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구조물의 동특성 및 동적응답 해석을 위한 효율적인 방법

Efficient Methods for Eigenvalue and Dynamic Response Analysis of Structures

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ABSTRACT

This dissertation proposes an efficient eigenvalue solution method for structures by improving Lanczos method. An improved Lanczos vector superposition method for efficient dynamic response analysis of structures is also proposed in this disseration.

The Lanczos method is an efficient eigenvalue solution method. In the field of quantum physics, the modified Lanczos algorithm using the technique of matrix power was presented to more efficiently obtain the eigenstate of quantum systems. Similar power technique was also applied to the subspace iteration method. However, the matrix-powered algorithm has not been applied to the Lanczos method for structural eigenproblems. This dissertation proposes an improved Lanczos method for eigenvalue analysis of structures by applying the power of the dynamic matrix. The convergence of proposed matrix-powered Lanczos method is better than that of the conventional Lanczos method because the matrix-powered Lanczos algorithm can reduce the number of required Lanczos vectors. The number of operations of proposed method is also smaller than that of the conventional method. However, in some cases, high power value of the dynamic matrix causes numerical instability. Therefore, special care must be taken in the selection of power value. By analyzing four numerical examples such as a simple spring-mass system, a plane frame structure, a three-dimensional frame structure and a three-dimensional building structure, the efficiency of the proposed matrix-powered Lanczos method is verified and the suitable power value of the dynamic matrix is presented. The proposed matrix-powered Lanczos method is also compared with the matrix-powered subspace iteration method through numerical examples.

The Lanczos vector superposition method is efficient in the dynamic response analysis of structures. However, it has some drawback. For example, if multi-input loads such as moving loads on bridges, winds acting on high-rise buildings and wave forces applying to large offshore structures are applied to structures, the calculation of transformed force vector requires many computing operations. An improved Lanczos vector superposition method is proposed in this dissertation to overcome the shortcoming. Proposed method is obtained by introducing the modified Lanczos algorithm that generates stiffness-orthonormal Lanczos vectors. Since proposed Lanczos vector superposition method can reduce computing operations for transformed force vector, proposed method is more efficient than the conventional Lanczos vector superposition method in the analysis of structures under multi-input loads. Two numerical examples such as a simple span beam and a multi-span continuous bridge are analyzed to verify the effectiveness of the proposed Lanczos vector superposition method. The results of the proposed method are also compared with those of other vector superposition methods such as the eigenvector superposition method, the mode acceleration method, the Ritz vector superposition method and the conventional Lanczos vector superposition method.

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CHAPTER 1

INTRODUCTION

1.1 Background

The dynamic analysis of structures is important in many fields of engineering. The dynamic analysis is divided into two solution methods; the direct integration method and the vector superposition method. The choice for one method or the other is determined only by their numerical effectiveness. For loading of a relatively short duration such as impulse loading (i.e., for a few time steps), the direct integration method is usually the most effective method, because the number of operations required in the direct integration is directly proportional to the number of time steps used in the analysis. On the other hand, for loading of long duration such as in an earthquake (i.e., for many time steps), the vector superposition method is an appropriate method. The vector superposition method is divided into three methods; the eigenvector superposition method, the Ritz vector superposition method and the Lanczos vector superposition method.

In many practical dynamic analyses, not only responses but also dynamic characteristics of structures such as natural frequencies and mode shapes are required. In that case, the eigenvector superposition method must be selected because it calculates natural frequencies and mode shapes by eigenvalue analysis. However, the eigenvalue analysis is the most time consuming step in the eigenvector superposition method. Therefore, an improved eigensolution method is required to efficiently obtain dynamic response for a structure by the eigenvector superposition method.

If only dynamic responses are required, the Ritz and Lanczos vector superposition methods are more efficient than the eigenvector superposition method since the methods do not perform eigenvalue analysis of structures. Practical structures such as high-rise buildings, long-span bridges and large offshore structures generally have many degrees of freedom that requires many computing operations in response analyses. Therefore, further improvement of Ritz or Lanczos vector superposition methods will be very useful in dynamic analysis of practical large structures.

1.2 Literature Review

1.2.1 Eigenvalue Analysis Methods

Many solution methods have been developed for the eigenvalue analysis. Among them, the determinant search method, the subspace iteration method and the Lanczos method are widely used.

The determinant search method (Bathe and Wilson, 1973; Gupta, 1973) combines the polynomial iteration method, the Sturm sequence method and the inverse iteration method. By this method, eigenvalues in any range and corresponding eigenvectors can be found.

The subspace iteration method (Bathe and Wilson, 1972) is a combination of the simultaneous inverse iteration method and a Rayleigh-Ritz analysis. To improve the method many researchers have studied a variety of procedures as follows. Akl et al. (1979; 1982) employed over-relaxation method to accelerate the subspace iteration method. Bathe and Ramaswamy (1980) used over-relaxation method and shifting techniques and they showed that the accelerated method could be applied effectively to the solution of eigenproblems in which the matrices have small or large bandwidths. Wilson and Itoh (1983) proposed an efficient subspace iteration strategy for large-scaled eigenproblems. Leung (1995) extended the subspace iteration algorithm to complex eigenproblems. Jung et al. (1999) proposed a stable acceleration technique using side conditions for solving the singularity problem of the subspace iteration method with shifting. The concept of side conditions is based on the Newton-Raphson method for eigenvalue analysis (Lee and Robinson, 1979; Lee et al., 1997; 1998).

The Lanczos method was first proposed by Lanczos (1950). The method is very efficient when a few eigenpairs of a large matrix are calculated. It is now widely accepted as the method of solution of eigenproblems. Paige (1972) showed that the Lanczos method can be efficiently applied to symmetric eigenproblems. Parlett and Scott (1979) presented the Lanczos algorithm with selective orthogonalization. In theory, the simple Lanczos algorithm cannot compute multiple eigenpairs. Ruhe (1979) introduced the band Lanczos algorithm to get several eigenvectors corresponding to multiple eigenvalues. Ericsson and Ruhe (1980) applied shifting technique to accelerate convergence of the Lanczos method. Nour-Omid et al. (1983) systematically compares the Lanczos method and the subspace iteration method. Simon (1984) presented the partial reorthogonalization algorithm to efficiently retain the orthogonality of Lanczos vectors. Chen and Taylor (1988) extended the Lanczos method to eigenproblems for damped structures. Rajakumar (1993) used the Lanczos algorithm for the quadratic eigenvalue problem. He presented a new approach that employs the Lanczos two-sided recursion to solve the quadratic eigenvalue problem. Smith et al. (1993) presented and discussed a Lanczos-based eigensolution technique for evaluating the natural frequencies and modes from frequency-dependent and nonlinear eigenproblems in structural dynamics. Kim and Lee (1999) introduced a Lanczos-based algorithm for the solution of eigenproblems for non-classically damped structures. Komzsik (2001) employed the Lanczos algorithm in NASTRAN.

Recently, in the field of quantum physics, Grosso et al. (1993) modified the Lanczos algorithm using the technique of matrix power to more efficiently obtain the eigenstate of quantum systems. The matrix-powered algorithm has been adopted by Cordelli (1994), Grosso et al. (1995), Bevilacqua et al. (1996) and Fornari et al. (1997). In the field of engineering, the similar power technique was also applied to the accelerated subspace iteration method to more efficiently calculate eigenpairs of structures (Lam and Bertolini, 1994; Qian and Dhatt, 1995; Bertolini and Lam, 1998; Wang and Zhou, 1999). In the accelerated subspace iteration method, the power technique is applied to the dynamic matrix of simultaneous inverse iteration process. On the other hand, the power technique is not applied to the Lanczos method for structural eigenproblems yet. This paper applies the technique of matrix power to the Lanczos method for eigenvalue analysis of structures and rigorous investigation of applicability of the matrix-powered Lanczos method is also presented.

1.2.2 Vector Superposition Methods

Vector superposition method for dynamic response analysis of structures is divided into three methods; the eigenvector superposition method, the Ritz vector superposition method and the Lanczos vector superposition method. The terminology, vector superposition method, is due to Papadrakakis (1993).

The eigenvector superposition method is widely used for dynamic analysis of structures. In the method, the equations of motion are transformed to a reduced diagonal form through lower eigenvectors (lower mode shapes). The mode acceleration method (Cornwell et al., 1983) is also frequently used to correct errors of the eigenvector superposition method. The method employs a pseudo-static correction to include higher modes that are neglected in the eigenvector superposition method. Those two methods perform eigenvalue analysis of structures to obtain eigenvectors.

Wilson et al. (1982) proposed the Ritz vector superposition method that does not perform eigenvalue analysis of structures. They showed that the method yields better computing efficiency than the eigenvector superposition method. In the method, Ritz vectors that are orthonormal to mass matrix are generated. Then, a reduced stiffness matrix is obtained through transform by the Ritz vectors. To diagonalize stiffness matrix also, Ritz vectors are modified by solving standard eigenproblems for reduced stiffness matrix. Finally, the equations of motion are transformed to a reduced diagonal form through Ritz vectors. Detailed procedures are presented in some references (Papadrakakis, 1993; Chopra, 1995). Many researchers (Bayo and Wilson, 1984; Wilson, 1985; Leger and Wilson, 1987; Leger, 1988) have employed the method for dynamic analysis of structures. The Ritz vector superposition method has been extended for dynamic analysis of nonclassically damped structures (Ibrahimbegovic et al., 1990;

Mehai et al., 1995). The extended method uses real Ritz vectors or complex Ritz vectors that are generated from the algorithm by Zheng et al. (1989).

Nour-Omid and Clough (1984) proposed the Lanczos vector superposition method that does not also perform eigenvalue analysis of structures. They showed that the method has better computing efficiency than the eigenvector superposition method. In the method, Lanczos vectors that are orthonormal to mass matrix are generated through Lanczos recursion. Then, an order-reduced tridiagonal matrix is obtained during the generation process of Lanczos vectors. Finally, the equations of motion are transformed to a reduced tridiagonal form through Lanczos vectors. The Lanczos vector superposition method has been extended for dynamic analysis of nonclassically damped structures (Nour-Omid and Regelbrugge, 1989; Chen and Taylor, 1990; Ibrahimbegovic et al., 1990; Mehai et al., 1995). The extended Lanczos vector method is based on the augmented Lanczos recursion (Chen and Taylor, 1988).

Although the Lanczos vector superposition method is very efficient, it has some drawback. For example, if multi-input loads such as moving loads on bridges, winds acting on high-rise buildings and wave forces applying to large offshore structures are applied to structures, the calculation of transformed force vector is somewhat costly in the method. Nour-Omid and Clough (1985) used the block Lanczos algorithm for the dynamic analysis of structures under multi-input loads. However, they explained that the method would be effective when the number of input loads is less than 10. This means that the method is not suitable for many practical structures under multi-input loads. Therefore, improvement of the Lanczos vector superposition method is required to overcome the above shortcoming.

1.3 Objectives and Scopes

The purpose of this study is to improve the Lanczos method for efficient eigenvalue analysis of structures and to improve the Lanczos vector superposition method for efficient dynamic response analysis of structures.

First, the objectives and scopes of the study on an improved Lanczos method can be summarized as follows:

(1) *Improvement of the Lanczos method for eigenvalue analysis of structures by applying matrix-powered algorithm*:
Power technique is applied to the dynamic matrix in the Lanczos recursion to improve convergence of the Lanczos method. By improving convergence, the number of operations can be reduced. By analyzing four numerical examples, it is verified that the proposed matrix-powered Lanczos method has better convergence and smaller number of operations than the conventional Lanczos method. The proposed method is also compared with the matrix-powered subspace iteration method through numerical examples.

(2) *Determination of suitable power value of the dynamic matrix of proposed method*:
Proposed algorithm can improve convergence of the Lanczos method. However, increasing power value of the dynamic matrix may cause numerical instability, resulting in failure in convergence. Therefore, special care must be taken in the selection of power value. This paper numerically evaluates the appropriate power value of the dynamic matrix. By analyzing the numerical examples, the suitable power value of the dynamic matrix is presented.

Next, the objectives and scopes of the study on an improved Lanczos vector superposition method can be summarized as follows:

(1) *Improvement of the Lanczos vector superposition method for dynamic response analysis of structures by using modified Lanczos algorithm*:

An improved Lanczos vector superposition method is proposed to efficiently solve the dynamic equation of motion of structures under multi-input loads. Proposed method, of course, can be also applied to single-input-loaded structures. Proposed method is derived from the modified Lanczos algorithm using Lanczos vectors that satisfy the stiffness-orthonormality condition. To show the effectiveness of proposed method, two numerical examples are presented and the results are also compared with those of the eigenvector superposition method, the mode acceleration method, the Ritz vector superposition method and the conventional Lanczos vector superposition method.

1.4 Organization

This dissertation consists of four chapters. Chapter 1 discusses the background, the literature review, and the objectives and scopes of this study.

In Chapter 2, an improved Lanczos method for eigenvalue analysis of structures is discussed. The conventional Lanczos method is reviewed in Section 2.1. In Section 2.2, an improved Lanczos method is proposed. Proposed method is obtained by applying matrix-powered Lanczos algorithm. The formulation procedure of the matrix-powered Lanczos algorithm is also described in this section. To show the effectiveness of the proposed method, four structures such as a simple spring-mass system, a plane frame structure, a three-dimensional frame structure and a three-dimensional building structure are considered in Section 2.3.

In Chapter 3, an improved Lanczos vector superposition method for dynamic response analysis of structures is discussed. The conventional Lanczos vector superposition method is reviewed in Section 3.1. In Section 3.2, an improved Lanczos vector superposition method is proposed. Proposed method is obtained by applying modified Lanczos algorithm. The formulation procedure of the reduced dynamic equation of motion from the modified Lanczos algorithm is also explained in this section. To show the effectiveness of the proposed method, two structures such as a simple span beam and a multi-span continuous bridge are analyzed in Section 3.3.

Finally, the conclusions of this study are summarized and some recommendations for the further study are also presented in Chapter 4.

CHAPTER 2
IMPROVED LANCZOS METHOD

2.1 Conventional Method

The eigenproblem of structure frequently encountered in various engineering fields can be expressed as

$$\mathbf{K}\boldsymbol{\phi}_j = \lambda_j \mathbf{M}\boldsymbol{\phi}_j \quad (j = 1,2,\cdots,n) \tag{2.1}$$

where $\mathbf{M}$ and $\mathbf{K}$ are symmetric mass and stiffness matrices of order *n*, respectively, and $\boldsymbol{\phi}_j$ are the *j*th eigenvalue and associated eigenvector of the system. The eigenvectors satisfy the orthogonal condition

$$\boldsymbol{\phi}_i^T \mathbf{M}\boldsymbol{\phi}_j = \delta_{ij} \tag{2.2}$$

where δ_{ij} is the Kronecker delta.

The Lanczos algorithm is equivalent to obtaining the Rayleigh-Ritz approximation with the vectors in the Krylov sequence as the trial vectors (Hughes, 1987). Given one arbitrary starting vector as $\mathbf{x}_0$, we can get the Krylov sequence $\mathbf{v}_i$ used to obtain the best approximation to the wanted eigenvectors as

$$\mathbf{v}_i = (\mathbf{K}^{-1}\mathbf{M})^i \mathbf{x}_0 \quad (i = 1,2,\cdots) \tag{2.3}$$

where $\mathbf{K}^{-1}\mathbf{M}$ is called the dynamic matrix (Clough and Penzien, 1993).

2.1.1 Lanczos Recursion

The Lanczos method involves supplementing the Krylov sequence with Gram-Schmidt orthogonalization process at each step (Hughes, 1987) as

$$\mathbf{x}_{i+1} = \mathbf{v}_i - \sum_{j=1}^{i} \upsilon_j \mathbf{x}_j \tag{2.4}$$

where $\mathbf{x}_j$ is *j*th Lanczos vector and υ_j is the component of $\mathbf{v}_i$ along $\mathbf{x}_j$. In this chapter, we depict the facts that the orthogonalization is required only with respect to two preceding vectors, and the resulting Lanczos vectors of the Krylov sequence leads a general eigenproblem to a standard eigenproblem with a tridiagonal matrix.

To derive the Lanczos algorithm, assume for a moment that the first *i* Lanczos vectors, $\{\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_i\}$, are founded, and the next Lanczos vectors, $\mathbf{x}_{i+1}$, will be constructed. From the definition of Kryolv sequence, (2.3) can be rearranged by

$$\mathbf{v}_i = (\mathbf{K}^{-1}\mathbf{M})^i \mathbf{x}_0 = (\mathbf{K}^{-1}\mathbf{M})(\mathbf{K}^{-1}\mathbf{M})^{i-1} \mathbf{x}_0 = (\mathbf{K}^{-1}\mathbf{M})\mathbf{v}_{i-1} \tag{2.5}$$

and, ($i-1$)th Kryolve sequence has the following form according to (2.4)

$$\mathbf{v}_{i-1} = \sum_{j=1}^{i} \hat{\upsilon}_j \mathbf{x}_j \tag{2.6}$$

Substituting (2.6) into (2.5), we obtain

$$\mathbf{v}_i = \sum_{j=1}^{i} \hat{\upsilon}_j \mathbf{K}^{-1}\mathbf{M}\mathbf{x}_j \tag{2.7}$$

By (2.7), ($i-1$)th Kryolve sequence has another form as

$$\mathbf{v}_{i-1} = \sum_{j=1}^{i-1} \hat{\upsilon}_j \mathbf{K}^{-1}\mathbf{M}\mathbf{x}_j \tag{2.8}$$

From (2.6) and (2.8), we can obtain following relation

$$\sum_{j=1}^{i} \hat{\upsilon}_j \mathbf{x}_j = \sum_{j=1}^{i-1} \hat{\upsilon}_j \mathbf{K}^{-1}\mathbf{M}\mathbf{x}_j \tag{2.9}$$

Equation (2.9) means that each $\mathbf{K}^{-1}\mathbf{M}\mathbf{x}_j$ can be written as a linear combination of the first (*j*+1) Lanczos vectors. Therefore, (2.7) can be rewritten by

$$\mathbf{v}_i = \sum_{j=1}^{i} \hat{\upsilon}_j \mathbf{K}^{-1}\mathbf{M}\mathbf{x}_j = \hat{\upsilon}_i \mathbf{K}^{-1}\mathbf{M}\mathbf{x}_i + \sum_{j=1}^{i-1} \hat{\upsilon}_j \mathbf{K}^{-1}\mathbf{M}\mathbf{x}_j = \hat{\upsilon}_i \mathbf{K}^{-1}\mathbf{M}\mathbf{x}_i + \sum_{j=1}^{i} \overline{\upsilon}_j \mathbf{x}_j \tag{2.10}$$

Substituting (2.10) into (2.4), then (2.4) becomes

$$\mathbf{x}_{i+1} = \hat{\upsilon}_i \mathbf{K}^{-1}\mathbf{M}\mathbf{x}_i + \sum_{j=1}^{i} \overline{\upsilon}_j \mathbf{x}_j - \sum_{j=1}^{i} \upsilon_j \mathbf{x}_j = \hat{\upsilon}_i \mathbf{K}^{-1}\mathbf{M}\mathbf{x}_i - \sum_{j=1}^{i} (\upsilon_j - \overline{\upsilon}_j)\mathbf{x}_j \tag{2.11}$$

Equation (2.11) means that the orthogonalization of Krylov sequence is equivalent to that of $\mathbf{K}^{-1}\mathbf{M}\mathbf{x}_i$. Therefore, next Lanczos vector $\mathbf{x}_{i+1}$ is obtained by first computing preliminary vectors $\overline{\mathbf{x}}_i$ as

$$\overline{\mathbf{x}}_i = \mathbf{K}^{-1}\mathbf{M}\mathbf{x}_i \tag{2.12}$$

In general, these preliminary vectors can be expressed as a linear combination of all the previous Lanczos vectors and a residual vector as

$$\overline{\mathbf{x}}_i = \tilde{\mathbf{x}}_i + \gamma_i \mathbf{x}_i + \delta_{i-1}\mathbf{x}_{i-1} + \varepsilon_i \mathbf{x}_{i-2} + \cdots \tag{2.13}$$

with

$$\tilde{\mathbf{x}}_i = \delta_i \mathbf{x}_{i+1} \tag{2.14}$$

where the residual vector, $\tilde{\mathbf{x}}_i$, is the pure components of $\bar{\mathbf{x}}_i$ orthogonal to all the previous Lanczos vectors, and δ_i is the pseudo length of $\tilde{\mathbf{x}}_i$. $\gamma_i, \delta_{i-1}, \varepsilon_i, \cdots$ are the components of the previous Lanczos vectors contained in $\bar{\mathbf{x}}_i$. The new Lanczos vectors satisfy the following orthogonal conditions to all the previous Lanczos vectors

$$\mathbf{x}_j^T \mathbf{M} \mathbf{x}_{i+1} = 0 \quad (j = 1, 2, \cdots, i) \tag{2.15}$$

$$\mathbf{x}_{i+1}^T \mathbf{M} \mathbf{x}_{i+1} = 1 \tag{2.16}$$

Here, the pseudo length of the new Lanczos vectors is normalized to be 1.

These component coefficients can be evaluated by imposing the orthogonal conditions, (2.15) and (2.16), between the Lanczos vectors. To obtain the coefficient γ_i , we premultiply both sides of (2.13) by $\mathbf{x}_i^T \mathbf{M}$. Then using the orthogonal relationships, we can get the following result

$$\gamma_i = \mathbf{x}_i^T \mathbf{M} \bar{\mathbf{x}}_i \tag{2.17}$$

The component δ_{i-1} may be obtained similarly by premultiplying $\mathbf{x}_{i-1}^T \mathbf{M}$. Then using the orthogonal conditions, we can have

$$\delta_{i-1} = \mathbf{x}_{i-1}^T \mathbf{M} \bar{\mathbf{x}}_i \tag{2.18}$$

Using (2.12) to eliminate $\bar{\mathbf{x}}_i$ in (2.18), this gives

$$\delta_{i-1} = \mathbf{x}_{i-1}^T \mathbf{M} \mathbf{K}^{-1} \mathbf{M} \mathbf{x}_i \tag{2.19}$$

and applying the transpose of (2.12) to $\mathbf{x}_{i-1}^T$, we can get

$$\delta_{i-1} = \overline{\mathbf{x}}_{i-1}^T \mathbf{M} \mathbf{x}_i \tag{2.20}$$

Finally, expanding $\overline{\mathbf{x}}_{i-1}$ in terms of (2.13), and then using the orthogonality relationships given (2.15) and (2.16), we can get the following result

$$\delta_{i-1} = \tilde{\mathbf{x}}_{i-1}^T \mathbf{M} \mathbf{x}_i \tag{2.21}$$

or rewriting δ_i for the (i+1)st vector

$$\delta_i = \tilde{\mathbf{x}}_i^T \mathbf{M} \mathbf{x}_{i+1} \tag{2.22}$$

The new Lanczos vectors are obtained simply by scaling the pure vectors, $\tilde{\mathbf{x}}_i$, as

$$\mathbf{x}_{i+1} = \frac{\tilde{\mathbf{x}}_i}{\delta_i} \tag{2.23}$$

where δ_i is the pseudo length of $\tilde{\mathbf{x}}_i$. Therefore, using the expressions for $\mathbf{x}_{i+1}$ in (2.22), we can obtain δ_i as follows

$$\delta_i = (\tilde{\mathbf{x}}_i^T \mathbf{M}\, \tilde{\mathbf{x}}_i)^{1/2} \tag{2.24}$$

Continuing in the same ways as for finding the expression in (2.21), the coefficient ε_i is obtained to be

$$\varepsilon_i = \tilde{\mathbf{x}}_{i-2}^T \mathbf{M}\mathbf{x}_i \tag{2.25}$$

Substituting (2.14) for $\tilde{\mathbf{x}}_{i-2}$ in (2.25), we can obtain

$$\varepsilon_i = \delta_{i-2} \mathbf{x}_{i-1}^T \mathbf{M}\mathbf{x}_i = 0 \tag{2.26}$$

A corresponding procedure could be used to demonstrate that all further terms in the expansions for $\bar{\mathbf{x}}_i$ will be zero. Therefore, (2.13) can be rewritten as the three-term recursive formulas for deriving the pure components, $\tilde{\mathbf{x}}_i$, as

$$\tilde{\mathbf{x}}_i = \mathbf{K}^{-1}\mathbf{M}\mathbf{x}_i - \gamma_i \mathbf{x}_i - \delta_{i-1}\mathbf{x}_{i-1} \tag{2.27}$$

2.1.2 Reduction to Tridiagonal System

After *m* steps, we have a set of Lanczos vectors, $\mathbf{X} = [\mathbf{x}_1\ \mathbf{x}_2 \cdots \mathbf{x}_m]$. $\mathbf{X}$ satisfies the following relation from orthonormal conditions

$$\mathbf{X}^T \mathbf{M}\mathbf{X} = \mathbf{I} \tag{2.28}$$

where $\mathbf{I}$ is identity matrix of order *m*. Lanczos recursion, (2.27), can be rearranged in matrix form

$$\mathbf{K}^{-1}\mathbf{M}\mathbf{X} - \mathbf{X}\mathbf{T} = \delta_m \mathbf{x}_{m+1} \mathbf{e}_m^T \tag{2.29}$$

where $\mathbf{e}_m$ is the last column $\mathbf{I}$ and $\mathbf{T}$ is a tridiagonal matrix of the form

$$\mathbf{T} = \begin{bmatrix} \gamma_1 & \delta_1 & & & \\ \delta_1 & \gamma_2 & \delta_2 & & \\ & & \ddots & & \\ & & \delta_{m-2} & \gamma_{m-1} & \delta_{m-1} \\ & & & \delta_{m-1} & \gamma_m \end{bmatrix} \tag{2.30}$$

Premultiplying (2.29) by $\mathbf{X}^T\mathbf{M}$ and applying the orthonormal condition, we get

$$\mathbf{X}^T\mathbf{M}\mathbf{K}^{-1}\mathbf{M}\mathbf{X} = \mathbf{T} \tag{2.31}$$

Here, we employ the Rayleigh-Ritz method to obtain the reduced eigenproblem from the eigenproblem as

$$\boldsymbol{\phi}_j = \mathbf{X}\tilde{\boldsymbol{\phi}}_j \tag{2.32}$$

Equation (2.1) can be rewritten in the form

$$\mathbf{M}\mathbf{K}^{-1}\mathbf{M}\boldsymbol{\phi}_j = \frac{1}{\lambda_j}\mathbf{M}\boldsymbol{\phi}_j \tag{2.33}$$

Substituting (2.32) into (2.33), premultiplying the results by $\mathbf{X}^T$ and using (2.28) and (2.31), we can obtain the tridiagonalized standard eigenproblem of order $m << n$.

$$\mathbf{T}\tilde{\boldsymbol{\phi}}_j = \frac{1}{\lambda_j}\tilde{\boldsymbol{\phi}}_j \tag{2.34}$$

We solve the reduced eigenproblem, (2.34). For the eigenvalues of reduced system, QR iteration is effectively used because $\mathbf{T}$ is tridiagonal (Bathe, 1996). Once the eigenvalues are computed, eigenvectors of reduced system can be calculated by inverse iteration with shift (Bathe, 1996) or explicit formulation (Wilkinson, 1965).

2.1.3 Loss of Orthogonality

The Lanczos algorithm, involving orthogonalization with only the two preceding vectors at each step, is subjected to loss of orthogonality with respect to earlier vectors due to round-off errors. If such errors are not corrected when they reach a critical size, the Lanczos vectors may become linearly dependent. A remedy to prevent the loss of orthogonality is to use the Gram-Schmidt orthogonalization process at each step (Lanczozs, 1950; Matthies, 1985). Such process is called full reorthogonalization (Hughes, 1987). In some cases, selective or partial reorthogonalization (Parlett, 1979; Simon, 1984) may be sufficient. However, the Gram-Schmidt process is also sensitive to round-off errors (Bathe, 1996). In this paper, full reorthogonalization process is used to retain the orthogonality of the Lanczos vectors as

$$\hat{\mathbf{x}}_i = \tilde{\mathbf{x}}_i - \sum_{k=1}^{i} (\tilde{\mathbf{x}}_i^T \mathbf{M} \mathbf{x}_k) \mathbf{x}_k \tag{2.35}$$

2.2 Proposed Method

This paper applies the power technique to the dynamic matrix in (2.3) and the following modified Krylov sequence can be obtained.

$$\mathbf{v}_i = ((\mathbf{K}^{-1}\mathbf{M})^{\delta})^i \mathbf{x}_0 \quad (i = 1, 2, \cdots) \tag{2.36}$$

where δ is positive integer. Then, Gram-Schmidt orthogonalization process of Krylov sequence can be modified as

$$\mathbf{x}_{i+1} = \mathbf{v}_i - \sum_{j=1}^{i} \upsilon_j \mathbf{x}_j = ((\mathbf{K}^{-1}\mathbf{M})^{\delta})^i \mathbf{x}_0 - \sum_{j=1}^{i} \upsilon_j \mathbf{x}_j \tag{2.37}$$

Equation (2.37) means that an approximated eigenvector, whose number of iteration is δi, is contained in (i+1) Lanczos vectors. Whereas, in (2.4), (i+1) Lanczos vectors contain an approximated eigenvector whose number of iterations is i. Since δi is larger than i, Lanczos vectors in (2.37) give a better solution than those in (2.4). In this paper, an improved Lanczos method, which is based on such concept of power of the dynamic matrix, is proposed.

2.2.1 Modified Lanczos Recursion

To derive improved Lanczos algorithm, assume for a moment that the first i Lanczos vectors, $\{\mathbf{x}_1, \mathbf{x}_2, \cdots, \mathbf{x}_i\}$, are founded, and the next Lanczos vectors, $\mathbf{x}_{i+1}$, will be constructed. From the modified Kryolv sequence, (2.36) can be rearranged by

$$\mathbf{v}_i = ((\mathbf{K}^{-1}\mathbf{M})^{\delta})^i \mathbf{x}_0 = (\mathbf{K}^{-1}\mathbf{M})^{\delta}((\mathbf{K}^{-1}\mathbf{M})^{\delta})^{i-1} \mathbf{x}_0 = (\mathbf{K}^{-1}\mathbf{M})^{\delta} \mathbf{v}_{i-1} \tag{2.38}$$

and, (i−1)th Kryolve sequence has the following form according to (2.37)

$$\mathbf{v}_{i-1} = \sum_{j=1}^{i} \hat{\upsilon}_j \mathbf{x}_j \tag{2.39}$$

Substituting (2.39) into (2.38), we obtain

$$\mathbf{v}_i = \sum_{j=1}^{i} \hat{\upsilon}_j (\mathbf{K}^{-1}\mathbf{M})^{\delta} \mathbf{x}_j \tag{2.40}$$

By (2.40), (i–1)th Kryolve sequence has another form as

$$\mathbf{v}_{i-1} = \sum_{j=1}^{i-1} \hat{\upsilon}_j (\mathbf{K}^{-1}\mathbf{M})^{\delta} \mathbf{x}_j \tag{2.41}$$

From (2.39) and (2.41), we can obtain following relation

$$\sum_{j=1}^{i} \hat{\upsilon}_j \mathbf{x}_j = \sum_{j=1}^{i-1} \hat{\upsilon}_j (\mathbf{K}^{-1}\mathbf{M})^{\delta} \mathbf{x}_j \tag{2.42}$$

Equation (2.42) means that each $(\mathbf{K}^{-1}\mathbf{M})^{\delta} \mathbf{x}_j$ can be written as a linear combination of the first (j+1) Lanczos vectors. Therefore, (2.40) can be rewritten by

$$\mathbf{v}_i = \sum_{j=1}^{i} \hat{\upsilon}_j (\mathbf{K}^{-1}\mathbf{M})^{\delta} \mathbf{x}_j = \hat{\upsilon}_i (\mathbf{K}^{-1}\mathbf{M})^{\delta} \mathbf{x}_i + \sum_{j=1}^{i-1} \hat{\upsilon}_j (\mathbf{K}^{-1}\mathbf{M})^{\delta} \mathbf{x}_j = \hat{\upsilon}_i (\mathbf{K}^{-1}\mathbf{M})^{\delta} \mathbf{x}_i + \sum_{j=1}^{i} \overline{\upsilon}_j \mathbf{x}_j \tag{2.43}$$

Substituting (2.43) into (2.37), then (2.37) becomes

$$\mathbf{x}_{i+1} = \hat{\upsilon}_i (\mathbf{K}^{-1}\mathbf{M})^{\delta} \mathbf{x}_i + \sum_{j=1}^{i} \overline{\upsilon}_j \mathbf{x}_j - \sum_{j=1}^{i} \upsilon_j \mathbf{x}_j = \hat{\upsilon}_i (\mathbf{K}^{-1}\mathbf{M})^{\delta} \mathbf{x}_i - \sum_{j=1}^{i} (\upsilon_j - \overline{\upsilon}_j) \mathbf{x}_j \tag{2.44}$$

Equation (2.44) means that the orthogonalization of Krylov sequence is equivalent to that of $(\mathbf{K}^{-1}\mathbf{M})^{\delta}\mathbf{x}_i$. Therefore, next Lanczos vector $\mathbf{x}_{i+1}$ is obtained by first computing preliminary vectors $\overline{\mathbf{x}}_i$ as

$$\overline{\mathbf{x}}_i = (\mathbf{K}^{-1}\mathbf{M})^{\delta}\mathbf{x}_i \tag{2.45}$$

In general, these preliminary vectors can be expressed as a linear combination of all the previous Lanczos vectors and a residual vector as

$$\overline{\mathbf{x}}_i = \tilde{\mathbf{x}}_i + \gamma_i\mathbf{x}_i + \delta_{i-1}\mathbf{x}_{i-1} + \varepsilon_i\mathbf{x}_{i-2} + \cdots \tag{2.46}$$

with

$$\tilde{\mathbf{x}}_i = \delta_i\mathbf{x}_{i+1} \tag{2.47}$$

where the residual vector, $\tilde{\mathbf{x}}_i$, is the pure components of $\overline{\mathbf{x}}_i$ orthogonal to all the previous Lanczos vectors, and δ_i is the pseudo length of $\tilde{\mathbf{x}}_i$. $\gamma_i, \delta_{i-1}, \varepsilon_i, \cdots$ are the components of the previous Lanczos vectors contained in $\overline{\mathbf{x}}_i$. The new Lanczos vectors satisfy the following orthogonal conditions to all the previous Lanczos vectors

$$\mathbf{x}_j^T\mathbf{M}\mathbf{x}_{i+1} = 0 \quad (j = 1, 2, \cdots, i) \tag{2.48}$$

$$\mathbf{x}_{i+1}^T\mathbf{M}\mathbf{x}_{i+1} = 1 \tag{2.49}$$

Here, the pseudo length of the new Lanczos vectors is normalized to be 1.

These component coefficients can be evaluated by imposing the orthogonal conditions, (2.48) and (2.49), between the Lanczos vectors. To obtain the coefficient γ_i, we premultiply both sides of (2.46) by $\mathbf{x}_i^T\mathbf{M}$. Then using the orthogonal relationships, we can get the following result

$$\gamma_i = \mathbf{x}_i^T \mathbf{M}\,\overline{\mathbf{x}}_i \tag{2.50}$$

The component δ_{i-1} may be obtained similarly by premultiplying $\mathbf{x}_{i-1}^T\mathbf{M}$. Then using the orthogonal conditions, we can have

$$\delta_{i-1} = \mathbf{x}_{i-1}^T \mathbf{M}\,\overline{\mathbf{x}}_i \tag{2.51}$$

Using (2.45) to eliminate $\overline{\mathbf{x}}_i$ in (2.51), this gives

$$\delta_{i-1} = \mathbf{x}_{i-1}^T \mathbf{M}(\mathbf{K}^{-1}\mathbf{M})^{\delta}\mathbf{x}_i = \mathbf{x}_{i-1}^T(\mathbf{M}\mathbf{K}^{-1})^{\delta}\mathbf{M}\mathbf{x}_i \tag{2.52}$$

and applying the transpose of (2.45) to $\mathbf{x}_{i-1}^T$, we can get

$$\delta_{i-1} = \overline{\mathbf{x}}_{i-1}^T \mathbf{M}\mathbf{x}_i \tag{2.53}$$

Finally, expanding $\overline{\mathbf{x}}_{i-1}$ in terms of (2.46), and then using the orthogonality relationships given (2.48) and (2.49), we can get the following result

$$\delta_{i-1} = \tilde{\mathbf{x}}_{i-1}^T \mathbf{M}\mathbf{x}_i \tag{2.54}$$

or rewriting δ_i for the (i+1)st vector

$$\delta_i = \tilde{\mathbf{x}}_i^T \mathbf{M}\mathbf{x}_{i+1} \tag{2.55}$$

The new Lanczos vectors are obtained simply by scaling the pure vectors, $\tilde{\mathbf{x}}_i$, as

$$\mathbf{x}_{i+1} = \frac{\tilde{\mathbf{x}}_i}{\delta_i} \tag{2.56}$$

where δ_i is the pseudo length of $\tilde{\mathbf{x}}_i$. Therefore, using the expressions for $\mathbf{x}_{i+1}$ in (2.55), we can obtain δ_i as follows

$$\delta_i = (\tilde{\mathbf{x}}_i^T \mathbf{M}\, \tilde{\mathbf{x}}_i)^{1/2} \tag{2.57}$$

Continuing in the same ways as for finding the expression in (2.54), the coefficient ε_i is obtained to be

$$\varepsilon_i = \tilde{\mathbf{x}}_{i-2}^T \mathbf{M}\mathbf{x}_i \tag{2.58}$$

Substituting (2.47) for $\tilde{\mathbf{x}}_{i-2}$ in (2.58), we can obtain

$$\varepsilon_i = \delta_{i-2}\mathbf{x}_{i-1}^T \mathbf{M}\mathbf{x}_i = 0 \tag{2.59}$$

A corresponding procedure could be used to demonstrate that all further terms in the expansions for $\bar{\mathbf{x}}_i$ will be zero. Therefore, (2.46) can be rewritten as the three-term recursive formulas for deriving the pure components, $\tilde{\mathbf{x}}_i$, as

$$\tilde{\mathbf{x}}_i = (\mathbf{K}^{-1}\mathbf{M})^{\delta}\, \mathbf{x}_i - \gamma_i \mathbf{x}_i - \delta_{i-1}\mathbf{x}_{i-1} \tag{2.60}$$

2.2.2 Reduction to Modified Tridiagonal System

After m steps, we have a set of Lanczos vectors, $\mathbf{X} = [\mathbf{x}_1\ \mathbf{x}_2 \cdots \mathbf{x}_m]$. $\mathbf{X}$ satisfies the following relation from orthonormal conditions

$$\mathbf{X}^T\mathbf{M}\mathbf{X} = \mathbf{I} \tag{2.61}$$

where $\mathbf{I}$ is identity matrix of order *m*. Lanczos recursion, (2.60), can be rearranged in matrix form

$$(\mathbf{K}^{-1}\mathbf{M})^{\delta}\mathbf{X} - \mathbf{X}\mathbf{T} = \delta_m \mathbf{x}_{m+1}\mathbf{e}_m^T \tag{2.62}$$

where $\mathbf{e}_m$ is the last column $\mathbf{I}$ and $\mathbf{T}$ is a tridiagonal matrix of the form

$$\mathbf{T} = \begin{bmatrix} \gamma_1 & \delta_1 & & & \\ \delta_1 & \gamma_2 & \delta_2 & & \\ & & \ddots & & \\ & & \delta_{m-2} & \gamma_{m-1} & \delta_{m-1} \\ & & & \delta_{m-1} & \gamma_m \end{bmatrix} \tag{2.63}$$

Premultiplying (2.62) by $\mathbf{X}^T\mathbf{M}$ and applying the orthonormal condition, we get

$$\mathbf{X}^T\mathbf{M}(\mathbf{K}^{-1}\mathbf{M})^{\delta}\mathbf{X} = \mathbf{T} \tag{2.64}$$

Here, we employ the Rayleigh-Ritz method to obtain the reduced eigenproblem from the eigenproblem as

$$\boldsymbol{\phi}_j = \mathbf{X}\tilde{\boldsymbol{\phi}}_j \tag{2.65}$$

Equation (2.1) can be rewritten in the form

$$\mathbf{M}(\mathbf{K}^{-1}\mathbf{M})^{\delta}\boldsymbol{\phi}_j = \frac{1}{\lambda_j^{\delta}}\mathbf{M}\boldsymbol{\phi}_j \tag{2.66}$$

Substituting (2.65) into (2.66), premultiplying the results by $\mathbf{X}^T$ and using (2.61) and (2.64), we can obtain the tridiagonalized standard eigenproblem of order $m << n$.

$$\mathbf{T}\tilde{\boldsymbol{\phi}}_j = \frac{1}{\lambda_j^{\delta}}\tilde{\boldsymbol{\phi}}_j \tag{2.67}$$

2.2.3 Loss of Orthogonality

Full reorthogonalization process, discussed in Section 2.1.3, is also applied to the proposed algorithm.

2.2.4 Summary of Algorithm for Proposed Method

The steps for proposed matrix-powered Lanczos method with the operations are summarized in Table 2.1. The algorithm with $\delta = 1$ leads to the conventional Lanczos algorithm. In the algorithm, $\overline{\mathbf{x}}_i$ is not calculated by direct δth power of the dynamic matrix, $\mathbf{K}^{-1}\mathbf{M}$. It is obtained by δ-time forward reduction and back-substitution. As δ increases in the matrix-powered Lanczos algorithm, the number of operations of forward reduction and back-substitution increases. However, since the power of dynamic matrix can reduce Lanczos steps, operation counts for reorthogonalization and eigenanalysis of reduced system are reduced. Generally, the degree of cost reduction in reorthogonalization and eigenanalysis of reduced system is larger than that of cost increase in forward reduction and back-substitution. For the eigenvalues of reduced tridiagonal system, QR iteration is effectively used (Bathe, 1996). In QR iteration, submatrix of order k is diagonalized by iteration in kth step (k = 2, 3, ···, i). Each iteration requires 6k operations in kth step (Wilkinson, 1965). Therefore, the number of total operations will be $\sum_{k=2}^{i} 6ks_k$ if the number of iterations in kth step is s_k . Once the eigenvalues are computed, eigenvectors of reduced system can be calculated by inverse iteration with shift (Bathe, 1996) or explicit formulation (Wilkinson, 1965) that has very simple procedure. In this disseration, explicit formulation is used. The number of operations of explicit formulation is $6i^2$. If the number of desired eigenpairs is *p*, eigenvalue analysis of reduced system (step 2.g and 2.h in Table 2.1) is performed for $i \geq p$.

The summary of the matrix-powered subspace iteration method is also presented in Table 2.2. In the table, $\mathbf{X}$ is the subspace for approximate eigenvectors. As the number of iterations increases, the first *p* columns of $\mathbf{X}$ converges to $\mathbf{\Phi} = [\boldsymbol{\phi}_1 \, \boldsymbol{\phi}_2 \cdots \boldsymbol{\phi}_p]$ and the *p* by *p* submatrix of $\mathbf{\Lambda}$ will be $diag(\lambda_1, \lambda_2, \cdots, \lambda_p)$. For the reduced eigensystem (step 2.d in Table 2.2), the Jacobi iteration method is used. In the Jacobi iteration method, one sweep requires $(3q^3 + 6q^2)$ operations (Bathe, 1996). Therefore, the number of operations for the Jacobi iteration will be $s_k(3q^3 + 6q^2)$ if the number of sweeps in *k*th step is s_k.

Table 2.1 Summary of algorithm for improved Lanczos method

Calculation	Number of operations
1. $\mathbf{K} = \mathbf{LDL}^T$	$(1/2)nh^2 + (3/2)nh$
2. Iteration $i = 1,2,\cdots,q$	
a. $\bar{\mathbf{x}}_i = (\mathbf{K}^{-1}\mathbf{M})^{\delta}\mathbf{x}_i$	$\delta\{n(2h+1)+2nh\}$
b. $\gamma_i = \mathbf{x}_i^T\mathbf{M}\bar{\mathbf{x}}_i$	$n(2h+1)+n$
c. $\tilde{\mathbf{x}}_i = \bar{\mathbf{x}}_i - \gamma_i\mathbf{x}_i - \delta_{i-1}\mathbf{x}_{i-1}$ $(\mathbf{x}_0 = \mathbf{0})$	$2n$
d. $\hat{\mathbf{x}}_i = \tilde{\mathbf{x}}_i - \sum_{k=1}^{i}(\tilde{\mathbf{x}}_i^T\mathbf{M}\mathbf{x}_k)\mathbf{x}_k$	$n(2h+1)+i(n+n)$
e. $\delta_i = (\hat{\mathbf{x}}_i^T\mathbf{M}\hat{\mathbf{x}}_i)^{1/2}$	$n(2h+1)+n+1$
f. $\mathbf{x}_{i+1} = \hat{\mathbf{x}}_i / \delta_i$	n
g. Solve $\mathbf{T}\tilde{\boldsymbol{\phi}}_j = (1/\lambda_j^{\delta})\tilde{\boldsymbol{\phi}}_j$ $(j = 1,2,\cdots,i)$	$\sum_{k=2}^{i} 6ks_k + 6i^2$
h. $\boldsymbol{\phi}_j = \mathbf{X}\tilde{\boldsymbol{\phi}}_j$ $(j = 1,2,\cdots,i)$	ni^2

Total operation:

$$(1/2)nh^2 + (4q\delta + 6q + 3/2)nh + \{q(q+1)(2q+1)/6 - (p-1)p(2p-1)/6$$
$$+ q^2 + q\delta + 9q\}n + q(q+1)(2q+1) - (p-1)p(2p-1) + q + \sum_{i=p}^{q}\sum_{k=2}^{i} 6ks_k$$

Note: n = order of **M** and **K**
h = half-bandwidth of **M** and **K**
p = the number of desired eigenpairs
q = the number of Lanczos vectors that make all p eigenpairs converge
s_k = the number of iterations of kth step in QR iteration

Table 2.2 Summary of algorithm for matrix-powered subspace iteration method

Calculation	Number of operations
1. $\mathbf{K} = \mathbf{LDL}^T$	$(1/2)nh^2 + (3/2)nh$
2. Iteration $k = 1,2,\cdots,r$	
a. $\overline{\mathbf{X}}_{k+1} = (\mathbf{K}^{-1}\mathbf{M})^{\delta}\mathbf{X}_k$	$\delta\{nq(2h+1) + 2nqh\}$
b. $\mathbf{K}_{k+1} = \overline{\mathbf{X}}_{k+1}^T\mathbf{K}\overline{\mathbf{X}}_{k+1}$	$nq(2h+1) + (1/2)nq(q+1)$
c. $\mathbf{M}_{k+1} = \overline{\mathbf{X}}_{k+1}^T\mathbf{M}\overline{\mathbf{X}}_{k+1}$	$nq(2h+1) + (1/2)nq(q+1)$
d. Solve $\mathbf{K}_{k+1}\mathbf{Q}_{k+1} = \mathbf{M}_{k+1}\mathbf{Q}_{k+1}\mathbf{\Lambda}_{k+1}$	$s_k(3q^3 + 6q^2)$
e. $\mathbf{X}_{k+1} = \overline{\mathbf{X}}_{k+1}\mathbf{Q}_{k+1}$	nq^2

Total operation:

$$(1/2)nh^2 + (4qr\delta + 4qr + 3/2)nh + (2q^2r + qr\delta + 3qr)n + (3q^3 + 6q^2)\sum_{k=1}^{r} s_k$$

Note: n = order of **M** and **K**
h = half-bandwidth of **M** and **K**
q = the size of subspace = min(2*p*, *p* + 8)
p = the number of desired eigenpairs
r = the number of subspace iterations that make all *p* eigenpairs converge
s_k = the number of sweeps of Jacobi iterations in *k*th step

2.3 Numerical Examples

To verify the effectiveness of proposed matrix-powered Lanczos method, four numerical examples such as a simple spring-mass system (Chen, 1993), a plane frame structure (Bathe and Wilson, 1972), a three-dimensional frame structure (Bathe and Wilson, 1972) and a three-dimensional building structure (Kim and Lee, 1999) are considered. The convergence and the number of operations of proposed method are compared with those of the conventional Lanczos method. Comparison with the matrix-powered subspace iteration method is also presented. The predetermined physical error norm is 10^{-6}, which yields a numerically stable eigenproblem solution and sufficient accuracy in the calculated eigenpairs for practical analysis (Bathe and Ramaswamy, 1980). The physical error norm (Bathe, 1996) is defined as

$$\varepsilon_j = \frac{\| \mathbf{K}\boldsymbol{\phi}_j - \lambda_j \mathbf{M}\boldsymbol{\phi}_j \|_2}{\| \mathbf{K}\boldsymbol{\phi}_j \|_2} \tag{2.68}$$

The maximum number of desired eigenpairs is set to be about 10 % of degrees of freedom in each example. To examine the suitable power value of the dynamic matrix, numerical examples are analyzed with varying the power value.

2.3.1 Simple Spring-Mass System

The first example is a simple spring-mass system as shown in Figure 2.1. The number of degrees of freedom is 100. Half-bandwidth of system matrices is 1.

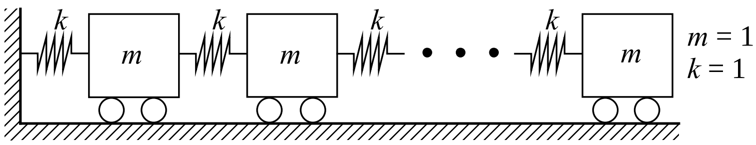


Figure 2.1 Geometry of simple spring-mass system

Some results are shown in Tables 2.3 and 2.4 and Figure 2.2. In the tables and the figure, δ represents the power value of the dynamic matrix. Results with the 1st power (δ = 1) correspond to the conventional Lanczos and subspace iteration methods. Tables 2.3 and 2.4 represent the number of required subspace iterations and the number of required Lanczos vectors, respectively, for calculating the desired eigenpairs. As seen from the tables, the convergence of the matrix-powered Lanczos and subspace iteration methods is better than that of the conventional Lanczos and subspace iteration methods. In the matrix-powered subspace iteration method, the 3rd and the 4th power give failure in convergence. In the matrix-powered Lanczos method, the 4th power gives failure in convergence. High power of matrix might cause numerical instability (Zill and Cullen, 1992). The failure of convergence in each method is due to the numerical instability.

Table 2.3 Number of required subspace iterations of simple spring-mass system

No. of desired eigenpairs	δ = 1 (ratio)	δ = 2 (ratio)	δ = 3 (ratio)	δ = 4 (ratio)
2	8 (1.00)	4 (0.50)	*	*
4	11 (1.00)	6 (0.55)	*	*
6	12 (1.00)	7 (0.58)	*	*
8	17 (1.00)	7 (0.41)	*	*
10	19 (1.00)	9 (0.47)	*	*

* : Failure in convergence

Table 2.4 Number of required Lanczos vectors of simple spring-mass system

No. of desired eigenpairs	δ = 1 (ratio)	δ = 2 (ratio)	δ = 3 (ratio)	δ = 4 (ratio)
2	9 (1.00)	7 (0.78)	6 (0.67)	5 (0.56)
4	14 (1.00)	11 (0.79)	9 (0.64)	8 (0.57)
6	18 (1.00)	14 (0.78)	12 (0.67)	11 (0.61)
8	21 (1.00)	17 (0.81)	15 (0.71)	14 (0.67)
10	25 (1.00)	20 (0.80)	18 (0.72)	*

* : Failure in convergence

Figure 2.2 summarizes the number of operations for calculating the desired eigenpairs in the matrix-powered Lanczos and subspace iteration methods. As seen from the figure, the number of operations of the matrix-powered Lanczos and subspace iteration methods is smaller than that of the conventional Lanczos and subspace iteration methods. Figure 2.2 shows that the number of operations of the matrix-powered Lanczos method is smaller than that of the matrix-powered subspace iteration method.

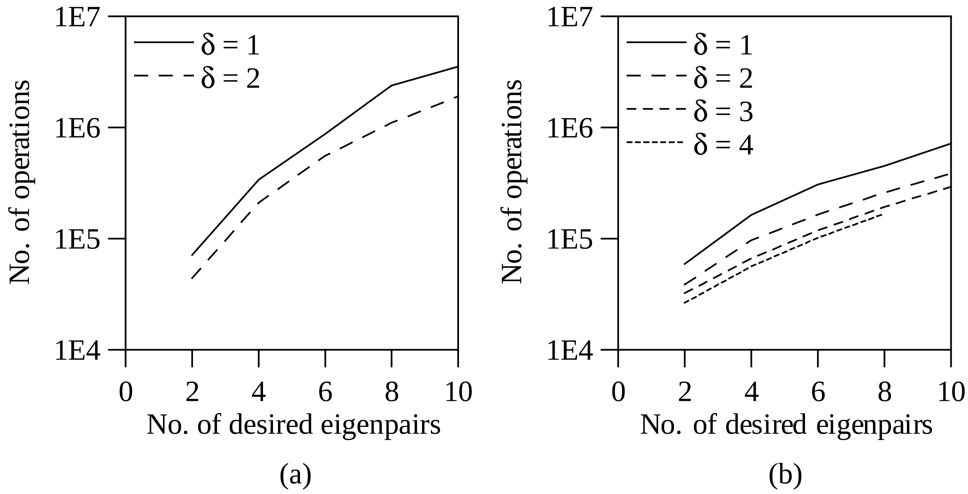


Figure 2.2 Number of operations of simple spring-mass system: (a) Matrix-powered subspace iteration method, (b) Matrix-powered Lanczos method

2.3.2 Plane Frame Structure

The second example is a plane frame structure. The geometric configuration, material properties and system data are shown in Figure 2.3 and Table 2.5.

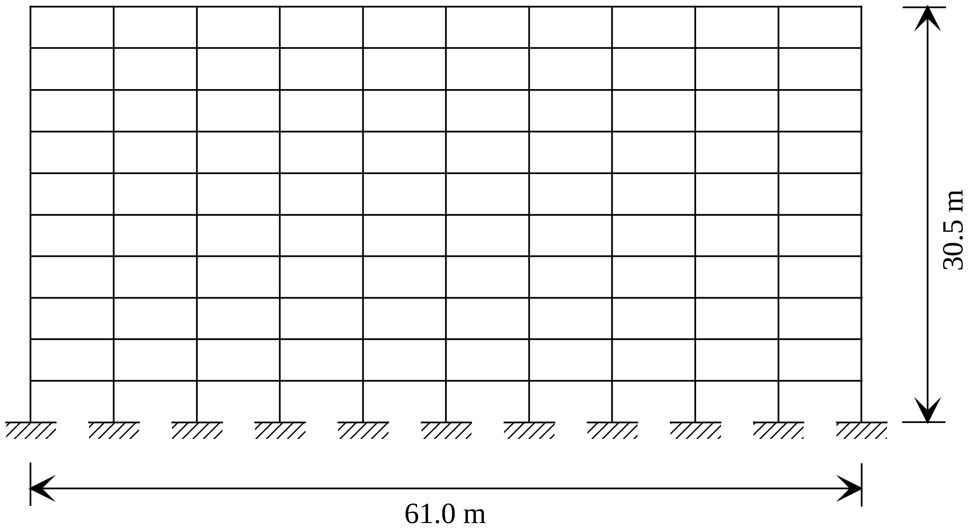


Figure 2.3 Geometry of plane frame structure

Table 2.5 Material properties and system data of plane frame structure

Material properties		System data	
Young's modulus (N/m^2)	2.068×10^{7}	Number of nodes	121
Section inertia (m^4)	8.631×10^{-3}	Number of beam elements	210
Section area (m^2)	0.2787	Number of degrees of freedom	330
Mass density (kg/m^3)	5.154×10^{2}	Half-bandwidth of system matrices	32

Results are summarized in Tables 2.6 and 2.7 and Figure 2.4. Tables 2.6 and 2.7 represent the number of required subspace iterations and the number of required Lanczos vectors, respectively, for calculating the desired eigenpairs. As seen from the tables, the convergence of the matrix-powered Lanczos and subspace iteration methods is better than that of the conventional Lanczos and subspace iteration methods. In the matrix-powered subspace iteration method, the 3rd and the 4th power give failure in convergence. In the matrix-powered Lanczos method, the 4th power gives failure in convergence.

Table 2.6 Number of required subspace iterations of plane frame structure

No. of desired eigenpairs	δ = 1 (ratio)	δ = 2 (ratio)	δ = 3 (ratio)	δ = 4 (ratio)
6	23 (1.00)	13 (0.57)	8 (0.35)	*
12	27 (1.00)	15 (0.56)	*	*
18	19 (1.00)	10 (0.53)	*	*
24	28 (1.00)	14 (0.50)	*	*
30	103 (1.00)	51 (0.50)	*	*

* : Failure in convergence

Table 2.7 Number of required Lanczos vectors of plane frame structure

No. of desired eigenpairs	δ = 1 (ratio)	δ = 2 (ratio)	δ = 3 (ratio)	δ = 4 (ratio)
6	29 (1.00)	21 (0.72)	18 (0.62)	16 (0.55)
12	39 (1.00)	29 (0.74)	25 (0.64)	23 (0.59)
18	45 (1.00)	35 (0.78)	31 (0.69)	29 (0.64)
24	49 (1.00)	39 (0.80)	35 (0.71)	*
30	92 (1.00)	71 (0.77)	62 (0.67)	*

* : Failure in convergence

Figure 2.4 represents the number of operations for calculating the desired eigenpairs in the matrix-powered Lanczos and subspace iteration methods. As seen from the figure, the number of operations of the matrix-powered Lanczos and subspace iteration methods is smaller than that of the conventional Lanczos and subspace iteration methods. It can be also seen that the number of operations of the matrix-powered Lanczos method is smaller than that of the matrix-powered subspace iteration method.

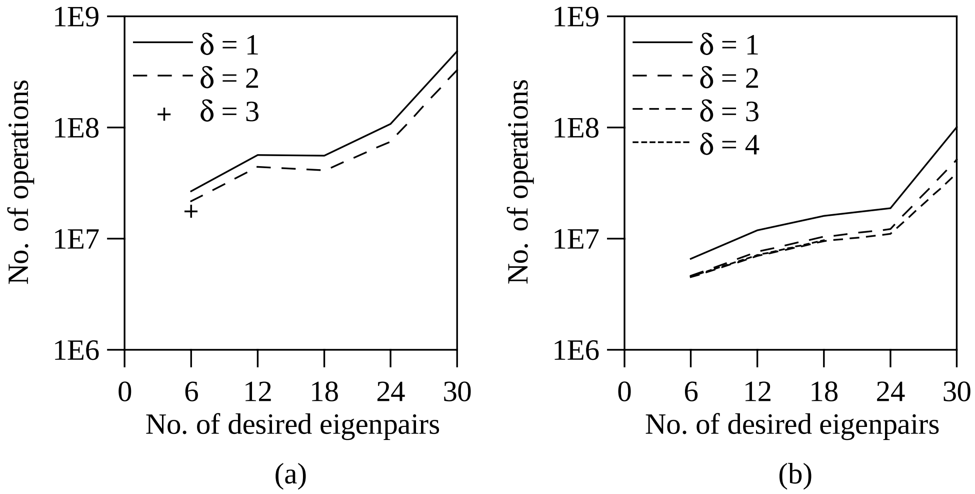


Figure 2.4 Number of operations of plane frame structure: (a) Matrix-powered subspace iteration method, (b) Matrix-powered Lanczos method

2.3.3 Three-Dimensional Frame Structure

The third example is a three-dimensional frame structure. The geometric configuration, the material properties and system data are shown in Figure 2.5 and Table 2.8.

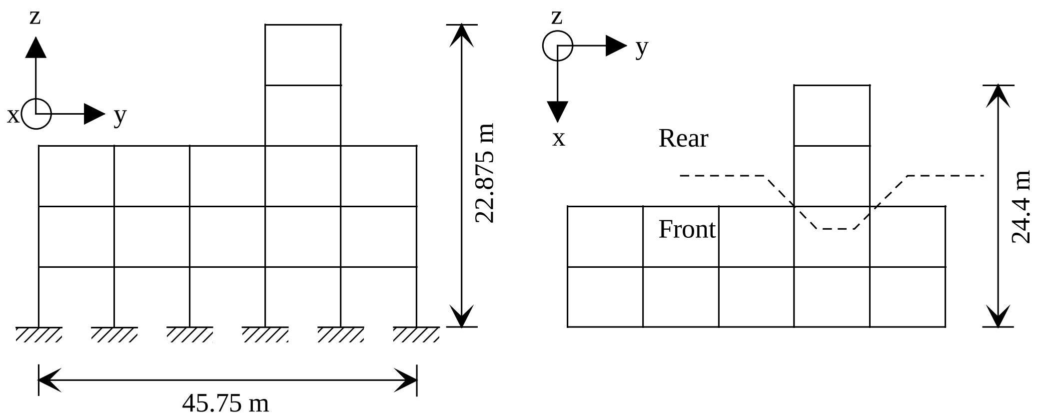


Figure 2.5 Geometry of three-dimensional frame structure

Table 2.8 Material properties and system data of three-dimensional frame structure

Material properties	Columns		Beams	
	Front	Rear	x-Direction	y-Direction
Young's modulus (N/m^2)	2.068×10^7	2.068×10^7	2.068×10^7	2.068×10^7
Section inertia (m^4)	8.631×10^{-3}	10.789×10^{-3}	6.473×10^{-3}	8.631×10^{-3}
Section area (m^2)	0.2787	0.3716	0.1858	0.2787
Mass density (kg/m^3)	5.154×10^2	5.154×10^2	5.154×10^2	5.154×10^2
System data				
Number of nodes	100			
Number of beam elements	191			
Number of degrees of freedom	468			
Half-bandwidth of system matrices	137			

Some results are shown in Tables 2.9 and 2.10 and Figure 2.6. Tables 2.9 and 2.10 represent the number of required subspace iterations and the number of required Lanczos vectors, respectively, for calculating the desired eigenpairs. The tables show that the convergence of the matrix-powered Lanczos and subspace iteration methods is better than that of the conventional Lanczos and subspace iteration methods. In the matrix-powered subspace iteration method, the 4th power gives failure in convergence.

Table 2.9 Number of required subspace iterations of three-dimensional frame structure

No. of desired eigenpairs	$\delta = 1$ (ratio)	$\delta = 2$ (ratio)	$\delta = 3$ (ratio)	$\delta = 4$ (ratio)
10	17 (1.00)	8 (0.47)	6 (0.35)	5 (0.29)
20	22 (1.00)	11 (0.50)	8 (0.36)	6 (0.27)
30	47 (1.00)	21 (0.45)	17 (0.36)	13 (0.28)
40	63 (1.00)	32 (0.51)	21 (0.33)	*
50	95 (1.00)	48 (0.51)	34 (0.36)	*

* : Failure in convergence

Table 2.10 Number of required Lanczos vectors of three-dimensional frame structure

No. of desired eigenpairs	$\delta = 1$ (ratio)	$\delta = 2$ (ratio)	$\delta = 3$ (ratio)	$\delta = 4$ (ratio)
10	28 (1.00)	21 (0.75)	19 (0.68)	17 (0.61)
20	48 (1.00)	37 (0.77)	34 (0.71)	31 (0.65)
30	64 (1.00)	51 (0.80)	46 (0.72)	43 (0.67)
40	99 (1.00)	77 (0.78)	69 (0.70)	64 (0.65)
50	121 (1.00)	94 (0.78)	84 (0.69)	78 (0.64)

Figure 2.6 summarizes the number of operations for calculating the desired eigenpairs in the matrix-powered Lanczos and subspace iteration methods. As seen from the figure, the number of operations of the matrix-powered Lanczos and subspace iteration methods is smaller than that of the conventional Lanczos and subspace iteration methods. It can be seen that the number of operations of the matrix-powered Lanczos method is smaller than that of the matrix-powered subspace iteration method.

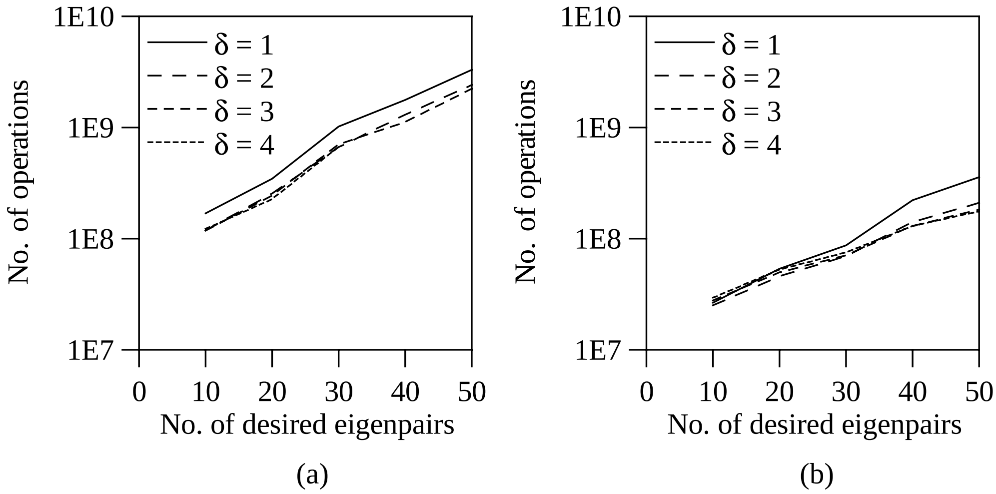


Figure 2.6 Number of operations of three-dimensional frame structure: (a) Matrix-powered subspace iteration method, (b) Matrix-powered Lanczos method

2.3.4 Three-Dimensional Building Structure

The last example is a three-dimensional building structure. The geometric configuration, the material properties and system data are shown in Figure 2.7 and Table 2.11.

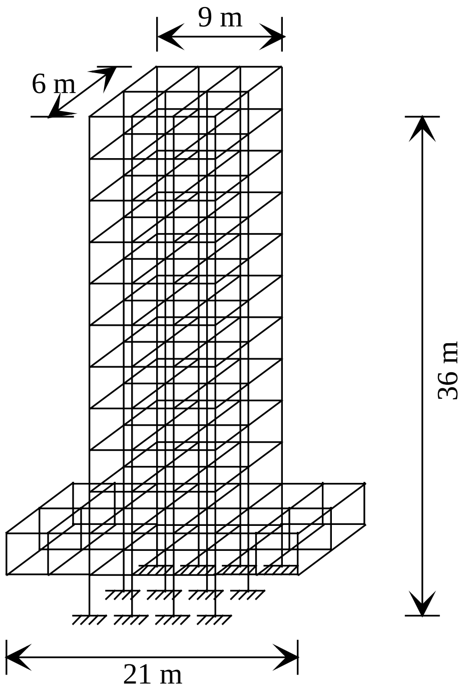


Figure 2.7 Geometry of three-dimensional building structure

Table 2.11 Material properties and system data of three-dimensional building structure

Material properties		System data	
Young's modulus (N/m^2)	2.1×10^{11}	Number of nodes	180
Section inertia (m^4)	8.3×10^{-6}	Number of beam elements	400
Section area (m^2)	0.01	Number of degrees of freedom	1008
Mass density (kg/m^3)	7.85×10^{3}	Half-bandwidth of system matrices	149

Tables 2.12 and 2.13 and Figure 2.8 shows the results. Tables 2.12 and 2.13 represent the number of required subspace iterations and the number of required Lanczos vectors, respectively, for calculating the desired eigenpairs. As seen from the tables, the convergence of the matrix-powered Lanczos and subspace iteration methods is better than that of the conventional Lanczos and subspace iteration methods. In the matrix-powered subspace iteration method, the 2nd, the 3rd and the 4th power give failure in convergence. In the matrix-powered Lanczos method, the 3rd and the 4th power give failure in convergence.

Table 2.12 Number of required subspace iterations of three-dimensional building structure

No. of desired eigenpairs	δ = 1 (ratio)	δ = 2 (ratio)	δ = 3 (ratio)	δ = 4 (ratio)
20	25 (1.00)	14 (0.56)	*	*
40	55 (1.00)	29 (0.53)	*	*
60	81 (1.00)	40 (0.49)	*	*
80	71 (1.00)	*	*	*
100	49 (1.00)	*	*	*

* : Failure in convergence

Table 2.13 Number of required Lanczos vectors of three-dimensional building structure

No. of desired eigenpairs	δ = 1 (ratio)	δ = 2 (ratio)	δ = 3 (ratio)	δ = 4 (ratio)
20	46 (1.00)	36 (0.78)	*	*
40	84 (1.00)	66 (0.79)	*	*
60	140 (1.00)	111 (0.79)	*	*
80	146 (1.00)	121 (0.83)	*	*
100	149 (1.00)	126 (0.85)	*	*

* : Failure in convergence

The number of operations for for calculating the desired eigenpairs in the matrix-powered Lanczos and subspace iteration methods is shown in Figure 2.8. As seen from the figure, the number of operations of the matrix-powered Lanczos and subspace iteration methods is smaller than that of the conventional Lanczos and subspace iteration methods. As shown in the figure, the number of operations of the matrix-powered Lanczos method is smaller than that of the matrix-powered subspace iteration method.

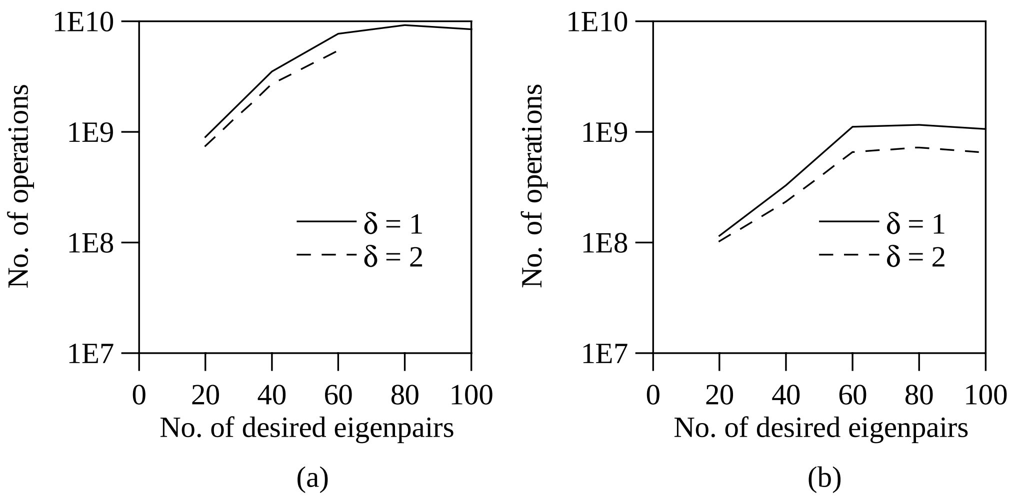


Figure 2.8 Number of operations of three-dimensional building structure: (a) Matrix-powered subspace iteration method, (b) Matrix-powered Lanczos method

Table 2.14 summarizes the number of operations of the matrix-powered Lanczos method for each example. From the table, it can be concluded that the suitable power value of the dynamic matrix that gives numerically stable solution is 2.

Table 2.14 Number of operations of matrix-powered Lanczos method

Structures	No. of desired eigenpairs	No. of operations ($\times 10^4$)			
		$\delta = 1$	$\delta = 2$	$\delta = 3$	$\delta = 4$
Simple spring-mass system	2	6	4	3	3
	4	16	10	7	6
	6	31	16	12	10
	8	45	26	19	17
	10	72	38	29	*
Plane frame structure	6	655	464	449	455
	12	1184	763	697	712
	18	1596	1041	949	965
	24	1875	1218	1102	*
	30	10089	5155	3851	*
3-dimensional frame structure	10	2653	2511	2774	2943
	20	5371	4581	4986	5268
	30	8672	6954	7083	7534
	40	22135	14155	13067	13018
	50	35811	20939	18249	17590
3-dimensional building structure	20	11485	10286	*	*
	40	33013	23448	*	*
	60	111578	65876	*	*
	80	115337	72156	*	*
	100	106151	64754	*	*

* : Failure in convergence

2.4 Summary of Results

By introducing the power of the dynamic matrix, the conventional Lanczos method has been improved. As shown in numerical analysis, the characteristics of proposed matrix-powered Lanczos method is identified as follows:

(1) The convergence of proposed matrix-powered Lanczos method is better than that of the conventional Lanczos method because the matrix-powered Lanczos algorithm can reduce the number of required Lanczos vectors. The number of operations of proposed method is also smaller than that of the conventional method.

(2) The number of operations of proposed matrix-powered Lanczos method is smaller than that of the matrix-powered subspace iteration method.

(3) The suitable power value of the dynamic matrix that that gives numerically stable solution in proposed matrix-powered Lanczos method is 2.

CHAPTER 3

IMPROVED LANCZOS VECTOR SUPERPOSITION METHOD

3.1 Conventional Method

In the analysis of dynamic responses of structural systems, the equation of motion can be expressed as

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f} \tag{3.1}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the n by n mass, damping and stiffness matrices, respectively, $\mathbf{f}$ is force vector and $\mathbf{u}$ is displacement vector. If Rayleigh damping is considered, (3.1) becomes

$$\mathbf{M}\ddot{\mathbf{u}} + (\alpha\mathbf{M} + \beta\mathbf{K})\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{f} \tag{3.2}$$

where α and β are the Rayleigh damping coefficients. Nour-Omid and Clough (1984) proposed the Lanczos vector superposition method to solve (3.2). The method is based on the conventional Lanczos recursion described in Section 2.1.1.

Performing transformation in (3.2) with $\mathbf{u} = \mathbf{X}\mathbf{q}$ and premultiplying both sides by $\mathbf{X}^T\mathbf{M}\mathbf{K}^{-1}$, (3.2) becomes

$$\mathbf{X}^T\mathbf{M}\mathbf{K}^{-1}\mathbf{M}\mathbf{X}\ddot{\mathbf{q}} + (\alpha\mathbf{X}^T\mathbf{M}\mathbf{K}^{-1}\mathbf{M}\mathbf{X} + \beta\mathbf{X}^T\mathbf{M}\mathbf{X})\dot{\mathbf{q}} + \mathbf{X}^T\mathbf{M}\mathbf{X}\mathbf{q} = \mathbf{X}^T\mathbf{M}\mathbf{K}^{-1}\mathbf{f} \tag{3.3}$$

where $\mathbf{X}(=[\mathbf{x}_1\,\mathbf{x}_2 \cdots \mathbf{x}_m])$ is a set of m (<< n) Lanczos vectors and $\mathbf{q}$ are generalized coordinates. From (2.28) and (2.31), this is rewritten in tridiagonal form

$$\mathbf{T}\ddot{\mathbf{q}} + (\alpha\mathbf{T} + \beta\mathbf{I})\dot{\mathbf{q}} + \mathbf{q} = \mathbf{X}^T\mathbf{M}\mathbf{K}^{-1}\mathbf{f} \tag{3.4}$$

where $\mathbf{I}$ is identity matrix of order *m*.

If single input loads are applied to structures, the force vector is of the form

$$\mathbf{f} = \mathbf{a}p \tag{3.5}$$

where $\mathbf{a}$ is the spatial load distribution vector and p is the time variation function. By taking $\mathbf{K}^{-1}\mathbf{a}$ as a starting vector in the Lanczos algorithm, the right side of (3.4) can be reduced as follows (see Table 3.1):

$$\mathbf{X}^T\mathbf{M}\mathbf{K}^{-1}\mathbf{f} = \mathbf{X}^T\mathbf{M}\mathbf{K}^{-1}\mathbf{a}p = \mathbf{X}^T\mathbf{M}\delta_0\mathbf{x}_1 p = \mathbf{X}^T\mathbf{M}\mathbf{X}\mathbf{e}_1\delta_0 p = \delta_0\mathbf{e}_1 p \tag{3.6}$$

where $\mathbf{e}_1$ is the first column of $\mathbf{I}$. Finally, (3.4) becomes

$$\mathbf{T}\ddot{\mathbf{q}} + (\alpha\mathbf{T} + \beta\mathbf{I})\dot{\mathbf{q}} + \mathbf{q} = \delta_0\mathbf{e}_1 p \tag{3.7}$$

When structures are under multi-input loads, the force vector takes the form

$$\mathbf{f} = \mathbf{A}\mathbf{p} \tag{3.8}$$

where $\mathbf{A}$ is the *n* by *k* spatial load distribution matrix, $\mathbf{p}$ is the *k* by 1 time variation function vector and *k* is the number of input loads. Then, (3.4) becomes

$$\mathbf{T}\ddot{\mathbf{q}} + (\alpha\mathbf{T} + \beta\mathbf{I})\dot{\mathbf{q}} + \mathbf{q} = \mathbf{X}^T\mathbf{M}\mathbf{K}^{-1}\mathbf{A}\mathbf{p} \tag{3.9}$$

In (3.9), the right side (transformed force vector) needs much computing time because forward reduction and back-substitution are required in the solution of $\mathbf{K}^{-1}\mathbf{A}$.

3.2 Proposed Method

3.2.1 Modified Lanczos Algorithm

In this dissertation, the modified Lanczos algorithm is proposed to improve the conventional Lanczos vector superposition method for structures under multi-input loads. Proposed algorithm introduces modified Lanczos vectors, $\mathbf{y}_i$'s that satisfy the following stiffness-orthonormality condition instead of mass-orthonormality condition.

$$\mathbf{y}_i^T \mathbf{K} \mathbf{y}_j = \delta_{ij} \tag{3.10}$$

where δ_{ij} Kronecker delta. Modified Lanczos recursion based on (3.10) can be derived as the following procedure. To derive the modified Lanczos recursion, assume that the first *i* Lanczos vectors, $\{\mathbf{y}_1, \mathbf{y}_2, \cdots, \mathbf{y}_i\}$, are founded, and the next Lanczos vectors, $\mathbf{y}_{i+1}$, will be constructed. As described in Section 2.1.1, next Lanczos vector $\mathbf{y}_{i+1}$ is obtained by computing preliminary vectors $\overline{\mathbf{y}}_i$ as

$$\overline{\mathbf{y}}_i = \mathbf{K}^{-1} \mathbf{M} \mathbf{y}_i \tag{3.11}$$

These preliminary vectors can be expressed as a linear combination of all the previous Lanczos vectors and a residual vector as

$$\overline{\mathbf{y}}_i = \tilde{\mathbf{y}}_i + \xi_i \mathbf{y}_i + \eta_{i-1} \mathbf{y}_{i-1} + \zeta_i \mathbf{y}_{i-2} + \cdots \tag{3.12}$$

with

$$\tilde{\mathbf{y}}_i = \eta_i \mathbf{y}_{i+1} \tag{3.13}$$

where the residual vector, $\tilde{\mathbf{y}}_i$, is the pure components of $\bar{\mathbf{y}}_i$ orthogonal to all the previous Lanczos vectors, and η_i is the pseudo length of $\tilde{\mathbf{y}}_i$. $\xi_i, \eta_{i-1}, \zeta_i, \cdots$ are the components of the previous Lanczos vectors contained in $\bar{\mathbf{y}}_i$. To obtain the coefficient ξ_i, we premultiply both sides of (3.12) by $\mathbf{y}_i^T\mathbf{K}$. Then using (3.10) and (3.11), we can get the following result

$$\xi_i = \mathbf{y}_i^T \mathbf{K}\, \bar{\mathbf{y}}_i = \mathbf{y}_i^T \mathbf{M}\, \mathbf{y}_i \tag{3.14}$$

The component η_{i-1} may be obtained similarly by premultiplying $\mathbf{y}_{i-1}^T\mathbf{K}$. Then using (3.10) and (3.11), we can have

$$\eta_{i-1} = \mathbf{y}_{i-1}^T \mathbf{K} \bar{\mathbf{y}}_i = \mathbf{y}_{i-1}^T \mathbf{M} \mathbf{y}_i \tag{3.15}$$

and applying the transpose of (3.11) to $\mathbf{y}_{i-1}^T$, we can get

$$\eta_{i-1} = \mathbf{y}_{i-1}^T \mathbf{M} \mathbf{y}_i = \mathbf{y}_{i-1}^T \mathbf{M} \mathbf{K}^{-1} \mathbf{K} \mathbf{y}_i = \bar{\mathbf{y}}_{i-1}^T \mathbf{K} \mathbf{y}_i \tag{3.16}$$

Finally, expanding $\bar{\mathbf{y}}_{i-1}$ in terms of (3.12), and then using the orthogonality relationships, we can get the following result

$$\eta_{i-1} = \tilde{\mathbf{y}}_{i-1}^T \mathbf{K} \mathbf{y}_i \tag{3.17}$$

or rewriting for the (*i*+1)st vector

$$\eta_i = \tilde{\mathbf{y}}_i^T \mathbf{K} \mathbf{y}_{i+1} \tag{3.18}$$

The new Lanczos vectors are obtained simply by scaling the pure vectors, $\tilde{\mathbf{y}}_i$, as

$$\mathbf{y}_{i+1} = \frac{\tilde{\mathbf{y}}_i}{\eta_i} \tag{3.19}$$

where η_i is the pseudo length of $\tilde{\mathbf{y}}_i$. Therefore, using the expressions for $\mathbf{y}_{i+1}$ in (3.18), we can obtain

$$\eta_i = (\tilde{\mathbf{y}}_i^T \mathbf{K}\, \tilde{\mathbf{y}}_i)^{1/2} \tag{3.20}$$

Continuing in the same ways as for finding the expression in (3.17), the coefficient ζ_i is obtained to be

$$\zeta_i = \tilde{\mathbf{y}}_{i-2}^T \mathbf{K}\mathbf{y}_i \tag{3.21}$$

Substituting (3.13) for $\tilde{\mathbf{y}}_{i-2}$ in (3.21), we can obtain

$$\zeta_i = \eta_{i-2} \mathbf{y}_{i-1}^T \mathbf{K}\mathbf{y}_i = 0 \tag{3.22}$$

A corresponding procedure could be used to demonstrate that all further terms in the expansions for $\bar{\mathbf{y}}_i$ will be zero. Therefore, modified Lanczos recursion can be written as

$$\tilde{\mathbf{y}}_i = \mathbf{K}^{-1}\mathbf{M}\mathbf{y}_i - \xi_i \mathbf{y}_i - \eta_{i-1}\mathbf{y}_{i-1} \tag{3.23}$$

Steps for the modified Lanczos algorithm are summarized in Table 3.1.

After m (<< n) steps, we have a set of Lanczos vectors, $\mathbf{Y} = [\mathbf{y}_1\, \mathbf{y}_2 \cdots \mathbf{y}_m]$, and $\mathbf{Y}$ satisfies following relations from (3.10)

$$\mathbf{Y}^T \mathbf{K}\mathbf{Y} = \mathbf{I} \tag{3.24}$$

where $\mathbf{I}$ is identity matrix of order *m*. Lanczos recursion, (3.13), can be rearranged in matrix form

$$\mathbf{K}^{-1}\mathbf{M}\mathbf{Y} - \mathbf{Y}\mathbf{S} = \eta_m \mathbf{y}_{m+1}\mathbf{e}_m^T \tag{3.25}$$

where $\mathbf{e}_m$ is the last column of $\mathbf{I}$ and $\mathbf{S}$ is a tridiagonal matrix of the form

$$\mathbf{S} = \begin{bmatrix} \xi_1 & \eta_1 & & & \\ \eta_1 & \xi_2 & \eta_2 & & \\ & & \ddots & & \\ & & \eta_{m-2} & \xi_{m-1} & \eta_{m-1} \\ & & & \eta_{m-1} & \xi_m \end{bmatrix} \tag{3.26}$$

Premultiplying (3.25) by $\mathbf{Y}^T\mathbf{K}$ and applying the stiffness-orthonormality condition, we get

$$\mathbf{Y}^T\mathbf{M}\mathbf{Y} = \mathbf{S} \tag{3.27}$$

3.2.2 Modified Tridiagonal Equation of Motion

Now, performing transformation in (3.2) with modified Lanczos coordinates, $\mathbf{u} = \mathbf{Y}\mathbf{r}$, and premultiplying both sides by $\mathbf{Y}^T$, (3.2) becomes

$$\mathbf{Y}^T\mathbf{M}\mathbf{Y}\ddot{\mathbf{r}} + (\alpha\mathbf{Y}^T\mathbf{M}\mathbf{Y} + \beta\mathbf{Y}^T\mathbf{K}\mathbf{Y})\dot{\mathbf{r}} + \mathbf{Y}^T\mathbf{K}\mathbf{Y}\mathbf{r} = \mathbf{Y}^T\mathbf{f} \tag{3.28}$$

From (3.24) and (3.27), this is rewritten in tridiagonal form

$$\mathbf{S}\ddot{\mathbf{r}} + (\alpha\mathbf{S} + \beta\mathbf{I})\dot{\mathbf{r}} + \mathbf{r} = \mathbf{Y}^T\mathbf{f} \tag{3.29}$$

If single input loads are applied to structures, the force vector takes the form of (3.5). By taking $\mathbf{K}^{-1}\mathbf{a}$ as a starting vector in the modified Lanczos algorithm, the right side of (3.29) can be reduced as follows (see Table 3.1):

$$\mathbf{Y}^T\mathbf{f} = \mathbf{Y}^T\mathbf{K}\mathbf{K}^{-1}\mathbf{a}p = \mathbf{Y}^T\mathbf{K}\eta_0\mathbf{y}_1 p = \mathbf{Y}^T\mathbf{K}\mathbf{Y}\mathbf{e}_1\eta_0 p = \eta_0\mathbf{e}_1 p \tag{3.30}$$

where $\mathbf{e}_1$ is the first column of $\mathbf{I}$. Finally, (3.29) will be

$$\mathbf{S}\ddot{\mathbf{r}} + (\alpha\mathbf{S} + \beta\mathbf{I})\dot{\mathbf{r}} + \mathbf{r} = \eta_0\mathbf{e}_1 p \tag{3.31}$$

When multi-input loads are applied to structures, (3.8) is used and (3.29) becomes

$$\mathbf{S}\ddot{\mathbf{r}} + (\alpha\mathbf{S} + \beta\mathbf{I})\dot{\mathbf{r}} + \mathbf{r} = \mathbf{Y}^T\mathbf{A}\mathbf{p} \tag{3.32}$$

The number of operations for computing $\mathbf{Y}^T\mathbf{A}$ in Equation (3.32) is *nmk*. In (3.9), $\mathbf{X}^T\mathbf{M}$ requires *nm*(2*h* + 1) operations (*h* is the half-bandwith of system matrices). Forward reduction and back-substitution in the calculation of $\mathbf{K}^{-1}\mathbf{A}$ need 2*nhk* operations. *nmk* operations are spent in multiplying $\mathbf{X}^T\mathbf{M}$ by $\mathbf{K}^{-1}\mathbf{A}$. So, total number of operations is 2*nh*(*m* + *k*) + *nm*(*k* + 1). It is clear that *nmk* is smaller than 2*nh*(*m* + *k*) + *nm*(*k* + 1). Therefore, (3.32) requires less computing time than (3.9).

Table 3.1 Algorithm for generation of Lanczos vectors

	Conventional algorithm		Proposed algorithm
1.	Pick starting vector $\mathbf{x}$	1.	Pick starting vector $\mathbf{y}$
2.	$\mathbf{x}_1 = \mathbf{x} / \delta_0,\ \delta_0 = (\mathbf{x}^T \mathbf{M} \mathbf{x})^{1/2}$	2.	$\mathbf{y}_1 = \mathbf{y} / \eta_0,\ \eta_0 = (\mathbf{y}^T \mathbf{K} \mathbf{y})^{1/2}$
3.	For i = 1, 2, ···, repeat:	3.	For i = 1, 2, ···, repeat:
	a. $\overline{\mathbf{x}}_i = \mathbf{K}^{-1} \mathbf{M} \mathbf{x}_i$		a. $\xi_i = \mathbf{y}_i^T \mathbf{M} \mathbf{y}_i$
	b. $\gamma_i = \mathbf{x}_i^T \mathbf{M} \overline{\mathbf{x}}_i$		b. If $i = m$, then terminate the loop
	c. If $i = m$, then terminate the loop		c. $\overline{\mathbf{y}}_i = \mathbf{K}^{-1} \mathbf{M} \mathbf{y}_i$
	d. $\tilde{\mathbf{x}}_i = \overline{\mathbf{x}}_i - \gamma_i \mathbf{x}_i - \delta_{i-1} \mathbf{x}_{i-1}\ (\mathbf{x}_0 = \mathbf{0})$		d. $\tilde{\mathbf{y}}_i = \overline{\mathbf{y}}_i - \xi_i \mathbf{y}_i - \eta_{i-1} \mathbf{y}_{i-1}\ (\mathbf{y}_0 = \mathbf{0})$
	e. $\hat{\mathbf{x}}_i = \tilde{\mathbf{x}}_i - \sum_{k=1}^{i} (\tilde{\mathbf{x}}_i^T \mathbf{M} \mathbf{x}_k) \mathbf{x}_k$		e. $\hat{\mathbf{y}}_i = \tilde{\mathbf{y}}_i - \sum_{k=1}^{i} (\tilde{\mathbf{y}}_i^T \mathbf{K} \mathbf{y}_k) \mathbf{y}_k$
	f. $\delta_i = (\hat{\mathbf{x}}_i^T \mathbf{M} \hat{\mathbf{x}}_i)^{1/2}$		f. $\eta_i = (\hat{\mathbf{y}}_i^T \mathbf{K} \hat{\mathbf{y}}_i)^{1/2}$
	g. $\mathbf{x}_{i+1} = \hat{\mathbf{x}}_i / \delta_i$		g. $\mathbf{y}_{i+1} = \hat{\mathbf{y}}_i / \eta_i$

Note: m = the number of Lanczos vectors to be used

3.3 Numerical Examples

A simple span beam (Pan and Li, 2002) and a multi-span continuous bridge (Park et al., 2002) are analyzed to verify the effectiveness of the proposed Lanczos vector superposition method. The results are compared with those of the eigenvector superposition method, the mode acceleration method, the Ritz vector superposition method and the conventional Lanczos vector superposition method. In the analysis by the eigenvector superposition method, eigenpairs are obtained the modified subspace iteration method that employs the Lanczos vectors as the starting vectors (Bathe, 1996) and removes converged eigenvectors from subspace (Wilson and Itoh, 1983). In each example, both single input and multi-input loads are considered. The accuracy and computing time are examined to compare each method. The following normalized RMS (root mean square) error is used in the examination of accuracy.

$$\varepsilon = \frac{\sqrt{\frac{1}{T_d}\int_0^{T_d} (\mathbf{u}_{exact} - \mathbf{u})^T (\mathbf{u}_{exact} - \mathbf{u})\, dt}}{\sqrt{\frac{1}{T_d}\int_0^{T_d} \mathbf{u}_{exact}^T \mathbf{u}_{exact}\, dt}} \tag{3.33}$$

where T_d is time duration. The results obtained by the Newmark method (Newmark, 1959) that is one of the direct integration methods are taken as the exact solutions.

3.3.1 Simple Span Beam

The first example is a simple span beam. The geometry, loading configurations, material properties and system data are shown in Figure 3.1 and Table 3.2. For the single input load, a concentrated sinusoidal force is used and a moving load is considered for the multi-input load. Nodal loading functions due to moving loads are generally modeled by triangular functions (Pan and Li, 2002). Since arrival times of a moving load are different at all nodal points, loading functions are also different at all nodal points. This means that a moving load is one of the multi-input loads. In this example, the number of input loads due to the moving load is 99 because the moving load passes 99 nodal points.

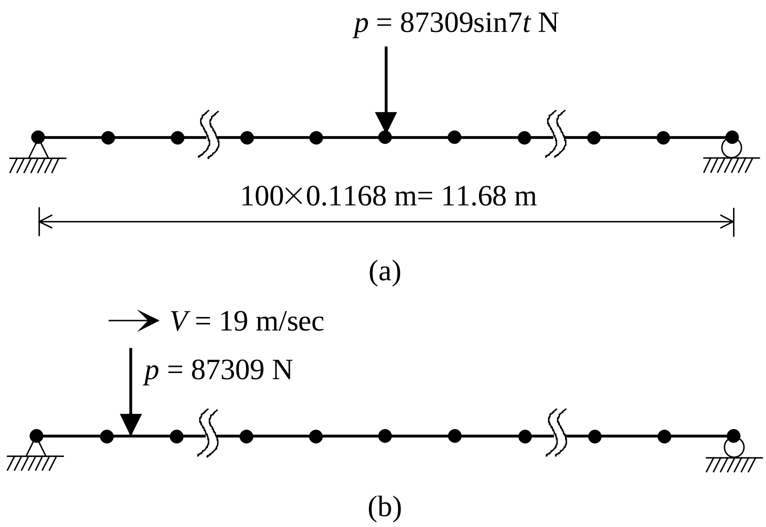


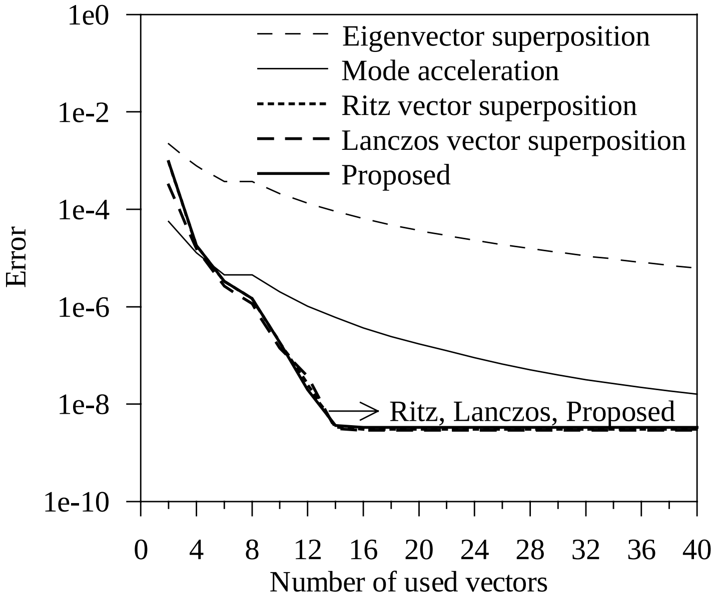
Figure 3.1 Simple span beam: (a) Single input load (sinusoidal load), (b) Multi-input load (moving load)

Table 3.2 Material properties and system data of simple span beam

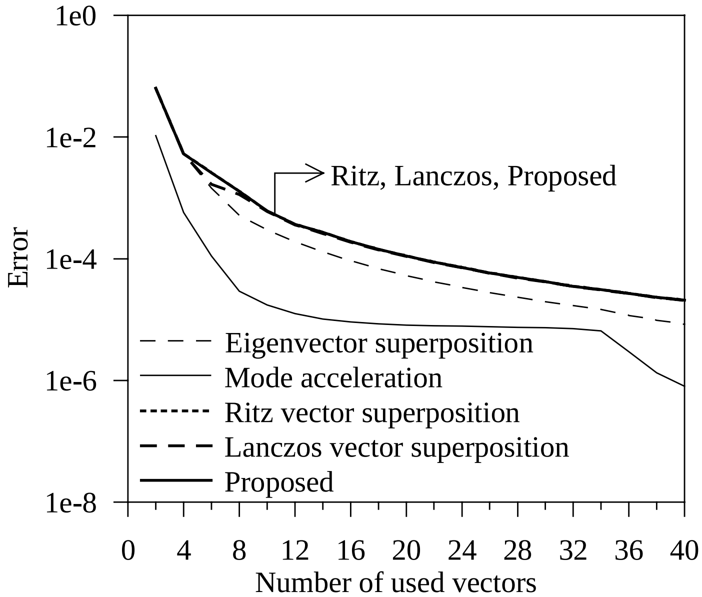
Material properties		System data	
Young's modulus (N/m^2)	1.72×10^8	Number of nodes	101
Section inertia (m^4)	1.00	Number of beam elements	100
Section area (m^2)	1.00	Number of degrees of freedom	200
Mass density (kg/m^3)	3105	Half-bandwidth of system matrices	3

Some results are shown in Figures 3.2 and 3.3. Figure 3.2 shows errors of each method with varying the number of used vectors. Proposed and conventional Lanczos vector superposition methods and the Ritz vector superposition method has almost the same accuracy. The three methods have better accuracy than the eigenvector superposition method and the mode acceleration method in the case of single input load. The reason the three methods have better accuracy is that the methods consider the spatial distribution of loading ($\mathbf{K}^{-1}\mathbf{a}$ in Equations (3.6) and (3.30)). On the other hand, the eigenvector superposition method and the mode acceleration method do not consider the loading distribution. For the multi-input loading case, the eigenvector superposition method and the mode acceleration method have better accuracy than the others. The accuracy of the mode acceleration method is the best.

The computing time of each method with varying the number of used vectors is presented in Figure 3.3. Since proposed and conventional Lanczos vector superposition methods and the Ritz vector superposition method do not perform eigenvalue analysis of structures, the three methods have less computing time than the eigenvector superposition method and the mode acceleration method. When the number of used vectors is large, the Ritz vector super position method is slightly more costly than proposed and conventional Lanczos vector superposition methods because it requires additional operation for computing eigensolution of reduced system. In the case of single input load, the computing efficiency of proposed and conventional Lanczos vector superposition methods is nearly the same. In the case of multi-input load, the conventional Lanczos vector superposition method is a little more costly than the Ritz vector superposition method when the number of used vectors is small. This is due to the calculation of transformed force vector (the right side of Equation (3.9)). It can bee seen that proposed method that reduces the operation for calculating transformed force vector has the best computing efficiency for multi-input loading case.



(a)



(b)

Figure 3.2 Accuracy of simple span beam: (a) Single input load, (b) Multi-input load

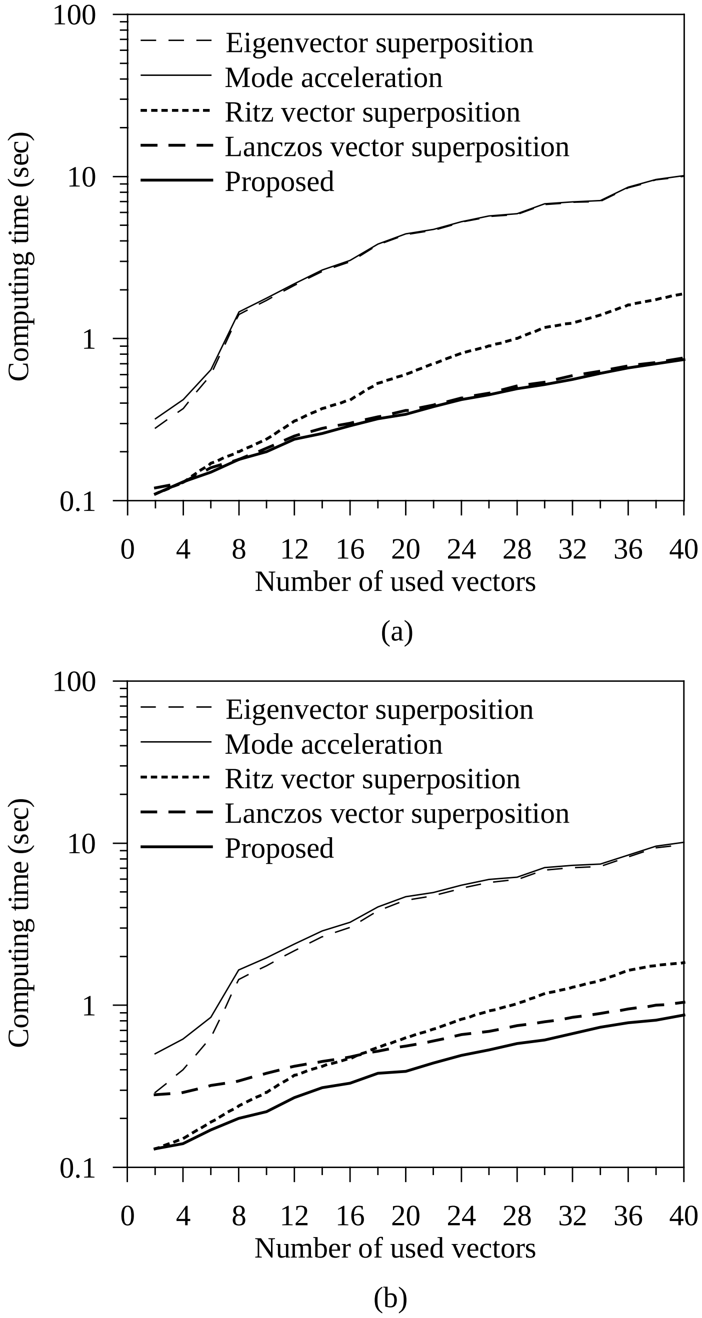
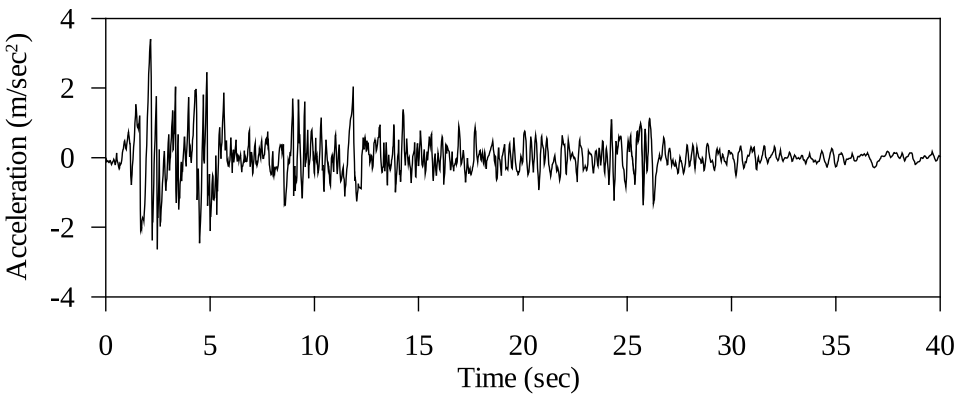


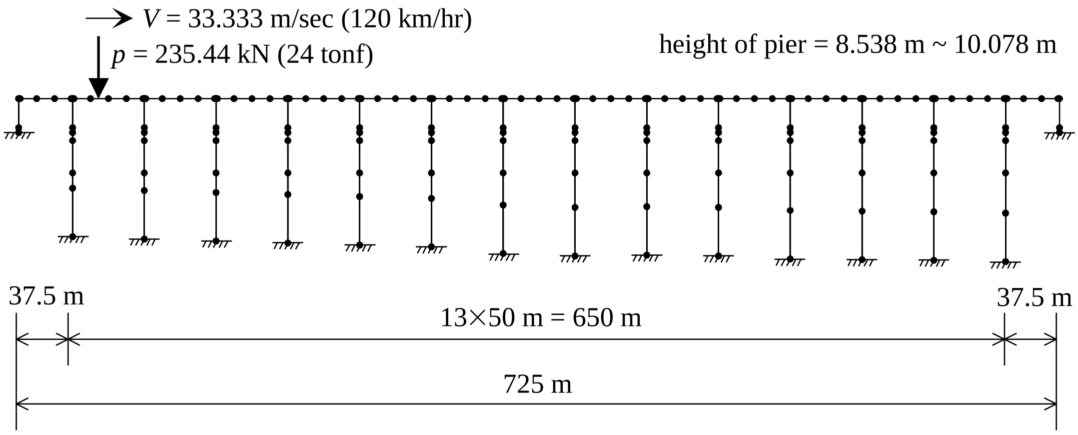
Figure 3.3 Computing time of simple span beam: (a) Single input load, (b) Multi-input load

3.3.2 Multi-Span Continuous Bridge

The second example is a multi-span continuous bridge. The deck type is PSC box girder. The geometry, loading configurations, material properties and system data are shown in Figure 3.4 and Table 3.3.



(a)



(b)

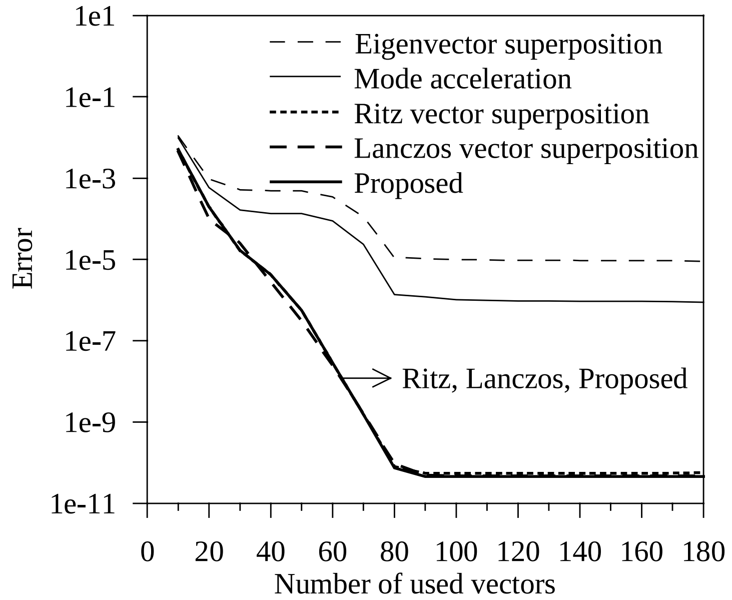
Figure 3.4 Multi-span continuous bridge: (a) Single input load (El Centro earthquake), (b) Multi-input load (moving load)

El Centro earthquake (N00W, 1940) is considered as a single input load and a moving load is considered for the multi-input load. The number of input loads due to the moving load is 89 because the moving load passes 99 nodal points on deck.

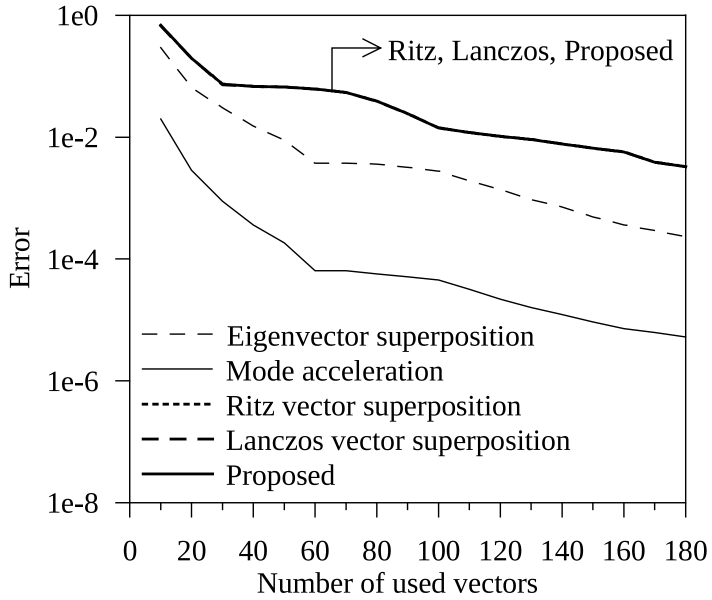
Numerical results are summarized in Figures 3.5 and 3.6. Figure 3.5 represents errors of each method with varying the number of used vectors. Figure 3.6 shows computing time for each method. It can bee seen that the results are similar to those of the previous example.

Table 3.3 Material properties and system data of multi-span continuous bridge

Material properties	Decks	Piers
Young's modulus (GN/m^2)	27.50	23.24
Section inertia (I_{xx}) (m^4)	80.67, 88.71, 105.70	92.17, 100.00, 84.42
Section inertia (I_{yy}) (m^4)	11.06, 11.54, 15.44	14.29, 25.01, 23.60
Torsion constant (m^4)	19.48, 20.87, 34.66	11.87, 66.73, 52.94
Section area (m^2)	8.62, 9.95, 18.24	14.00, 24.50, 17.00
Mass density (kg/m^3)	2300	2300
System data		
Number of nodes	177	
Number of beam elements	160	
Number of connection elements	16	
Number of degrees of freedom	870	
Half-bandwidth of system matrices	35	

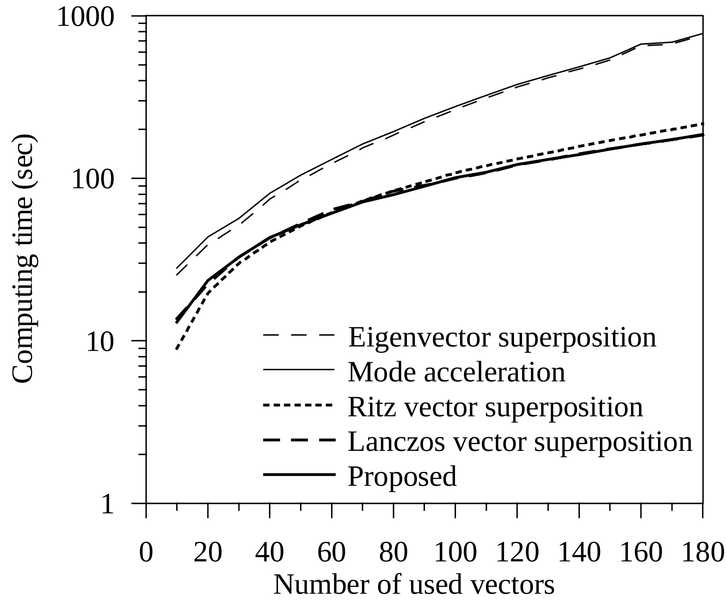


(a)

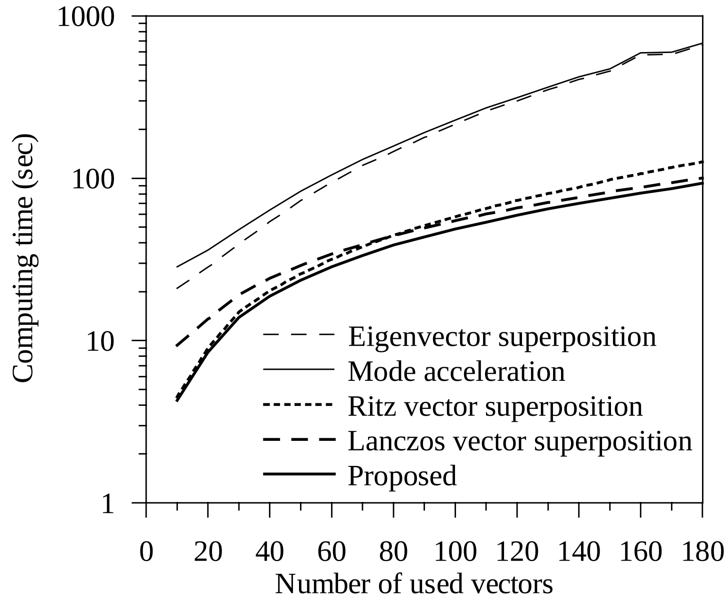


(b)

Figure 3.5 Accuracy of multi-span continuous bridge: (a) Single input load, (b) Multi-input load



(a)



(b)

Figure 3.6 Computing time of multi-span continuous bridge: (a) Single input load, (b) Multi-input load

3.4 Summary of Results

Improved Lanczos vector superposition method based on the modified Lanczos algorithm that generates stiffness-orthonormal Lanczos vectors is proposed for efficient dynamic analysis of structures under multi-input loads. Of course, proposed method is applicable to single-input-loaded structures. From numerical analysis, the characteristics of proposed Lanczos vector superposition method can be identified as follows:

(1) The accuracy of the Lanczos and Ritz vector superposition methods is almost equal.

(2) For the single input loading case, the Lanczos and Ritz vector superposition methods are more accurate than the eigenvector superposition and mode acceleration methods. For the multi-input loading case, the eigenvector superposition and mode acceleration methods are more accurate than the Lanczos and Ritz vector superposition methods.

(3) The Lanczos and Ritz vector superposition methods are more efficient than the eigenvector superposition and mode acceleration methods.

(4) For the single input loading case, the efficiency of proposed and conventional Lanczos vector superposition methods is almost equal. For the multi-input loading case, since proposed method can reduce computing operation for transformed force vector, proposed method is more efficient than the conventional Lanczos vector superposition method.

CHAPTER 4

CONCLUSIONS

4.1 Summary and Conclusions

This study proposes an improved Lanczos method by applying power of the dynamic matrix for efficient eigenvalue analysis of structures and an improved Lanczos vector superposition method for efficient dynamic response analysis of structures.

First, the characteristics of proposed matrix-powered Lanczos method by the analytical and the numerical results from examples are summarized as follows:

(1) The convergence of proposed matrix-powered Lanczos method is better than that of the conventional Lanczos method because the matrix-powered Lanczos algorithm can reduce the number of required Lanczos vectors. The number of operations of proposed method is also smaller than that of the conventional method.

(2) The number of operations of proposed matrix-powered Lanczos method is smaller than that of the matrix-powered subspace iteration method.

(3) The suitable power value of the dynamic matrix that that gives numerically stable solution in proposed matrix-powered Lanczos method is 2.

And, the characteristics of proposed Lanczos vector superposition method are summarized as follows by analytical investigations and numerical analyses of examples:

(1) The accuracy of the Lanczos and Ritz vector superposition methods is almost equal.

(2) For the single input loading case, the Lanczos and Ritz vector superposition methods are more accurate than the eigenvector superposition and mode acceleration methods. For the multi-input loading case, the eigenvector superposition and mode acceleration methods are more accurate than the Lanczos and Ritz vector superposition methods.

(3) The Lanczos and Ritz vector superposition methods are more efficient than the eigenvector superposition and mode acceleration methods.

(4) For the single input loading case, the efficiency of proposed and conventional Lanczos vector superposition methods is almost equal. For the multi-input loading case, since proposed method can reduce computing operation for transformed force vector, proposed method is more efficient than the conventional Lanczos vector superposition method.

4.2 Recommendations for Further Study

Most structures considered in analysis and design are assumed to be linear. Proposed methods in this dissertation can be efficiently used for eigenvalue and dynamic response analysis of linear structures. However, if factors such as plasticity, ductility, failure and large deflection are considered, structures will be nonlinear. Therefore, extension of proposed methods to nonlinear problems is required if nonlinear factors are considered. However, the practical modification of proposed methods for nonlinear problems is somewhat complicated. The research to extend proposed methods to nonlinear analysis is now in progress.

요 약 문

구조물의 동특성 및 동적응답 해석을 위한 효율적인 방법

본 논문은 Lanczos 방법의 개선을 통해 구조물의 효율적인 고유치 해법을 제안하였다. 아울러, 구조물의 효율적인 동적응답 해석을 위한 개선된 Lanczos 벡터 중첩법을 제안하였다.

Lanczos 방법은 효율적인 고유치 해법이다. 양자물리학 분야에서 양자의 고유상태를 더욱 효율적으로 계산하기 위해 행렬의 거듭제곱 기법을 도입한 수정된 Lanczos 알고리즘이 적용된 바 있다. 유사한 기법이 부분공간반복법에도 적용되었다. 반면, 그러한 행렬의 거듭제곱 기법이 구조공학 분야의 Lanczos 방법에 아직 적용되지 않았다. 본 논문은 동적행렬의 거듭제곱 기법을 도입한 개선된 Lanczos 방법을 제안하였다. 제안한 알고리즘이 요구되는 Lanczos 벡터의 수를 감소시키므로 제안방법은 기존의 방법보다 수렴성이 더욱 양호하며 연산회수가 더 적다. 한편, 동적행렬의 거듭제곱 시 수치적 불안정 문제가 발생할 수 있으므로 동적행렬의 적절한 거듭제곱값의 선택이 중요하다. 스프링-질량 시스템, 평면 뼈대 구조물, 삼차원 뼈대 구조물, 삼차원 빌딩 구조물의 수치해석을 통해 제안방법의 효율성을 검증하였으며 동적행렬의 적절한 거듭제곱값을 제시하였다. 수치예제로부터 제안방법과 행렬의 거듭제곱 기법을 도입한 부분공간반복법과의 비교를 아울러 제시하였다.

Lanczos 벡터중첩법은 구조물의 동적응답해석에 있어서 효율적이다. 그러나, 교량을 통과하는 이동하중, 고층빌딩에 작용하는 풍하중, 대형 해양구조물에 작용하는 파랑하중 등과 같은 다중입력하중의 해석 시 변환하중벡터 계산에 많은 수치연산이 요구된다는 단점이 있다. 본 논문은 그러한 단점을 극복한 개선된 Lanczos

벡터중첩법을 제안하였다. 제안방법은 기존의 질량 직교조건 대신 강성 직교조건을 만족하는 수정된 Lanczos 벡터를 생성하는 수정된 Lanczos 알고리즘에 기초하고 있다. 제안방법은 다중입력하중의 해석에 있어서 변환하중벡터의 계산시간을 감소시키므로 기존의 방법보다 더욱 효율적이다. 제안방법의 효율성을 검증하기 위해 단경간 보와 다경간 연속교량에 대한 수치해석을 수행하였다. 수치예제를 통해 제안방법과 고유벡터중첩법, 모드가속도법, Ritz 벡터중첩법, 기존의 Lanczos 벡터중첩법 등 다른 벡터중첩법과의 비교분석도 함께 수행하였다.

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감사의 글

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저보다 먼저 사회에 진출하신 실험실 선배님들께 깊은 감사드리며 먼저 사회에 진출한 실험실 후배들에게도 감사드립니다. 또한, 조지성 선배, 조상원, 박규식, 탄, 문영종, 최강민, 이헌재, 이성진, 장종우, 임현우, 장지은에게도 감사드립니다. 석사 동기들에게도 감사의 뜻을 전합니다. 대학교 선배님들에게 깊은 감사드리며 대학교 및 고등학교 동기들의 관심에 감사드립니다. 고향 친구들의 우정에 감사드리며 특히 박민에게 고맙다는 말을 전합니다.

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LIST OF PUBLICATIONS

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