTEMS

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Abstract

It is well known that many real systems have asymmetric mass, damping and stiffness matrices. In this case, the method for calculating eigenpair sensitivity is different from that of symmetric system. To determine the derivatives of the eigenpairs in asymmetric damped case, a modal method was recently developed by Adhikari. When a dynamic system has many degrees of freedom, only a few lower modes are available, and because the higher modes should be truncated to use the modal method, the errors may become significant. In this paper a procedure for determining the sensitivities of the eigenpairs of asymmetric damped system using a few lowest set of modes is proposed. Numerical examples show that proposed method achieves better calculating efficiency and highly accurate results when a few modes are used.

Introduction

Natural frequency and mode shape of system are essential to understand dynamic behavior of structure. But design parameters can be varied with damage, deterioration, corrosion etc. and this causes variation in natural frequency and mode shape. The variation of eigenpair brings about variation of dynamic behavior of systems and this affects the stability of structure directly. Therefore eigen-sensitivity analysis has played a central part in structural stability analysis and has emerged as an important area of research. And eigenpair sensitivity is used in many areas, the optimization of structure subject to natural frequency, system identification, finite modeling updating, structural control etc.

For symmetric systems, modal methods (Murthy & Haftka 1988, Lim & Junkins 1987) and its modified ones (Wang 1985, Liu et al. 1987) approximate the eigenvector derivatives by a linear combination of the eigenvector. The modal methods employ a modal superposition idea. Therefore, the accuracy is dependent on the number of modes used in calculation. To guarantee the accuracy, the classical modal method needs higher eigenvector. Recently Zeng presented modified modal methods such as multiple modal acceleration and shifted-poles method for the complex eigenvectors in symmetric viscous damping systems, which achieved highly accurate results when only a few modes are used.

However, in many problems in dynamics the inertia, stiffness and damping properties of the system cannot be represented by symmetric matrices. These kind of problems typically arise in the dynamics of actively controlled structures and in many general non-conservative dynamic systems, for example – moving vehicles on roads, missile following trajectories, ship motion in sea water or the study of aircraft flutter. The

asymmetric of damping and stiffness terms are often addressed in the context of gyroscopic and follower forces.
To calculate the eigenpair derivatives in this case, Adhikari and M. I. Friwell proposed a modal method by modifying the modal method for symmetric damped systems.
In this paper, by combining the modal method by Adhikari and M. I. Friswell and modal acceleration and shifted-pole method by Zeng modified modal methods for asymmetric damped system are presented. So highly accurate modal method for calculating the derivatives for asymmetric damped systems has been developed. And fewer eigenvectors required for the predetermined accuracy. As a result, the method is more efficient in computation than Adhikari's modal method.
Numerical examples show that proposed method achieves better calculating efficiency and highly accurate results when a few modes are used.

Previous modal method for asymmetric systems

The general equation of motion for an N-degree of freedom system with damping is

$$M\ddot{u}(t) + C\dot{u}(t) + Ku(t) = 0 \tag{1}$$

where M, C and K are mass, damping and stiffness matrices, respectively. The traditional restrictions of symmetry and positive definiteness are not imposed on M, C and K, however, it is assumed that M^{-1} exists.
Equation (1) can be rewritten in the following state-space form,

$$A\dot{x}(t) + Bx(t) = 0 \tag{2}$$

where

$$A = \begin{bmatrix} C & M \\ M & 0 \end{bmatrix},\ B = \begin{bmatrix} K & 0 \\ 0 & -M \end{bmatrix},\ x(t) = \begin{Bmatrix} u(t) \\ \dot{u}(t) \end{Bmatrix} \tag{3}$$

From the equation (2), we obtain following two equations because of the asymmetricity of the system.

$$(sA + B)z = 0 \qquad y^T(sA + B) = 0 \tag{4}$$

where s is the eigenvalue, z is the right eigenvector and y is the left eigenvector which is related to the right and left eigenvector of the second order system
For distinct eigenvalues, two normalizing conditions by Adhikari are as follows.

$$y_j^T A z_j = 2s_j \qquad \{u_j\}_{n_j} = \{v_j\}_{n_j} \tag{5}$$

where $\{*_j\}_{n_j}$ denotes the j-th element of a vector n_j is chosen so that the corresponding elements of the eigenvectors are as large as possible. Thus

$$\left|\{u_j\}_{n_j}\right|\left|\{v_j\}_{n_j}\right| = \max_n \left|\{u_j\}_n\right|\left|\{v_j\}_n\right| \tag{6}$$

Differentiate equation (4) with design parameter α. Using this equation, the derivative of eigenvalues can be obtain as.

$$s_{j,\alpha} = -\frac{y_j^T(s_j A_{,\alpha} + B_{,\alpha})z_j}{y_j^T A z_j} = -\frac{y_j^T(s_j A_{,\alpha} + B_{,\alpha})z_j}{2s_j} \tag{7}$$

To derive the eigenvector derivatives, we can expand $z_{j,\alpha}$ and $y_{j,\alpha}$ as complex linear combinations of z_i and y_i, for all $i=1,\cdots,2N$.

$$z_{j,\alpha}=\sum_{i=1}^{2N}a_{ji}z_i \qquad y_{j,\alpha}=\sum_{i=1}^{2N}b_{ji}y_i \tag{8}$$

Differentiate equation (4) and Substituting the equation (8) into that equation, we can obtain the coefficient a_{jk}, b_{jk}. So the derivatives of eigenvectors are

$$z_{j,\alpha}=\left\{\sum_{k=1,k\neq j}^{N}\left[\frac{z_k y_k^T}{2s_k(s_j-s_k)}+\frac{(z_k y_k^T)^*}{2s_k^*(s_j-s_k^*)}\right]+\frac{(z_j y_j^T)^*}{2s_j^*(s_j-s_j^*)}\right\}f_j+a_{jj}z_j \tag{9}$$

$$y_{j,\alpha}=\left\{\sum_{k=1,k\neq j}^{N}\left[\frac{y_k z_k^T}{2s_k(s_j-s_k)}+\frac{(y_k z_k^T)^*}{2s_k^*(s_j-s_k^*)}\right]+\frac{(y_j z_j^T)^*}{2s_j^*(s_j-s_j^*)}\right\}g_j+b_{jj}y_j \tag{10}$$

where

$$f_j=-(s_{j,\alpha}A+sA_{,\alpha}+B_{,\alpha})z_j,\quad g_j=-(s_{j,\alpha}A+sA_{,\alpha}+B_{,\alpha})^T y_j \tag{11}$$

Multiple modal acceleration method (MMA Method)

To get the high convergence rate, use the modal acceleration approach.
Separate the response $z_{,\alpha}$ into a pseudostatic response z_{s0} and a dynamic correction response z_{d0}

$$z_{,\alpha}=z_{s0}+z_{d0} \tag{12}$$

where

$$z_{s0}=B^{-1}f \tag{13}$$

$$z_{d0}=z_{,\alpha}-z_{s0} \tag{14}$$

or

$$\begin{aligned} z_{d0}&=(sA+B)^{-1}f-B^{-1}f \\ &=Z\begin{bmatrix}\ddots & & \\ & \dfrac{1}{2s_k(s-s_k)}\left(\dfrac{s}{s_k}\right) & \\ & & \ddots\end{bmatrix}Y^T f \end{aligned} \tag{15}$$

where Y and Z is the modal matrix to be formed by the right and left eigenvector, respectively.

By the similar procedure, separate the response $z_{,\alpha}$.

$$z_{,\alpha}=z_{s\,n-1}+z_{d\,n-1} \tag{16}$$

where

$$z_{sn-1}=B^{-1}f[I-sAz_{sn-2}]f \tag{17}$$

$$z_{d\,n-1} = z_{,\alpha} - z_{s\,n-1}$$
$$= Z \begin{bmatrix} \ddots & & \\ & \dfrac{1}{2s_k(s-s_k)}\left(\dfrac{s}{s_k}\right)^n & \\ & & \ddots \end{bmatrix} Y^T f \tag{18}$$

So the right eigenvector derivative is given as

$$z_{j,\alpha} = \left\{ B^{-1}\sum_{m=0}^{M-1}(-s_j AB^{-1})^m + \sum_{k=1,k\neq j}^{N}\left[\left(\frac{s_j}{s_k}\right)^M \frac{z_k y_k^T}{2s_k(s_j - s_k)} + \left(\frac{s_j}{s_k^*}\right)^M \frac{(z_k y_k^T)^*}{2s_k^*(s_j - s_k^*)}\right] + \left(\frac{s_j}{s_j^*}\right)^M \frac{(z_j y_j^T)^*}{2s_j^*(s_j - s_j^*)}\right\} f_j + a_{jj} z_j \tag{19}$$

By the similarly procedure, the left eigenvector derivatives is given as

$$y_{j,\alpha} = \left\{ B^{-T}\sum_{m=0}^{M-1}(-s_j A^T B^{-T})^m + \sum_{k=1,k\neq j}^{N}\left[\left(\frac{s_j}{s_k}\right)^M \frac{y_k z_k^T}{2s_k(s_j - s_k)} + \left(\frac{s_j}{s_k^*}\right)^M \frac{(y_k z_k^T)^*}{2s_k^*(s_j - s_k^*)}\right] + \left(\frac{s_j}{s_j^*}\right)^M \frac{(y_j z_j^T)^*}{2s_j^*(s_j - s_j^*)}\right\} g_j + b_{jj} y_j \tag{20}$$

Multiple modal accelerations with Shifted-Poles (MMAS Method)

For more high convergence rate, the term $(s_j A + B)^{-1}$ is expanded in Taylor's series at the position β as

$$\begin{aligned}(s_j A + B)^{-1} &= [(B + \beta A - (s_j - \beta)(-A)]^{-1} \\ &= (B+\beta A)^{-1}[I + (s_j - \beta)(B+\beta A)^{-1}A]^{-1} \\ &= (B+\beta A)^{-1}\sum_{m=0}^{M-1}[-(s_j-\beta)A(B+\beta A)^{-1}]^m\end{aligned} \tag{21}$$

Through the similar previous procedure using the equation (21), the j-th right eigenvector derivatives are formulated as

$$z_{j,\alpha} = \left\{ (B+\beta A)^{-1}\sum_{m=0}^{M-1}[-(s_j-\beta)A(B+\beta A)^{-1}]^m + \sum_{k=1,k\neq j}^{N}\left[\left(\frac{s_j-\beta}{s_k-\beta}\right)^M \frac{z_k y_k^T}{2s_k(s_j - s_k)} + \left(\frac{s_j-\beta}{s_k^*-\beta}\right)^M \frac{(z_k y_k^T)^*}{2s_k^*(s_j - s_k^*)}\right] + \left(\frac{s_j-\beta}{s_j^*-\beta}\right)^M \frac{(z_j y_j^T)^*}{2s_j^*(s_j - s_j^*)}\right\} f_j + a_{jj} z_j \tag{22}$$

By the similarly procedure, the left eigenvector derivatives is given as

$$y_{j,\alpha} = \left\{ (B+\beta A)^{-T} \sum_{m=0}^{M-1} [-(s_j - \beta) A^T (B+\beta A)^{-T}]^m + \sum_{k=1,k\neq j}^{N} \left[\left(\frac{s_j - \beta}{s_k - \beta} \right)^M \frac{y_k z_k^T}{2s_k (s_j - s_k)} + \left(\frac{s_j - \beta}{s_k^* - \beta} \right)^M \frac{(y_k z_k^T)^*}{2s_k^* (s_j - s_k^*)} \right] + \left(\frac{s_j - \beta}{s_j^* - \beta} \right)^M \frac{(y_j z_j^T)^*}{2s_j^* (s_j - s_j^*)} \right\} g_j + b_{jj} y_j \quad (23)$$

Numerical Examples

Whirling beam whose system matrices are asymmetric is considered as numerical example. This example is a gyroscopic system rotating with high speed and has a lumped mass in center of beam as figure 1.

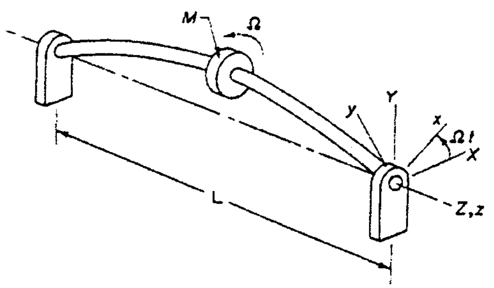

Figure 1. Whirling beam

$m_0 = 10kg/m, \; M = 10kg$

$L = 5m, \; EI_x = 4L^3/5\pi^2 Nm^2$

$EI_y = 9L^3/5\pi^2 Nm^2$

$K_1 = K_2 = L^2/20Nm$

$\Omega = \sqrt{21.6\pi} \; rad \; s^{-1}$

$c = h = 1/4Nsm^{-1}$

The equation of motion of gyroscopic system is as follows

$$M\ddot{u}(t) + (C+G)\dot{u}(t) + (K+H)u(t) = F(t)$$

Where M, C, K, F are mass, damping, stiffness and external force matrices respectively, G is gyroscopic matrix and H is circulatory matrix that make system matrix asymmetric. The detail information of this system is represented in Ref. 3.

Table 1 shows exact eigenvalues and eigenvectors and their derivatives with respect to length for design variable.

Table 1. Eigenvalues , first right eigenvector and their derivatives

Mode Number	Eigenvalues	Derivatives	Right Eigenvector	Derivative
1	1.678e-2 + 1.053e+1i	3.371e-3 + 3.457e-1i	-4.138e-2 – 5.596e-5i	7.416e-3 – 4.927e-5i
2	1.678e-2 – 1.053e+1i	3.371e-3 - 3.457e-1i	0	0
3	6.328e-2 + 1.253e+1i	8.804e-3 + 2.062e-1i	1.404e-1 + 7.721e-4i	5.578e-3 + 8.632e-5i
4	6.328e-2 - 1.253e+1i	8.804e-3 - 2.062e-1i	5.446e-5 – 4.221e-2i	4.942e-5 + 7.736e-3i
5	2.309e-1 + 1.408e+1i	2.190e-2 + 1.773e-2i	0	0
6	2.309e-1 - 1.408e+1i	2.190e-2 - 1.773e-2i	7.804e-4 – 1.433e-1i	-8.601e-5 + 6.276e-3i
7	-2.664e-1 + 1.512e+1i	-2.397e-2 – 1.773e-2i	-1.049e-4 – 4.356e-1i	5.231e-4 + 6.378e-2i
8	-2.664e-1 - 1.512e+1i	-2.397e-2 + 1.773e-2i	0	0
9	-1.133e-1 + 1.667e+1i	-8.803e-3 – 2.061e-1i	-1.049e-2 – 1.478e+0i	-1.555e-3 + 1.020e-2i
10	-1.133e-1 - 1.667e+1i	-8.803e-3 + 2.061e-1i	4.444e-1 – 1.345e-4i	-6.687e-2 + 5.267e-4i
11	-5.904e-2 + 1.867e+1i	-3.771e-3 – 3.445e-1i	0	0
12	-5.904e-2 - 1.867e+1i	-3.771e-3 + 3.445e-1i	1.509e+0 – 1.062e-2i	-1.656e-2 – 1.553e-3i

Table 2. Error comparison of proposed method using four modes

DOF Number	Modal Method (%)	MA (%)	MMA (%)	MMAS (%)
1	21.177	14.856	6.736	0.389
2	0.000	0.000	0.000	0.000
3	95.485	66.975	31.353	1.845
4	25.535	11.578	8.122	0.377
5	0.000	0.000	0.000	0.000
6	106.783	48.401	33.950	1.546

Table 3. Error comparison of MMAS using fewer modes

DOF Number	4 mode (%)	3 mode (%)	2 mode (%)
1	0.389	2.884	7.148
2	0.000	0.000	0.000
3	1.845	1.661	2.211
4	0.377	4.483	7.070
5	0.000	0.000	0.000
6	1.546	1.392	1.870

To demonstrate the effectiveness of modified modal methods, right eigenvector derivative is calculated using four modes. Table 2 shows resulting errors. (MMA : M=2, MMAS : β=eigenvalue-1, Ma=2)

As you can see in the table 4, multiple modal acceleration method with shifted poles is very effective. To demonstrate the efficiency of MMAS, right eigenvector derivative is calculated using fewer modes. Table 3 shows resulting errors.(β=eigenvalue-1, Ma=2)

Conclusion

The modified modal methods for the eigenpair derivatives of asymmetric damped system has been derived. It is assumed that the system does not possess any repeated eigenvalues. By analyzing the numerical example, it is verified that the proposed methods are more effective than previous method.

For accurate result, previous modal method is needed all eigenvalues and eigenvectors of system. But in practical case, only a few lower modes are available. So the errors may become significant. The modified modal methods is possible to calculate derivatives of eigenvlaues and eigenvectors of asymmetric damped system using a few lower modes.

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References

Sondipon Adhikari and Michael I. Friswell (2001), "Eigenderivative Analysis of Asymmetric Non-Conservative Systems," *International Journal for Numerical Method in Engineering*, **51**, 709-733.

Qing-Hua Zeng, "Highly Acurate Modal Method for Calculating Eigenvector Derivative in Viscous Damping Systems," *AIAA Journal*, **33**, 746-751.

L. Meirovitch and G. Ryland, "A Perturbation Technique for Gyroscopic Systems with Small International and External Damping," *Journal of Sound and Vibration*, **100**, 393-408.