

Smart Damping Strategy of Seismically Excited Cable-Stayed Bridges Using MR Fluid Dampers

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Abstract

This paper proposes a new smart damping strategy for seismic protection of cable-stayed bridges, focusing on inclusion of effects of control-structure interaction, by examining the ASCE first generation benchmark control problem for a seismically excited cable-stayed bridge. Magnetorheological (MR) fluid dampers, which belong to the class of controllable fluid dampers, are the supplemental damping devices proposed for this study, and a clipped-optimal control algorithm, shown to perform well in previous studies involving MR fluid dampers, is employed. A modified Bouc-Wen model, which is obtained from the comprehensive investigation of the full-scale MR fluid damper, is considered for the dynamic model of the MR fluid damper. Because the MR fluid damper is a controllable energy-dissipation device that cannot add mechanical energy to the structural system, the proposed control strategy is fail-safe in that bounded-input, bounded-output stability of the controlled structure is guaranteed. Numerical simulation results show that the performance of the proposed semi-active control strategy using MR fluid dampers is quite effective.

Introduction

In order to protect structures against natural hazards such as severe earthquakes and strong winds, many control algorithms and devices have been proposed over the last few decades. However, direct comparison of different control strategies has proven to be a challenging task because standard structures are generally not considered. This problem can be addressed by employing *testbed* structures, that is, by developing benchmark problems. To compare results of various control strategies such as passive, active, semi-active, or a combinations thereof and to direct future research efforts toward the most promising structural control strategies, benchmark control problems have been initiated and developed by the American Society of Civil Engineers and the International Association of Structural Control (Spencer *et al.* 1998; ASCE 1997; EESD 1998; IASC 1998; Ohtori *et al.* 2000). Until recently, however, all of the benchmark problems considered have focused on the control of buildings.

Because there are a growing number of cable-stayed bridges throughout the world, more research on the seismic protection of such structures is needed. These structures are very flexible, presenting unique and challenging problems. To effectively study the seismic response control of cable-stayed bridges, a benchmark problem for seismic protection has been developed and presented in Dyke *et al.* (2000). This first generation benchmark control problem for seismically excited cable-stayed bridges considers a bridge currently under construction in Cape Girardeau, Missouri, USA, which will be completed in 2003. Based on detailed drawings of this cable-stayed bridge, a three-dimensional linearized evaluation model has been developed to represent the complex behavior of the bridge. For the control design problem, evaluation criteria also have been provided.

Magnetorheological (MR) fluid dampers are the new class of semi-active control devices that utilize MR fluids to provide controllable damping forces. Because of their mechanical simplicity, high dynamic range, low power requirements, large force capacity, and robustness, MR fluid dampers are one of the most promising devices for structural vibration control. MR damper-based control strategies not only offer the reliability of passive control devices but also maintain the versatility and adaptability of fully active control systems. Also, these devices overcome many of the expenses and technical difficulties associated with semi-active devices previously considered. Recent studies indicate that, for certain applications, MR fluid dampers can achieve the majority of the performance of fully active systems (Dyke *et al.* 1996, Jansen and Dyke 2000, Yoshioka *et al.* 2002).

The purpose of this paper is to investigate the effectiveness of a smart damping strategy using MR fluid dampers for the seismic protection of cable-stayed bridges by using the benchmark bridge model provided by Dyke *et al.* (2000). In this study, a modified Bouc-Wen model, which is obtained from the comprehensive investigation of the full-scale MR fluid damper, is considered for the dynamic model of the MR fluid damper. Also, a clipped-optimal control algorithm, shown to perform well in previous studies involving MR fluid dampers (Dyke *et al.* 1996; Dyke and Spencer 1997), is employed. Since the MR fluid damper is an energy-dissipation device that cannot add mechanical energy to the structural system, the proposed control strategy is fail-safe, in that it guarantees the bounded-input, bounded-output stability of the controlled structure. Following a brief overview of the benchmark problem statement, a seismic control design strategy using MR fluid dampers is proposed. Numerical simulation results are then presented to demonstrate the effectiveness of the proposed control strategy.

Benchmark Problem Statement

Benchmark Bridge Model

This benchmark problem considers the cable-stayed bridge shown in Figure 1, which is scheduled for completion in Cape Girardeau, Missouri, USA in 2003. Note that because bearings at pier 4 do not restrict longitudinal motion and rotation about the longitudinal axis of the bridge, the Illinois approach has a negligible effect on the dynamics of the cable-stayed portion of the bridge. In this benchmark study, therefore, only the cable-stayed portion of the bridge is considered. Based on detailed drawings of the bridge, Dyke *et al.* (2000) developed and made available a three-dimensional linearized evaluation model that effectively represents the complex behavior of the full-scale benchmark bridge. Because this bridge is

assumed to be attached to bedrock, the effect of the soil-structure interaction has been neglected. A one-dimensional ground motion is applied in the longitudinal direction for the first generation benchmark problem of seismically excited cable-stayed bridges. More detailed information on the bridge model is explained in Dyke et al. (2000).

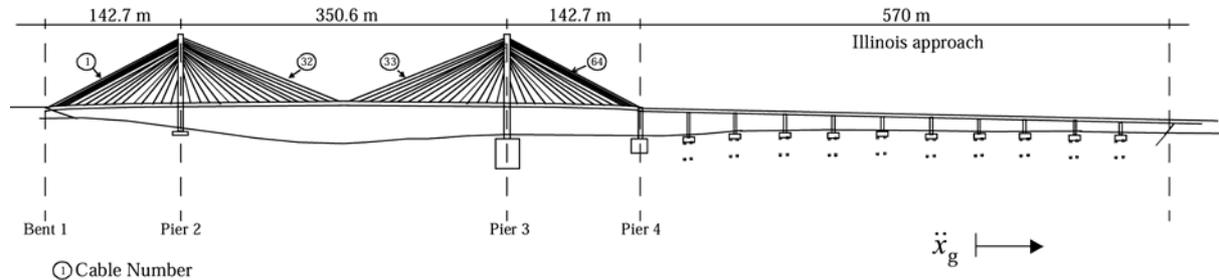


Figure 1. Schematic of the Cape Girardeau Bridge (Dyke *et al.* 2000).

Evaluation Criteria

Eighteen criteria have been defined (Dyke et al. 2000) to evaluate the capabilities of each proposed control strategy. Three historical earthquake records are considered, the 1940 El Centro NS, the 1985 Mexico City and the 1999 Gebze NS.

The first six evaluation criteria (J_1 - J_6) consider the ability of the controller to reduce peak responses. J_1 and J_2 are non-dimensionalized measures of the shear forces at the tower base and the deck level in the towers, respectively. J_3 and J_4 are non-dimensionalized measures of the moments in the towers at the same locations. J_5 is a non-dimensionalized measure of the deviation of the tension in the stay cables from the nominal pretension. J_6 is a measure of the peak deck displacement at piers 1 and 4.

The second five evaluation criteria (J_7 - J_{11}) consider normed (i.e., *rms*) responses over the entire simulation time. In this preliminary study, these criteria do not considered.

The last seven evaluation criteria (J_{12} - J_{18}) consider the requirements of each control system itself. In this stage, only two criteria (J_{12} and J_{13}) about the control system are considered. J_{12} deals with the maximum force generated by the control devices. J_{13} is based on the maximum stroke of the control devices.

Seismic Control System Using MR Fluid Dampers

In this section, a description of the proposed control system using MR fluid dampers is provided. Accelerometers, displacement transducers and force transducers are employed as sensors. MR fluid dampers are used as control devices. A clipped-optimal control algorithm, which has been successfully applied with MR fluid dampers in previous studies (Dyke *et al.* 1996, 1997; Dyke and Spencer 1997), is employed to determine the control action.

Sensors

Five accelerometers and four displacement transducers are used for feedback in the control algorithm. Two accelerometers are located at the top of each of the two towers, and one is located on the deck at mid span. Two displacement sensors are placed between the deck and pier 2, and the other two are placed between the deck

and pier 3. Because the clipped-optimal control algorithm considered herein requires measurement of the damper control forces applied to the structure, 24 force transducers are installed. All sensors employed in this study are assumed to be ideal, having a constant magnitude and phase (Dyke et al. 2000), and the sensitivity of accelerometers (G_a), the displacement transducers (G_d) and the force transducers (G_f) are 7/9.81 V/(m/sec²), 30 V/m and 0.01 V/kN, respectively. Thus, sensors can be modeled as

$$\mathbf{y}_s = \begin{bmatrix} G_a \mathbf{I}_{5 \times 5} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & G_a \mathbf{I}_{5 \times 5} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & G_a \mathbf{I}_{5 \times 5} \end{bmatrix} (\mathbf{y}_m + \mathbf{v}) = \mathbf{G}(\mathbf{y}_m + \mathbf{v}) \quad (1)$$

where \mathbf{y}_s is a vector of the measured responses, including noise, in Volts, \mathbf{y}_m is a vector of the noise-free responses in physical units, and \mathbf{v} is the measurement noise, which has an *rms* value of 0.003 V (Dyke et al. 2000).

Control Devices: MR Fluid Dampers

A total of 24 MR fluid dampers are considered as control devices. Four between the deck and pier 2, eight between the deck and pier 3, eight between the deck and bent 1, and four between the deck and pier 4 are placed. To accurately predict the behavior of the controlled structure, an appropriate modeling of MR fluid dampers is essential.

Yang et al. (2001) proposed a new mechanical model considering shear-thinning and inertial effects, which were observed in the experiment of the full-scale MR fluid damper. The new dynamic model of the overall MR fluid damper system is comprised of two parts: (i) a dynamic model of the power supply (i.e. the current driver), and (ii) a dynamic model of the MR fluid damper.

The current driver can dramatically reduce the MR fluid damper response time compared with a constant voltage power supply (Yang et al. 2000). The dynamic model of the current driver used in the experiment is identified as follows (Yang et al. 2001). The transfer function between the input reference signal \dot{i}_0 and current i is given by

$$i(s) = \frac{1001.45s + 1016.1}{s^2 + 503.7s + 508.05} \dot{i}_0(s) \quad (2)$$

Based on the damper response analysis (Yang et al. 2001), a mechanical model of MR fluid damper is shown in Figure 2. The damper resisting force is given by

$$f = \alpha z + kx + c(\dot{x})\dot{x} + m\ddot{x} + f_0 \quad (3)$$

where the evolutionary variable z is governed by

$$\dot{z} = -\gamma|\dot{x}|z|z|^{n-1} - \beta\dot{x}|z|^n + A\dot{x} \quad (4)$$

In this model, the fluid inertial effect is represented by an equivalent mass m ; the accumulator stiffness is represented by k ; friction force due to the damper seals as well as measurement bias are represented by f_0 ; and the post-yield plastic damping

coefficient is represented by $c(\dot{x})$. To describe the shear-thinning effect on damper resisting force at low velocities as observed in the experimental data, $c(\dot{x})$ is defined as a mono-decreasing function with respect to the absolute velocity $|\dot{x}|$. The post-yield damping coefficient is assumed to have a form of

$$c(\dot{x}) = a_1 e^{-(a_2 |\dot{x}|)^p} \quad (5)$$

where a_1, a_2 and p are positive constants.

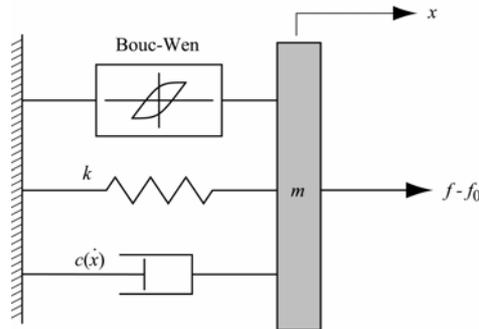


Figure 2. Modified Bouc-Wen model (Yang et al. 2001).

To determine a dynamic model that is valid under the fluctuating input current, the functional dependence of the parameters on the input current must be determined. Since the fluid yield stress is dependent on input current, α can then be assumed as a function of the input current i . Moreover, from the experiment results, a_1, a_2, m, n , and f_0 are also functions of the input current. In this preliminary study, however, m is not considered.

In order to obtain the relationship between the input current i and damper parameters α, a_1, a_2, n , and f_0 the damper was driven by band-limited random displacement excitations with a cutoff frequency of 2 Hz at various constant current levels. A constrained nonlinear least-squares optimization scheme based on the trust-region and preconditioned conjugated gradients (PCG) methods is then used. The results are shown in Table 1. A linear piecewise interpolation approach is utilized to estimate these damper parameters for current levels that are not listed in the above table. The rest damper parameters which are not varied with input current are chosen to be $\gamma = 25179.04 \text{ m}^{-1}$, $\beta = 27.1603 \text{ m}^{-1}$, $A = 1377.9788 \text{ m}^{-1}$, $k = 20.1595 \text{ N/m}$, and $p = 0.2442$.

Table 1. MR fluid damper parameters at various current levels under random displacement excitations.

Current (A)	α (10^5 N)	a_1 ($10^3 \text{ N}\cdot\text{sec/m}$)	a_2 (sec/m)	n	f_0 (N)
0.0237	1.4520	4359	686.72	1.000	1457.27
0.2588	2.2374	22830	2431.57	2.1058	2686.37
0.5124	2.2732	27457	2576.14	3.6951	4438.65

0.7625	2.1132	30122	2555.31	5.5195	4351.04
1.0132	2.1527	24926	1819.33	6.1414	2733.63
1.5198	2.1341	33606	2850.69	7.3619	5653.91
2.0247	2.2209	32307	2814.35	7.3092	5217.19

Note that a first order filter needs to be used to accommodate the dynamics involved in the MR fluid reaching rheological equilibrium (Yang et al. 2001)

$$H(s) = \frac{31.4}{s + 31.4} \quad (6)$$

The vector of forces \mathbf{f} produced by the dampers are thus written as

$$\mathbf{f} = \mathbf{K}_f [f_1 \ f_2 \ \dots \ f_n]^T \quad (7)$$

where f_i is the force generated by the i th damper installed in the structure (see (3)), and \mathbf{K}_f is a matrix of zeros and ones that accounts for the number of and location of the installed devices.

Control Design Model

Because the evaluation model is too large for control design and implementation, a reduced-order model (i.e., design model) of the system should be developed. The design model given by Dyke *et al.* (2000) was derived from the evaluation model by forming a balanced realization of the system and condensing out the states with relatively small controllability and observability grammians to obtain (Laub *et al.* 1987):

$$\dot{\mathbf{x}}_d = \mathbf{A}_d \mathbf{x}_d + \mathbf{B}_d \mathbf{f} + \mathbf{E}_d \ddot{\mathbf{x}}_g \quad (8)$$

$$\mathbf{z} = \mathbf{C}_d^z \mathbf{x}_d + \mathbf{D}_d^z \mathbf{f} + \mathbf{F}_d^z \ddot{\mathbf{x}}_g \quad (9)$$

$$\mathbf{y}_s = \mathbf{G}(\mathbf{C}_d^y \mathbf{x}_d + \mathbf{D}_d^y \mathbf{f} + \mathbf{F}_d^y \ddot{\mathbf{x}}_g + \mathbf{v}) \quad (10)$$

where \mathbf{x}_d is the design state vector with a dimension $d = 30$, $\ddot{\mathbf{x}}_g$ is the ground acceleration, \mathbf{f} is the applied control force, \mathbf{z} the regulated output vector including shear forces and moments in the towers, deck displacements, and cable tension forces, and \mathbf{y}_s is the output responses from the sensors that are used for control signal determination (see (1)).

Control Schemes for MR Fluid Dampers

The strategy of a clipped-optimal control algorithm (Dyke *et al.* 1996; Dyke and Spencer 1997) for seismic protection using MR fluid dampers is as follows: First, an “ideal” active control device is assumed, and an appropriate *primary* controller for this active device is designed. Then a *secondary* bang-bang-type controller causes the MR fluid damper to generate the desired active control force, so long as this force is dissipative. This approach is adopted for control of the cable-stayed bridge.

In this study, an H_2/LQG control design (Spencer *et al.* 1994) is adopted as the

primary controller. The ground excitation is taken to be a stationary white noise, and an infinite horizon performance index is chosen that weights appropriate parameters of the structure, i.e.,

$$J = \lim_{\tau \rightarrow \infty} \frac{1}{\tau} E \left[\int_0^{\tau} \{ \mathbf{z}^T \mathbf{Q} \mathbf{z} + \mathbf{f}^T \mathbf{R} \mathbf{f} \} dt \right] \quad (11)$$

where \mathbf{R} is an identity matrix, and \mathbf{Q} is the response weighting matrix. A stochastic response analysis has been performed to determine appropriate values of the weighting parameters. Through the preliminary parametric study, the following combination of weighting parameters is considered:

$$\mathbf{Q}_{\text{om\&d}} = \begin{bmatrix} q_{\text{om}} \mathbf{I}_{4 \times 4} & \mathbf{0} \\ \mathbf{0} & q_{\text{d}} \mathbf{I}_{4 \times 4} \end{bmatrix} \quad (12)$$

where q_{om} and q_{d} weight the overturning moments and the deck displacements, respectively. By employing the above weighting matrix in the H_2 /LQG to obtain the *primary* controller $\mathbf{K}_c(s)$, a “desired” active control command is obtained. This desired control force vector $\mathbf{f}_c = [f_{c1} \ f_{c2} \ \dots \ f_{cn}]^T$ can then be written as

$$\mathbf{f}_c = L^{-1} \left\{ -\mathbf{K}_c(s) L \begin{Bmatrix} \mathbf{y}_m \\ \mathbf{y}_f \end{Bmatrix} \right\} \quad (13)$$

where f_{ci} is the desired control force signal for the i th MR damper, \mathbf{y}_m is the measured structural response vector, \mathbf{y}_f is the measured control force vector, and $L^{-1}\{\}$ is the inverse Laplace transform operator.

Because the force generated in the i th MR damper are dependent on the responses of the structural system, the MR damper cannot always produce the desired optimal control forces. Only the control current i_i can be directly controlled. Thus, a force feedback loop is incorporated to induce the force in the MR damper f_i to generate approximately the desired optimal control force f_{ci} . To this end, the i th command signal i_i is selected according to the control law

$$i_i = I_{\text{max}} H[(f_{ci} - f_i) f_i] \quad (14)$$

where I_{max} is the current to the current driver associated with saturation of the MR effect in the physical device, and $H(\cdot)$ is the Heaviside step function.

Numerical Simulation Results

To verify the effectiveness of the proposed control design, a set of simulations is performed for the three earthquakes specified in the benchmark problem statement (Dyke *et al.* 2000). Simulation results for the proposed control design are compared to those of a fully active control design, which employs the H_2 /LQG method as control algorithm, those of an ideal semi-active control design, which does not

considered the dynamics of control devices, and those of two passive cases in which the MR fluid damper is used. *Passive-off* and *passive-on* refer to the cases in which the current to the MR fluid damper is held at a constant value of $i = 0$ A and $i = i_{\max} = 2$ A, respectively. In this preliminary study, appropriate values of weighting parameters for the proposed semi-active control design and the active control design are determined to be (see (12)) $q_{om}=6\times 10^{-9}$ and $q_d=6\times 10^3$.

Table 2 shows the maximum values of the 18 evaluation criteria for all the three earthquakes. Note that in this study ideal hydraulic actuators are considered as active control devices, (i.e., actuator dynamics are neglected), which is consistent with the sample controller provided by Dyke *et al.* (2000).

As shown in Table 2, the performance of the proposed smart damping system considering the dynamics of MR fluid dampers is not quite different from that of ideal semi-active control system, which does not consider the dynamics of control devices. Some of the maximum evaluation criteria in the case of considering the dynamics of MR fluid dampers are worse than those in the case of not considering the dynamics (e.g., J_3 (14.8 %) and J_5 (8.3 %)), whereas some else of the criteria in the device dynamics-considering case are better than those in the device dynamics-neglecting case (e.g., J_4 (8.3 %), J_6 (30.4 %) and J_{13} (30.3 %)). Although including the dynamics of control devices would generally worsen the achievable results, some results in the case of including the dynamics show better than those in the ideal case. That might be because the proposed smart damping devices still act as passive-type viscous dampers when the calculated semi-active control force is zero, as opposed to the ideal smart damping system. This behavior of the damper could affect the performance of the proposed control system affirmatively. Moreover, the clipped-optimal controller shows the better performance than the passive modes (i.e. *passive-off* and *-on*).

Table 2. Maximum evaluation criteria for all the three earthquakes.

Criterion	Ideal active control	Smart damping			
		Ideal smart damping	Modified Bouc-Wen model		
			Passive-off	Passive-on	Clipped-optimal
J_1	0.4992	0.4558	0.4437	0.6364	0.4597
J_2	1.1988	1.1944	1.4969	1.3191	1.1228
J_3	0.4461	0.4761	0.5211	0.7564	0.5586
J_4	0.8692	0.8281	2.1176	0.6533	0.7647
J_5	0.1571	0.1782	0.2216	0.2131	0.1943
J_6	2.0181	1.9666	6.4827	0.6827	1.5086
J_{12}	1.9608e-3	1.9608e-3	4.2543e-4	1.9608e-3	1.9608e-3
J_{13}	1.1065	1.0782	3.5543	0.4483	0.8271

To demonstrate the feasibility of these controllers, peak values of the force, stroke, and velocity are provided for each earthquake in Table 3. The force, stroke, and velocity requirements presented by Dyke *et al.* (2000) are 1000 kN, 0.2 m, and 1 m/sec, respectively. As seen from Table 3, all the three maximum responses satisfy the actuator constraints in the proposed smart damping case. In the *passive-off* case,

however, the maximum stroke of the damper is larger than the required value under the 1999 Gebze NS earthquake.

Table 3. Actuator requirements for control strategies.

Earthquake	Max.	Ideal active control	Smart damping			
			Ideal smart damping	Modified Bouc-Wen model		
				Passive-off	Passive-on	Clipped-optimal
1940 EI Centro NS	Force (kN)	1000	1000	216.94	1000	1000
	Stroke (m)	0.0956	0.0803	0.1222	0.0666	0.0740
	Velocity (m/s)	0.6301	0.5637	0.6093	0.5140	0.5713
1985 Mexico City	Force (kN)	865.31	691.05	199.12	1000	766.48
	Stroke (m)	0.0437	0.0387	0.0497	0.0126	0.0314
	Velocity (m/s)	0.2766	0.2213	0.2084	0.1745	0.1874
1999 Gebze NS	Force (kN)	1000	1000	216.97	1000	1000
	Stroke (m)	0.1451	0.1414	0.4662	0.0430	0.1085
	Velocity (m/s)	0.4324	0.4439	0.6690	0.3394	0.5922

Conclusions

In this paper, a semi-active control strategy using MR fluid dampers has been proposed by investigating the ASCE first benchmark control problem for seismic responses of cable-stayed bridges. The proposed control design employs five accelerometers, four displacement transducers and 24 force transducers as sensors, a total of 24 MR fluid dampers as control devices, and the controller has 30 states. The modified Bouc-Wen model, which is obtained from the comprehensive investigation of the full-scale MR damper, is considered for the dynamic model of the MR fluid damper. A clipped-optimal control algorithm is used to determine the control action for each MR fluid damper. The numerical results demonstrate that the performance of the proposed control design is quite effective. In addition, semi-active control strategy has many attractive features, such as the bounded-input, bounded-output stability and small energy requirements. The results of this preliminary investigation indicate that MR fluid dampers could effectively be used for control of seismically excited cable-stayed bridges.

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