Implementation of Modal Control for Seismically Excited Structures using Magnetorheological Dampers

Sang-Won Cho¹; Byoung-Wan Kim²; Hyung-Jo Jung³; and In-Won Lee, M.ASCE⁴

Abstract: This paper proposes an implementation of modal control for seismically excited structures using magnetorheological (MR) dampers. Many control algorithms such as clipped-optimal control, decentralized bang-bang control, and the control algorithms based on Lyapunov stability theory have been adopted for semiactive systems including MR dampers. In spite of good features, some algorithms have drawbacks such as poor performance or difficulties in designing the weighting matrix of the controller. However, modal control reshapes the motion of a structure by merely controlling a few selected vibration modes. Hence a modal control scheme is more convenient to design the controller than other control algorithms. Although modal control has been investigated for several decades, its potential for semiactive control, especially for the MR damper, has not been exploited. Thus, in order to study the effectiveness for a MR damper system, a modal control scheme is implemented to seismically excited structures. A Kalman filter is included in a control scheme to estimate modal states from measurements by sensors. Three cases of the structural measurement are considered by a Kalman filter to verify the effect of each measurement; displacement, velocity, and acceleration, respectively. Moreover, a low-pass filter is applied to eliminate the spillover problem. In a numerical example, a six-story building model with the MR dampers on the bottom two floors is used to verify the proposed modal control scheme. The El Centro earthquake is used to excite the system, and the reduction in the drifts, accelerations, and relative displacements throughout the structure is examined. The performance of the proposed modal control scheme is compared with that of other control algorithms previously studied. The numerical results indicate that the motion of the structure is effectively suppressed by merely controlling a few lowest modes, although resulting responses varied greatly depending on the choice of measurements available and weightings.

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Introduction

Magnetorheological (MR) dampers are semiactive control devices which use MR fluids to provide controllable damping forces. MR dampers are quite promising for civil engineering applications, since they have many attractive features such as small power requirements, reliability, and are inexpensive to manufacture (Dyke and Spencer 1996; Ginder et al. 1996; Kamath and Werley 1997; Spencer et al. 1997; Spencer and Sain 1997; Dyke et al. 1998). A number of control algorithms have been adopted for semiactive systems including the MR damper: Jansen and Dyke (2000) discussed recently proposed semiactive control algorithms including the decentralized bang-bang controller (McClamroch and Gavin 1995), the controller based on Lyapunov stability theory (Brogan 1991; Leitmann 1994); the clipped-optimal controller (Sack and Patten 1994; Sack et al. 1994; Dyke 1996a,b,c,d); the modulated homogeneous friction controller (Inaudi 1997); and the maximum energy dissipation algorithm. They also formulated these algorithms for use with MR dampers and evaluated and compared the performance of each algorithm.

Modal control represents one control class, in which the motion of a structure is reshaped by merely controlling some selected vibration modes. Modal control is especially desirable for the vibration control of a civil engineering structure, which is usually a large structural system, that may involve hundreds or even thousands of degrees of freedom. Its vibration is usually dominated by the first few modes. Therefore the motion of the structure can be effectively suppressed by merely controlling these few modes (Yang 1982; Lu 2001; Lu and Chung 2001). To date, numerous procedures and algorithms concerning modal control or pole assignment have been proposed in literature. A modal control method using full state feedback may not be practical for a structural system involving a large number of degrees of freedom (DOFs), since the control implementation may require a large amount of sensors. Thus a modal control scheme, which uses modal state estimation, is desirable. To estimate the modal states from the sensor output, a Luenberberg observer (Luenberger 1971; Meirovitch 1990) and a Kalman-Bucy filter (Meirovitch 1967) can be used for the case of low noise-to-signal ratios and for high noise-to-signal ratios, respectively. The troublesome part

¹PhD, Dept. of Civil and Environmental Engineering, Korea Advanced Institute of Science and Technology, 373-1 Guseong-dong, Daejeon 305-701, Korea (corresponding author). E-mail: s.w.cho@kaist.ac.kr
²PhD, KORDI, Yuseong P.O. Box 23, Daejeon 305-343, Korea. E-mail: kimbw@kriso.re.kr
³Professor, Dept. of Civil and Environmental Engineering, Sejong Univ., 98 Gunja-dong, Gwangjin-gu, Seoul 143-747, Korea. E-mail: hjung@sejong.ac.kr
⁴Professor, Dept. of Civil and Environmental Engineering, Korea Advanced Institute of Science and Technology, 373-1 Guseong-dong, Daejeon 305-701, Korea. E-mail: iwlee@kaist.ac.kr

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of estimating the modal states for feedback in modal control is the problem of spillover. Note, however, that a small amount of damping inherent in the structure is often sufficient to overcome the observation spillover effect (Meirovitch and Baruh 1983). At any rate, observation spillover can be eliminated if the sensor signals are prefiltered so as to screen out the contribution of the uncontrolled modes. Recently, a modal control scheme was applied for vibration mitigation of seismic structures using variable friction dampers, a kind of semiactive control device (Lu 2004).

The purpose of this study is to implement modal control for seismically excited structures that use MR dampers and to compare the performance of the proposed method with that of other control algorithms already studied. A modal control scheme with a Kalman filter and a low-pass filter is applied. A Kalman filter is included in a control scheme to estimate modal states from measurements by sensors. Three cases of the structural measurement are considered by a Kalman filter to verify the effect of each measurement; displacement, velocity, and acceleration, respectively. Moreover, a low-pass filter is applied to eliminate the spillover problem. In a numerical example, a six-story building model with the MR dampers on the bottom two floors is used to verify the effectiveness of the proposed modal control scheme, which was selected with the same test structure in Jansen and Dyke (2000) for the direct comparison between the control algorithms. An experimentally verified phenomenological model based on the Bouc–Wen model (Yi et al. 1998; Dyke et al. 1999; Yi et al. 1999; Jansen and Dyke 2000) is used to describe the behavior of the MR damper. The responses of the excited system are examined for each algorithm, and the performance of the various control algorithms on the multi-input system are compared.

Modal Control Scheme for Magnetorheological Dampers

In this section, a modal control scheme with a Kalman filter and a low-pass filter is implemented to seismically excited structure. A Kalman is included in a control scheme to estimate modal states from various measurements. Moreover, a low-pass filter is applied to eliminate the spillover problem. After the implementation of a modal control scheme, numerical simulation is presented in a subsequent section for comparisons between control algorithms.

Modal Control

Consider a seismically excited structure controlled with \( n \) MR dampers. Assuming that the forces provided by the control devices are adequate to keep the response of the primary structure from exiting the linear region, the equations of motion can be written

\[
M \ddot{x}(t) + C \dot{x}(t) + Kx(t) = \Lambda f(t) - M \Gamma \ddot{x}_g
\]

where \( M, C, \) and \( K = n \times n \) mass, damping, and stiffness matrices, respectively; \( x = n \)-dimensional vector of the relative displacements of the floors of the structure; \( f = [f_1, f_2, \ldots, f_n]^T \) = vector of measured control forces generated by \( m \) MR dampers; \( \ddot{x}_g \) = ground acceleration; \( \Gamma = \) column vector of ones; and \( \Lambda = \) matrix determined by the placement of MR dampers in the structure. This equation can be written in state-space form as

\[
\dot{z} = Fz + Gf + N \ddot{x}_g
\]

where \( y = Hz + Mf + v \) \hspace{1cm} (2b)

where \( z = \) state vector; \( y = \) vector of measured outputs; and \( v = \) measurement noise vector. The displacement can be expressed as the linear combination

\[
x(t) = \sum_{r=1}^{n} \phi_r \eta_r(t) = \Phi \eta, \ r = 1, 2, \ldots, n
\]

where \( \eta_r(t) = \) rth modal displacement; \( \phi_r = \) rth eigenvector; \( \Phi = \) eigenvector set; and \( \eta = \) modal displacement vector. The eigenvectors are orthogonal and can be normalized so as to satisfy the orthonormality conditions

\[
\phi_r^T M \phi_r = \delta_{r,s}, \ \phi_r^T K \phi_r = \omega_r^2 \delta_{r,s}, \ r = 1, 2, \ldots, n
\]

where \( \delta_{r,s} = \) Kronecker delta and \( \omega_r = \) natural frequency. Thus inserting Eq. (3) into Eq. (1), multiplying by \( \phi_r^T \), and considering orthogonal condition between eigenvectors, we obtain

\[
\ddot{\eta}_r + 2 \zeta_r \omega_r \dot{\eta}_r + \omega_r^2 \eta_r = \phi_r^T \dot{f} - \phi_r^T \Gamma \ddot{x}_g, \ r = 1, 2, \ldots, n
\]

where \( \zeta_r = \) modal damping ratios. Eq. (5) can be written in the matrix form as

\[
\ddot{\eta}(t) + \Delta \dot{\eta}(t) + \Omega^2 \eta(t) = B^T \dot{f}(t) + E^T \ddot{x}_g
\]

where \( \Delta = \) diagonal matrix listing \( 2 \zeta_r \omega_r \); \( \Omega = \) diagonal matrix listing \( \omega_1^2; \ldots; \omega_n^2 \); \( B^T = \Phi^T \Lambda \); and \( E^T = \Phi^T \Gamma \). Eq. (6) can be written in the modal space-state form as

\[
\dot{w}(t) = Aw(t) + Bf(t) + E \ddot{x}_g \hspace{1cm} (7a)
\]

\[
y(t) = Cw(t) \hspace{1cm} (7b)
\]

where \( w(t) = [\eta^T; \dot{\eta}^T]^T = \) modal state vector and

\[
A = \begin{bmatrix} 0 & I \\ -\Omega^2 & -\Delta \end{bmatrix}, \ B = \begin{bmatrix} 0 \\ B^T \end{bmatrix}, \ E = \begin{bmatrix} 0 \\ E^T \end{bmatrix}
\]

In modal control, only a limited number of lower modes are controlled. Hence, \( l \) controlled modes can be selected with \( l < n \) and the displacement may be partitioned into controlled and uncontrolled parts as

\[
x(t) = x_c(t) + x_R(t)
\]

where \( x_c = m = \) \( 2l \)-dimensional modal state vector by the controlled modes and

\[
x_c(t) = A_c w_c(t) + B_c f(t) + E_c \ddot{x}_g
\]

\[
y_c(t) = C_c w_c(t)
\]

where \( A_c = \) \( 2l \times 2l \) matrix, \( B_c = \) \( 2l \times 1 \) matrix, and \( E_c = \) \( 2l \times 1 \) matrix. For feedback control, the control vector is related to the modal state vector according to

\[
f(t) = -K_c w_c(t)
\]

where \( K_c = m \times 2l \) control gain matrix. The control gain matrix \( K_c \) can be determined by various control laws, however, \( H_2 / LQG \) (Linear Quadratic Gaussian) methods are advocated in the following section because of their successful application in previous
always produce the desired optimal control force to the responses of the structural system, the MR damper cannot = Heaviside step function.

Consider a clipped-optimal control law given by Eq. (12), where

\[ F_{ci} = \max \{k_{Q} v_{i} - f_{ci}, 0\} \]

To this end, the \( i \)th command signal \( v_{i} \) is selected according to the control law

\[ v_{i} = V_{\text{max}} H[(f_{ci} - f_{i})/f_{i}] \]  

where \( V_{\text{max}} \) = voltage to the current driver associated with saturation of the MR effect in the physical device, and \( H(\cdot) \) = Heaviside step function.

**Design of Optimal Controller**

Referring to the discussions in above section, control gain matrix \( k_{C} \) should be decided. Although a variety of approaches may be used to design the optimal controller, \( H_{r}/LQG \) (Linear Quadratic Gaussian) methods are advocated because of their successful application in previous studies (Dyke et al. 1996a,b,c,d). For the controller design, \( \hat{x}_{k} \) is taken to be a stationary white noise, and an infinite horizon performance index is chosen that weights the modal states by controlled modes such as

\[ J = \lim_{T \rightarrow \infty} \frac{1}{T} \mathbb{E} \left[ \int_{0}^{T} (w_{i}^{T}Q_{W} + u^{T}R_{u}) dt \right] \]  

where \( R = 2 \times 2 \) identity matrix because the numerical example has two MR dampers, and \( Q = 2 \times 2 \) diagonal matrix. It should be noted that the size of \( Q \) is reduced from \( 2n \times 2n \) to \( 2 \times 2 \) because the limited lower modes are controlled. Therefore it can be said that it is more convenient to design the smaller weighting matrix of modal control. For example, when the lowest one mode is controlled for calculating the modal control action, \( Q \) is a \( 2 \times 2 \) diagonal matrix such as

\[ Q = \begin{bmatrix} q_{md} & 0 \\ 0 & q_{me} \end{bmatrix} \]  

where \( q_{md} \) = weighting element for a modal displacement and \( q_{me} \) is for a modal velocity. When the lowest two modes are controlled, \( Q = 4 \times 4 \) diagonal matrix.

\[ Q = \begin{bmatrix} q_{md1} & 0 \\ 0 & q_{md2} \\ q_{me1} & 0 \\ 0 & q_{me2} \end{bmatrix} \]  

The measurement noise is assumed to be identically distributed, statistically independent Gaussian white noise processes, and \( \sigma_{w_{i}}^{2} / \sigma_{u_{i}}^{2} = \gamma = 100 \). Then, the controller is

\[ G_{C}(s) = K_{C}(sI - (A_{C} - LQ_{C}C)^{-1}B_{C} \]

where

\[ \hat{\omega}_{C}(t) = A_{C}\hat{\omega}_{C}(t) + B_{C}f(t) + E_{C}\hat{x}_{k} + L[y(t) - C_{C}\hat{\omega}_{C}(t) - D_{C}f(t)] \]

where \( \hat{\omega}_{C}(t) \) = estimated controlled modal state and \( L = \) optimally chosen observer gain matrix by solving a matrix Riccati equation, which assumes that the noise intensities associated with an earthquake and sensors are known. \( C_{C} \) is changeable according to the signals which are used for the feedback and \( D_{C} \) is generally zero except the acceleration feedback. For modal state estimation from the displacements, \( C_{C} \) in Eq. (22) is as follows:

\[ C_{C} = [\Phi_{C} 0] \]  

For control with the velocity feedback

\[ C_{C} = [0 \Phi_{C}] \]  

For control with the acceleration feedback

\[ PA_{C} + A_{C}P - PB_{C}B_{C}P + C_{C}Q_{C}C_{C} = 0 \]  

and

\[ L = (C_{C}S)' \]  

where \( S = \) solution of the algebraic Riccati equation given by

\[ SA_{C}' + A_{C}S - SC_{C}'C_{C}S + \gamma E_{C}E_{C}' \]
To examine the effect of the control forces on the uncontrolled modes, the sensor signals will include contributions from all the modes, so that the output vector is corrected to

$$y = Cw(t) = C_\mathcal{C} \mathcal{C}w(t) + C_Kw_R(t)$$

(27)

To examine the effect of the control forces on the uncontrolled modes, residual modes can be written

$$\hat{w}_R(t) = A_Kw_R(t) + B_Ky(t) + E_R\ddot{x}\dot{x}$$

(28)

where $w_R$ = residual state vector by uncontrolled modes. Substituting Eq. (26) into Eq. (10a) and considering Eq. (28), we obtain

$$\dot{w}_c(t) = A_Kw_R(t) - B_K\dot{w}_c(t) + E_C\ddot{x}\dot{x}$$

(29a)

$$\dot{w}_R(t) = A_Kw_R(t) - B_K\dot{w}_c(t) + E_R\ddot{x}\dot{x}$$

(29b)

Moreover, substituting Eqs. (26) and (27) into Eq. (22), we can write the observed equation in the form

$$\hat{w}_c(t) = (A_c - B_cK_c)\hat{w}_c(t) + LC_c[w_c(t) - \hat{w}_c(t)] + LC_Rw_R(t)
+ E_C\ddot{x}\dot{x}$$

(30)

Then the error vector is defined

$$e_c(t) = \hat{w}_c(t) - w_c(t)$$

(31)

so that Eqs. (29) and (30) can be rearranged

$$\dot{w}_c(t) = (A_c - B_cK_c)\hat{w}_c(t) - B_cK_c e_c(t) + E_C\ddot{x}\dot{x}$$

$$\dot{w}_R(t) = -B_K\hat{w}_c(t) + A_Kw_R(t) - B_K\hat{w}_c(t) + E_R\ddot{x}\dot{x}$$

(32)

$$\dot{e}_c(t) = (A_c - LC_c)e_c(t) + LC_Rw_R(t) + E_C\ddot{x}\dot{x}$$

(33)

Note that the term $-B_KK_c$ in Eq. (33) is responsible for the excitation of the residual modes by the control forces and is known as control spillover (Balas 1978). If $C_R$ is zeros, which means the sensor signal only includes controlled modes, the term $-B_KK_c$ has no effect on the eigenvalues of the closed-loop system. Hence we conclude that control spillover cannot destabilize the system, although it may cause some degradation in the system performance. Normally, however, the above system cannot satisfy the separate principle because the term $LC_R$ affects eigenvalues of the controlled system by the observer. This effect is known as observation spillover and can produce instability in the residual modes. However, a small amount of damping inherent in the structure is often sufficient to overcome the observation spillover effect. (Meirovitch and Baruh 1983). At any rate, observation spillover can be eliminated if the sensor signals are prefiltered so as to screen out the contribution of the uncontrolled modes (Meirovitch 1990)

**Elimination of Observable Spillover**

Eq (33) in the above section should be further improved for eliminating the observable spillover. A low-pass filter is introduced to measure the filtered response vector $y_f$ defined as

$$\dot{y}_f(t) = F_0\dot{z}(t) + G_0y(t)$$

(34)

or in the frequency domain

$$y_f(j\omega) = H_j(j\omega)y(j\omega)$$

(35)

where $H_j(j\omega) = [H_j(j\omega - j\omega)^{-1}G_0 + M_j]$. Substituting Eq. (27) into Eq. (35), the new sensor dynamics becomes

$$y_f(j\omega) = H_j(j\omega)[C_Rw_c(j\omega) + C_Kw_R(j\omega)]$$

(36)

If the low-pass filter dynamics $H_j(j\omega)$ can be selected as a diagonal matrix, Eq. (36) becomes

$$y_f(j\omega) = C_I[H_j(j\omega)w_c(j\omega) + C_Kw_R(j\omega)]$$

(37)

The pole of the low-pass filter dynamics can be placed by proper selection of the parameters, $H_j, F_j, G_j, M_j$, then the roll-off can occur for the lowest modal frequency of the residual dynamics. The second term of the right-hand side of Eq. (37), which represents the residual modal state, may have the following characteristics:

$$|H_j(j\omega)w_R(j\omega)| \equiv \epsilon_2 |w_R(j\omega)|$$

(38)

where $\epsilon_2 \equiv 0$. Otherwise, the first term of the right-hand side of Eq. (37), which represents the controlled modal state, may also have the following characteristics:

$$|H_j(j\omega)w_c(j\omega)| \equiv |w_c(j\omega)|$$

(39)

From Eqs. (38) and (39), the new sensor dynamics $y_f$ can be rewritten as

$$y_f(j\omega) \equiv C_Iw_c(j\omega) + \epsilon_2 C_Kw_R(j\omega) + O(\epsilon)$$

(40)

or in time domain

$$y_f(t) = C_Iw_c(t) + \epsilon_2 C_Kw_R(t) + O(\epsilon)$$

(41)

Substituting Eq. (41) into Eq. (34), the controlled system matrix in Eq. (33) becomes

$$\begin{bmatrix}
\dot{w}_c(t) \\
\dot{w}_R(t) \\
\dot{e}_c(t)
\end{bmatrix} =
\begin{bmatrix}
A_c - B_cK_c & 0 & -B_cK_c \\
-\epsilon_2 & 0 & -\epsilon_2 & -B_cK_c
\end{bmatrix}
\begin{bmatrix}
w_c(t) \\
w_R(t) \\
e_c(t)
\end{bmatrix}
+ \begin{bmatrix}
E_C \\
E_R \\
E_C
\end{bmatrix}\ddot{x}\dot{x}$$

(42)

where $\epsilon_2 = \epsilon_1 C_K \equiv 0$. Thus the separate principle can be applied in the design of observer gain since the term $L$ in $\epsilon_2$ no longer contributes to the characteristics of the system. In other words, the observable spillover does not occur in this controlled system. Hence the controlled modal states in Eq. (22) may be suppressed by a well-designed control input, and the residual modal states may be also attenuated by their natural damping.

**Numerical Example**

To evaluate the proposed modal control scheme for use with the MR damper, a numerical example is considered in which a model of a six-story building is controlled with four MR dampers (Fig. 1). This numerical example is the same as that of Jansen and
Dyke (2000) and is adopted for direct comparisons between the proposed modal control scheme and other control algorithms. Two MR dampers are rigidly connected between the ground and the first floor, and two MR dampers are rigidly connected between the first and second floors. Each MR damper is capable of producing a force equal to 1.8% the weight of the entire structure, and the maximum voltage input to MR devices is $V_{max} = 5$ V. The governing equations can be written in the form of Eq. (7) by defining the mass of each floor, $m_i$, as 0.227 N/(cm/s^2), the stiffness of each floor, $k_i$, as 297 N/cm, and a damping ratio for each mode of 0.5%. MR damper parameters used in this study are $c_{a0} = 0.0064$ Ns/cm, $c_{b0} = 0.0052$ Ns/cm V, $a_0 = 8.66$ N/cm, $a_1 = 8.86$ N/cm V, $g = 300$ cm^2, $b = 300$ cm^2, $A = 120$, and $n = 2$. In simulation, the model of the structure is subjected to the NS component of the 1940 El Centro earthquake. Because the building system considered is a scaled model, the amplitude of the earthquake was scaled to 10% of the full-scale earthquake.

Fig. 2 shows the uncontrolled responses of the sixth floor in frequency domain. We can find that the first mode is dominant in all responses of the sixth floor. Thus it will be possible to reduce the responses through modal control that control using the lowest one or two modes.

The various control algorithms were evaluated using a set of evaluation criteria based on those used in the second generation linear control problem for buildings (Spencer et al. 1997). The first evaluation criterion is a measure of the normalized maximum floor displacement relative to the ground, given as

$$J_1 = \max_{i,j} \left| \frac{x_i(t)}{x_{max}} \right|$$

where $x_i(t)$ is relative displacement of the $i$th floor over the entire response, and $x_{max}$ denotes the uncontrolled maximum displacement. The second evaluation criterion is a measure of the reduction in the interstory drift. The maximum of the normalized interstory drift is

$$J_2 = \max_{i,j} \left( \frac{|d_i(t)/h|}{d_{max}} \right)$$

where $h_i$ = height of each floor (30 cm), $d_i(t) =$ interstory drift of the above ground floors over the response history, and $d_{max}$ denotes the normalized peak interstory drift in the uncontrolled response. The third evaluation criterion is a measure of the normalized peak floor accelerations, given by
J_3 = \max_{t,i} \left( \frac{\ddot{x}_a(t)}{x_{\text{max}}^a} \right) \quad (45)

where the absolute accelerations of the i-th floor, \( \ddot{x}_a(t) \), are normalized by the peak uncontrolled floor acceleration, denoted \( x_{\text{max}}^a \). The final evaluation criteria considered in this study is a measure of the maximum control force per device, normalized by the weight of the structure, given by

\[
J_4 = \max_{t,i} \left( \frac{f_i(t)}{W} \right) \quad (46)
\]

where \( W \) = total weight of the structure (1,335 N). The corresponding uncontrolled responses are as follows: \( x_{\text{max}} = 1.313 \) cm, \( \ddot{x}_{\text{max}} = 0.00981 \) cm, \( \ddot{x}_{\text{max}} = 146.95 \) cm/s^2.

The resulting evaluation criteria were presented by Jansen and Dyke (2000) for the control algorithms previously studied.

For modal control, three cases of the structural measurements are considered: accelerations, displacements, and velocities. Using each structural measurement, a Kalman filter estimates the modal states. Fig. 3 represents the results of the stochastic response analysis for the acceleration feedback case. The variations of each evaluation criteria for increasing weighting parameters are shown in a three-dimensional plot. Previously mentioned, \( J_1 \) is evaluation criteria for the maximum displacement, \( J_2 \) is for the maximum interstory drift and \( J_3 \) is for the maximum acceleration. In Fig. 3, \( J_T \) is the summation of evaluation criteria, \( J_1, J_2, \) and \( J_3 \). From the variations of \( J_T \), we can find the weighting for reduction of overall structural responses whereas from \( J_1, J_2, \) and \( J_3 \) we can find the weighting for reduction of related responses. In Fig. 3, it can be seen that \( J_1 \) is minimum at \( q_{md}=400 \) and \( q_{mv}=1,500 \), \( J_2 \) is at \( q_{md}=1 \) and \( q_{mv}=500 \), \( J_3 \) is at \( q_{md}=2,200 \) and \( q_{mv}=100 \), and \( J_4 \) is at \( q_{md}=500 \) and \( q_{mv}=600 \). A designer can decide which to use according to control objectives. By using the controller \( (H_2/LQG) \) with designed weighting matrices from Fig. 3, we can get the results in Table 1. The numbers in parentheses indicate the percent reduction as compared to the best passive case in the previous study (Jansen and Dyke 2000). For the displacement and velocity feedback cases, weighting matrices are designed similarly to the acceleration feedback case in Fig. 3.

For each feedback case, in Table 1 four modal control designs with different capabilities are considered. The modal controllers \( A_{J1}, A_{J2}, A_{J3}, \) and \( A_{JT} \) with acceleration feedback use a weighting that minimizes the evaluation criteria \( J_1, J_2, J_3, \) and \( J_T \), respectively. The modal controllers \( D_{J1}, D_{J2}, D_{J3}, \) and \( D_{JT} \) with displacement feedback and \( V_{J1}, V_{J2}, V_{J3}, \) and \( V_{JT} \) with velocity feedback use a weighting which minimizes the evaluation criteria \( J_1, J_2, J_3, \) and \( J_T \), respectively. For each weighting, the lowest one and two modes cases are given. In the lowest two modes case, we place identical weighting on each mode: \( q_{md1} = q_{md2} = q_{md} \) and \( q_{mv1} = q_{mv2} = q_{mv} \).
The calculated evaluation criteria for various control strategies are compared in Table 1. The performance of the proposed modal control scheme is generally better than that of other control strategies. The results show that the modal controllers $A$ and $V$ appear to be quite effective in achieving significant reductions in both the maximum displacement and interstory drift over the passive case. In fact, the modal controller $A_{11}$ achieves a 39\% reduction in the relative displacement as compared to the better passive case. If further reductions in interstory drift and acceleration are desired in the controller, modal controllers $A_{12}$ and $A_{13}$ can achieve the reductions in the interstory drift and absolute acceleration of 30 and 23\%, respectively, over the best passive cases, although the maximum displacement increased. The reduction by modal controller $A_{12}$ is resulting in the lowest interstory drift of all cases considered here. The modal controller $V_{11}$ using the lowest two modes and $V_{13}$ achieve reductions in relative displacement and absolute acceleration of 41 and 30\%, respectively, resulting in the lowest values of all cases considered here. The modal controllers $A_{12}$ and $V_{12}$ do not achieve any lowest value of evaluation criteria, but have competitive performance in all evaluation criteria. Notice that the designer has some versatility depending on the control objectives for the particular structure under consideration. The modal controller $D$ compared with modal controllers $A$ and $V$ appears to be worse in achieving reductions, which agrees with the fact that the variations of evaluation criteria are more sensitive to weighting parameter $q_{m}$ than $q_{mv}$.

Comparing the lowest one mode case with a two mode case, every lowest value of evaluation criteria occurs at the lowest one mode case, except the modal controller $V_{11}$ that achieves further reductions by 6\% from the one mode case (reductions of 41\% over the best passive case) in the relative displacement.

### Table 1. Normalized Controlled Maximum Responses of the Various Feedback Cases due to the Scaled El Centro Earthquake

<table>
<thead>
<tr>
<th>Control strategy</th>
<th>$J_1$</th>
<th>$J_2$</th>
<th>$J_3$</th>
<th>$J_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modal control $A_{11}$ ($q_{mv}= 400, q_{m}= 1,500$)</td>
<td>1 mode</td>
<td>0.310(-39)</td>
<td>0.529(-24)</td>
<td>1.07(+18)</td>
</tr>
<tr>
<td>Modal control $A_{12}$ ($q_{mv}= 400, q_{m}= 500$)</td>
<td>2 modes</td>
<td>0.392(-23)</td>
<td>0.543(-22)</td>
<td>1.05(+16)</td>
</tr>
<tr>
<td>Modal control $A_{13}$ ($q_{mv}= 2,200, q_{m}= 100$)</td>
<td>1 mode</td>
<td>0.398(-21)</td>
<td>0.485(-30)</td>
<td>0.870(-4)</td>
</tr>
<tr>
<td>Modal control $A_{14}$ ($q_{mv}= 500, q_{m}= 600$)</td>
<td>2 modes</td>
<td>0.413(-18)</td>
<td>0.510(-27)</td>
<td>0.781(-14)</td>
</tr>
<tr>
<td>Modal control $A_{15}$ ($q_{mv}= 100, q_{m}=4,900$)</td>
<td>1 mode</td>
<td>0.549(+8)</td>
<td>0.618(-11)</td>
<td>0.697(-23)</td>
</tr>
<tr>
<td>Modal control $A_{16}$ ($q_{mv}=100, q_{m}=4,900$)</td>
<td>2 modes</td>
<td>0.548(+8)</td>
<td>0.585(-16)</td>
<td>0.741(-18)</td>
</tr>
<tr>
<td>Modal control $D_{11}$ ($q_{mv}= 3,300, q_{m}=4,700$)</td>
<td>1 mode</td>
<td>0.380(-25)</td>
<td>0.488(-30)</td>
<td>0.823(-9)</td>
</tr>
<tr>
<td>Modal control $D_{12}$ ($q_{mv}= 700, q_{m}=800$)</td>
<td>2 modes</td>
<td>0.423(-16)</td>
<td>0.533(-23)</td>
<td>0.876(-3)</td>
</tr>
<tr>
<td>Modal control $V_{11}$ ($q_{mv}= 1, q_{m}=400$)</td>
<td>1 mode</td>
<td>0.403(-20)</td>
<td>0.560(-20)</td>
<td>0.765(-15)</td>
</tr>
<tr>
<td>Modal control $V_{12}$ ($q_{mv}=1,300, q_{m}=100$)</td>
<td>2 modes</td>
<td>0.325(-36)</td>
<td>0.504(-28)</td>
<td>1.06(+17)</td>
</tr>
<tr>
<td>Modal control $V_{13}$ ($q_{mv}= 600, q_{m}=500$)</td>
<td>1 mode</td>
<td>0.702(+39)</td>
<td>0.728(+5)</td>
<td>0.671(-26)</td>
</tr>
<tr>
<td>Modal control $V_{14}$ ($q_{mv}= 200, q_{m}=4,900$)</td>
<td>2 modes</td>
<td>0.678(+34)</td>
<td>0.689(-1)</td>
<td>0.796(-12)</td>
</tr>
</tbody>
</table>

### Conclusions

In this paper, modal control was implemented to seismically excited structures using MR dampers. To this end, a modal control scheme was applied together with a Kalman filter and a low-pass filter. A Kalman filter considered three cases of the structural measurement to estimate modal states: displacement, velocity, and acceleration, respectively. Moreover, a low-pass filter was used to eliminate the spillover problem. In a numerical example, a six-story structure was controlled using MR dampers on the lower two floors. The responses of the system to a scaled El Centro earthquake excitation were found for each controller through a simulation of the system.

Modal control reshapes the motion of a structure by merely controlling a few selected vibration modes. Hence in the designing phase of a controller, the size of weighting matrix $Q$ was reduced because the lowest one or two modes were controlled. Therefore it is more convenient to design the smaller weighting matrix of modal control. This is one of the important benefits of the proposed modal control scheme.

The numerical results show that the motion of the structure was effectively suppressed by merely controlling a few lowest modes, although resulting responses varied greatly depending on the choice of measurements available and weightings. The modal controllers $A$ and $V$ achieved significant reductions in the responses. The modal controllers $A_{12}$, $A_{11}$, and $A_{13}$ achieve reductions (30, 41, and 30\%) in evaluation criteria $J_1$, $J_2$, and $J_3$, respectively, resulting in the lowest values of all cases considered here. The modal controllers $A_{12}$ and $V_{12}$ fail to achieve any lowest value of evaluation criteria, but have competitive performance in all evaluation criteria. Based on these results, the proposed
modal control scheme is found to be suited for use with MR dampers in a multi-input control system. Further studies are underway to examine the influence of the number of controlled modes on the control performance.

References


Spencer, B. F., Jr., et al. (1997).


