

## THREE-DIMENSIONAL FINITE ELEMENT ANALYSES OF STRUCTURES UNDER WIND LOADS

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### Abstract

Simplified beam analyses are commonly used to obtain the responses of wind-loaded structures, and sometimes three-dimensional plate analyses are adopted to more exactly calculate the responses of structures under wind loads. However, there is no systematic comparison of simplified beam analyses and three-dimensional plate analyses so far. In this paper, simplified beam analyses and three-dimensional plate analyses are systematically compared by considering a plate structure under wind loads. Numerical analysis shows that torsional moments from beam analyses are quite different from those from plate analyses and member forces along width directions cannot be exactly calculated from beam analyses. Therefore, three-dimensional plate analyses are required to exactly calculate torsional moments and width-directional member forces of structures under wind loads, instead of simplified beam analyses.

### Introduction

The development of construction techniques has resulted in the emergence of large-scaled structures such as high-rise buildings, tall slender towers and long-span bridges. Those structures are often sensitive to wind loads due to their flexibility. Accordingly, analyses of wind-loaded structures have become more important in structural designs. Simplified beam analyses using lift coefficients, drag coefficients and moment coefficients are commonly used to obtain the responses of wind-loaded structures. In the field of wind engineering, many researches on the analyses of wind-loaded structures have still been performed by using beam modeling (Phongkumsing *et al.*, 2001; Chen *et al.*, 2001; Huynh and Thoft-Christensen, 2001). Three-dimensional plate analyses using pressure coefficients are adopted to more exactly calculate the responses of structures under wind loads (Zahlten and Borri, 1998). It is commonly recognized that three-dimensional plate analyses give more exact results than simplified beam analyses. However, comparison of simplified beam analyses and three-dimensional plate analyses has not been investigated systematically. In addition, disadvantages of conventional simplified beam analyses have not been addressed clearly. This paper systematically compares conventional simplified beam analyses with exact three-dimensional plate analyses by considering a plate structure under wind loads, and shows disadvantages of conventional simplified beam analyses of wind-loaded structures.

## Beam analyses of wind-loaded structures

The dynamic equation of motion of wind-loaded structures can be expressed as follows:

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{f\} \quad (1)$$

where  $[M]$ ,  $[C]$  and  $[K]$  represent mass, damping and stiffness matrices, respectively, and  $\{u\}$  is displacement vector and  $\{f\}$  is force vector due to wind loads.

The procedure of beam analyses for wind-loaded structures is as follows. First, a structure is modeled with finite beam elements. Second, lift coefficients ( $C_L$ ), drag coefficients ( $C_D$ ) and moment coefficients ( $C_M$ ) are evaluated according to the shapes of beam sections through experimental or analytical methods. Then, distributed forces are given by

$$L = (1/2)C_L\rho V^2 B, \quad D = (1/2)C_D\rho V^2 B, \quad M = (1/2)C_M\rho V^2 B^2 \quad (2)$$

where  $L$ ,  $D$  and  $M$  are lift force, drag force and moment per unit length.  $\rho$  is wind density,  $V$  wind speed and  $B$  the width of beam section (Simiu and Scanlan, 1996). Finally, the responses can be obtained by solving Eq. (1) with  $\{f\}$  which is composed of  $L$ ,  $D$  and  $M$ .

## Plate analyses of wind-loaded structures

The procedure of plate analyses for wind-loaded structures is summarized as follows. First, a structure is modeled with triangular finite plate elements. Next, a pressure coefficient at each element is evaluated through experimental or analytical methods. Then a wind pressure at each element can be given by

$$p = (1/2)C_p\rho V^2 \quad (3)$$

where  $p$  is the wind pressure and  $C_p$  is the pressure coefficient. By multiplying  $p$  by the area of plate element, the magnitude of a wind load acting on each element can be calculated. The direction of the wind load agrees with that of normal vector of a plate element. Therefore, the wind load can be calculated by

$$\vec{f}_w = (1/2)C_p\rho V^2 A\vec{n} \quad (4)$$

where  $\vec{f}_w$  is the wind load,  $A$  is the area of plate element and  $\vec{n}$  is the normal vector of plate element. By distributing such wind load to three nodal points of plate element equally, wind load vector can be composed as in

$$\{f\}_w = (1/6)C_p\rho V^2 A[n_x, n_y, n_z, n_x, n_y, n_z, n_x, n_y, n_z]^T \quad (5)$$

where  $\{f\}_w$  is wind load vector and  $n_x$ ,  $n_y$  and  $n_z$  are components of  $\vec{n}$ . Finally, the responses of wind-loaded structures can be calculated by using Eq. (1) with  $\{f\}$  which is composed of  $\{f\}_w$ .

## Numerical example

In order to compare beam analyses with plate analyses, a numerical example is introduced. The geometric and material properties of an example structure, which is a plate with seven hinge supports at both edges, are shown in Fig. 1. Each hinge has a rotational restraint about x-axis so that a torsional moment can be induced. For calculation, Young's modulus is  $2.5 \times 10^5 \text{ kgf/cm}^2$ , mass density  $2.446 \times 10^{-6} \text{ kgf-sec}^2/\text{cm}^4$ , thickness 60 cm, Poisson ratio 0.17 and damping ratio 5 %.

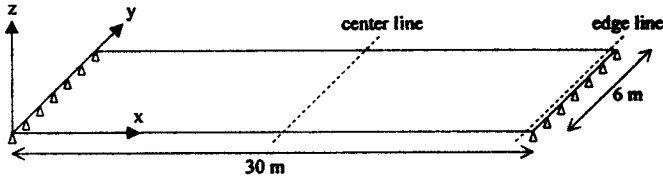


Fig. 1. The properties of an example structure.

The properties of an example wind are shown in Fig. 2. The velocity history of the example wind is generated artificially. In the generation of the example wind, Kaimal spectrum (Simiu and Scanlan, 1996) is used as a reference spectrum. In this study, analyses are performed for the angle of attack  $\alpha = 15, 30, 45, 60, 75$  and  $90$  degree.

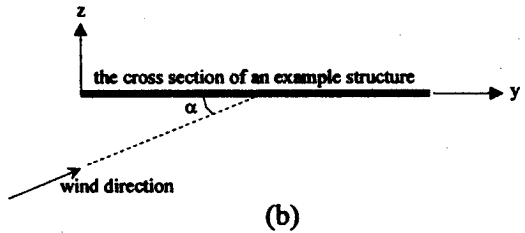
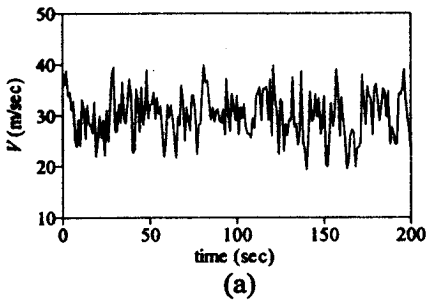


Fig. 2. The properties of an example wind: (a) Velocity history; (b) Angle of attack

$C_p$  for plate analyses are analytically calculated from the formulas in Abernathy's paper (Abernathy, 1962).  $C_L$ ,  $C_D$  and  $C_M$  for beam analyses can be calculated respectively by integrating  $C_p$  as follows:

$$C_L = \left( \int_0^B C_p dy / B \right) \cos \alpha, \quad C_D = \left( \int_0^B C_p dy / B \right) \sin \alpha, \quad C_M = \int_0^B C_p (B/2 - y) dy / B^2 \quad (6)$$

Fig. 3 represents  $C_p$  along the width direction of the example structure.  $C_L$ ,  $C_D$  and  $C_M$  of the example structure, which are calculated from Eqs. (6), are shown in Table 1.

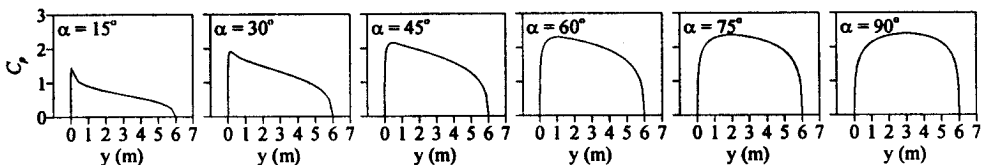
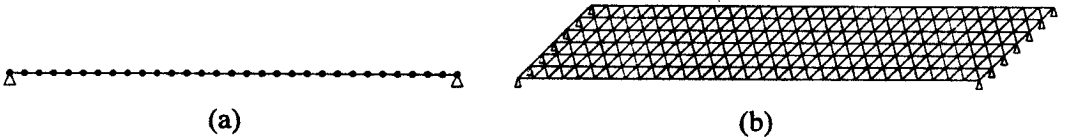


Fig. 3. Pressure coefficients along the width direction of the example structure

**Table 1: Lift coefficients, drag coefficients and moment coefficients of the example structure**

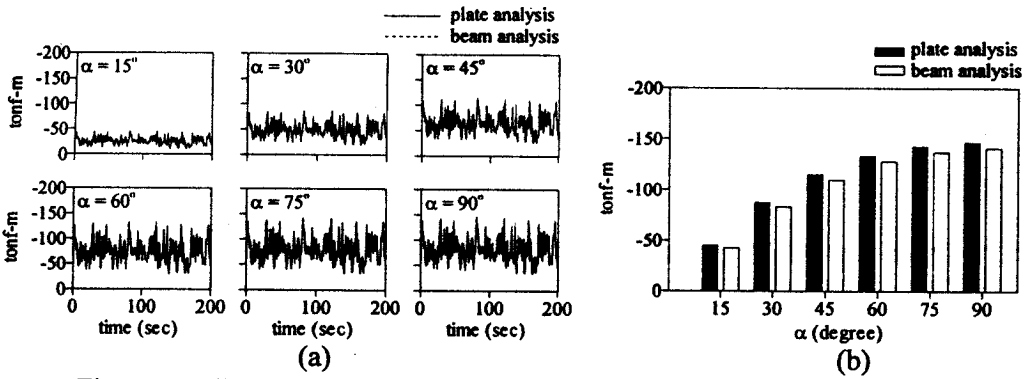
| $\alpha$ (degree) | $C_L$ | $C_D$ | $C_M$ |
|-------------------|-------|-------|-------|
| 15                | 0.605 | 0.162 | 0.057 |
| 30                | 1.055 | 0.609 | 0.086 |
| 45                | 1.139 | 1.139 | 0.090 |
| 60                | 0.937 | 1.622 | 0.074 |
| 75                | 0.520 | 1.941 | 0.041 |
| 90                | 0.000 | 2.066 | 0.000 |

In beam analyses, by using distributed loads ( $L$ ,  $D$  and  $M$ ) calculated from Eqs. (2), Eq. (1) is solved with the beam model shown in Fig. 4(a). In plate analyses, wind load vectors  $\{f\}_w$  are calculated from Eq. (5) with  $C_p$  shown in Fig. 3. Eq. (1) is solved with the plate model shown in Fig. 4(b).

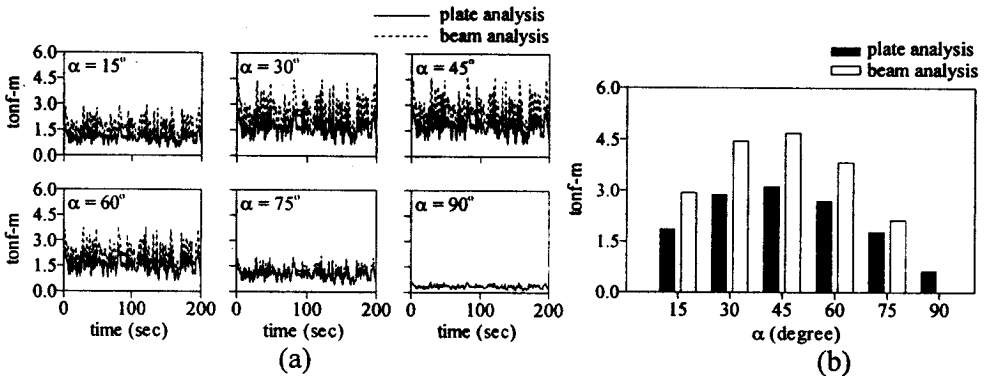


**Fig. 4. Finite element model: (a) Beam element model; (b) Plate element model**

Fig. 5, Fig. 6 and Fig. 7 represents bending moments at the center line, torsional moments at the edge line and principal stresses at the center line of the example structure, respectively. As can be seen from the above results, torsional moments from beam analyses are quite different from those from plate analyses.



**Fig. 5. Bending moments at the center line: (a) Time history; (b) Maxima**



**Fig. 6. Torsional moments at the edge line: (a) Time history; (b) Maxima**

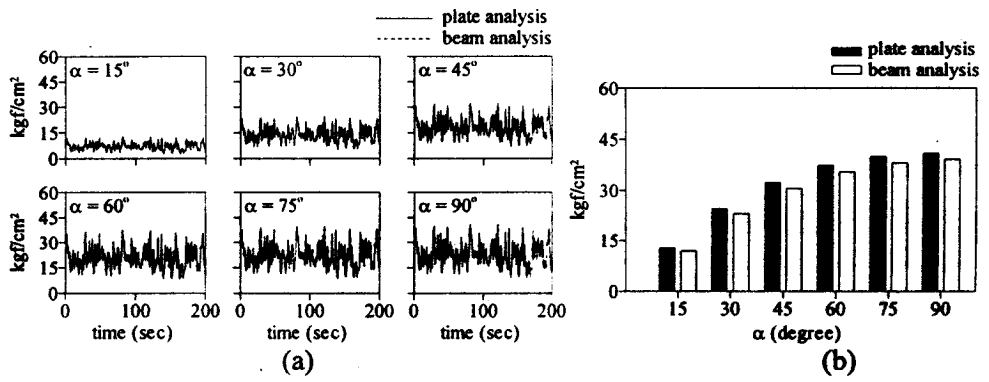


Fig. 7. Principal stresses at the center line: (a) Time history; (b) Maxima

Fig. 8 and Table 2 represent member forces along width directions of the example structure at the time of maximum point. These results verify that member forces which vary along width directions can be obtained from plate analyses, whereas those cannot be exactly calculated from beam analyses due to the limitation of the analyses that constant member forces are obtained along width directions.

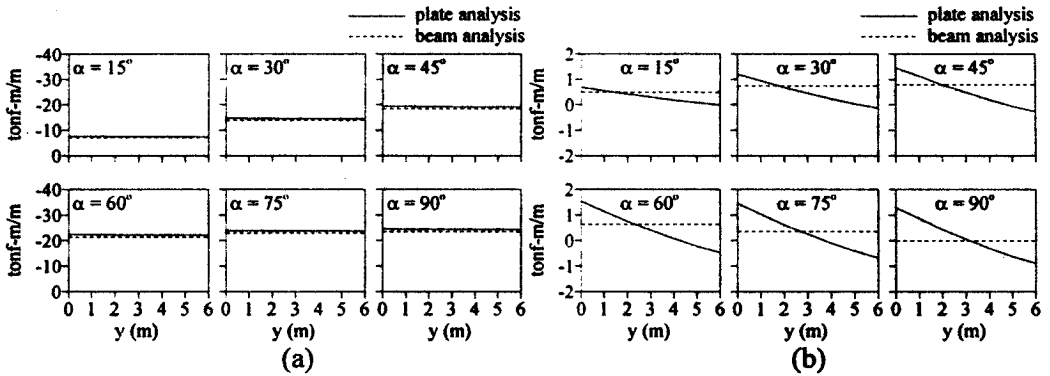


Fig. 8. Member forces along width directions (time = 154 sec):

(a) Bending moments along the center line; (b) Torsional moments along the edge line

Table 2: Member forces along width directions (unit = tonf-m/m, time = 154 sec):

(a) Bending moments along the center line; (b) Torsional moments along the edge line

|     | y (m) | $\alpha = 15^\circ$ |       | $\alpha = 30^\circ$ |        | $\alpha = 45^\circ$ |        | $\alpha = 60^\circ$ |        | $\alpha = 75^\circ$ |        | $\alpha = 90^\circ$ |        |
|-----|-------|---------------------|-------|---------------------|--------|---------------------|--------|---------------------|--------|---------------------|--------|---------------------|--------|
|     |       | plate               | beam  | plate               | beam   | plate               | beam   | plate               | beam   | plate               | beam   | plate               | beam   |
| (a) | 0     | -7.67               | -7.12 | -14.79              | -13.85 | -19.45              | -18.32 | -22.47              | -21.30 | -23.98              | -22.85 | -24.58              | -23.49 |
|     | 1     | -7.62               | -7.12 | -14.70              | -13.85 | -19.34              | -18.32 | -22.35              | -21.30 | -23.87              | -22.85 | -24.48              | -23.49 |
|     | 2     | -7.58               | -7.12 | -14.63              | -13.85 | -19.26              | -18.32 | -22.28              | -21.30 | -23.81              | -22.85 | -24.43              | -23.49 |
|     | 3     | -7.54               | -7.12 | -14.58              | -13.85 | -19.20              | -18.32 | -22.23              | -21.30 | -23.77              | -22.85 | -24.41              | -23.49 |
|     | 4     | -7.52               | -7.12 | -14.54              | -13.85 | -19.15              | -18.32 | -22.19              | -21.30 | -23.75              | -22.85 | -24.41              | -23.49 |
|     | 5     | -7.50               | -7.12 | -14.51              | -13.85 | -19.13              | -18.32 | -22.18              | -21.30 | -23.75              | -22.85 | -24.43              | -23.49 |
|     | 6     | -7.47               | -7.12 | -14.47              | -13.85 | -19.08              | -18.32 | -22.12              | -21.30 | -23.70              | -22.85 | -24.39              | -23.49 |
| (b) | 0     | 0.68                | 0.49  | 1.20                | 0.74   | 1.46                | 0.78   | 1.53                | 0.64   | 1.45                | 0.35   | 1.29                | 0.00   |
|     | 1     | 0.55                | 0.49  | 0.94                | 0.74   | 1.12                | 0.78   | 1.14                | 0.64   | 1.04                | 0.35   | 0.86                | 0.00   |
|     | 2     | 0.42                | 0.49  | 0.68                | 0.74   | 0.78                | 0.78   | 0.75                | 0.64   | 0.62                | 0.35   | 0.43                | 0.00   |
|     | 3     | 0.30                | 0.49  | 0.46                | 0.74   | 0.49                | 0.78   | 0.41                | 0.64   | 0.25                | 0.35   | 0.06                | 0.00   |
|     | 4     | 0.18                | 0.49  | 0.23                | 0.74   | 0.20                | 0.78   | 0.07                | 0.64   | -0.10               | 0.35   | -0.30               | 0.00   |
|     | 5     | 0.08                | 0.49  | 0.03                | 0.74   | -0.06               | 0.78   | -0.22               | 0.64   | -0.41               | 0.35   | -0.62               | 0.00   |
|     | 6     | -0.01               | 0.49  | -0.13               | 0.74   | -0.27               | 0.78   | -0.46               | 0.64   | -0.67               | 0.35   | -0.88               | 0.00   |

## 5. Conclusions

Systematic comparison of simplified beam analyses and three-dimensional plate analyses is presented to verify disadvantages of conventional simplified beam analyses of structures under wind loads. The results from the numerical example can be summarized as follows:

(1) Bending moments and principal stresses from beam analyses are similar to those from plate analyses. On the other hand, torsional moments from beam analyses are quite different from those from plate analyses.

(2) Member forces which vary along width directions can be obtained from plate analyses, whereas those cannot be exactly calculated from beam analyses due to the limitation of the analyses that constant member forces are obtained along width directions.

Therefore, three-dimensional plate analyses are required to exactly calculate torsional moments and width-directional member forces of structures under wind loads, instead of simplified beam analyses.

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