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감쇠시스템의 개선된 **Sturm** 수열 성질

Modified Sturm Sequence Property for Damped Systems

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ABSTRACT

Most of the eigenvalue analysis methods for the undamped or proportionally damped systems use the well-known Sturm sequence property to check the missed eigenvalues when only a set of the lowest modes is to be used for large structures.

However, in the case of the non-proportionally damped systems such as the soil-structure interaction system, the structural control system and the composite structures, no counterpart of the Sturm sequence property for undamped systems has been developed yet. Hence, when some important modes are missed for those systems, it may lead to poor results in dynamic analysis.

In this study, a numerical method for calculating the number of eigenvalues in an open disk of arbitrary radius for the eigenproblem with a damping matrix is proposed by applying Gleyse's theorem. To verify the applicability of the proposed method, two numerical examples are considered.

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CHAPTER 1

INTRODUCTION

1.1 Background

To obtain the dynamic response of a large civil structure, it is economic and efficient to superpose the results of a few lowest modes. Therefore, there have been proposed many eigensolution techniques which can find only a set of the lowest modes. The Lanczos and subspace methods are belong to this type of technique. In these techniques, however, some important modes can be missed in the calculation process, which may lead to poor results in dynamic analysis. Hence, a checking technique for missed eigenvalues is required in finding the missed ones. For the case of the undamped or proportionally damped systems, they can be easily found by using the well-known Sturm sequence property (Meirovitch 1980; Hughes 1987; Petyt 1990; Bathe 1996).

However, in the case of the non-proportionally damped systems such as the soil-structure interaction system, the structural control system and composite structures and so on, no counterpart of the Sturm sequence property for undamped systems has been developed yet (Newland 1989). Hence, when some important modes are missed for those systems, it may lead to poor results in dynamic analysis. A number of researchers (Rajakumar 1993; Kim and Lee 1999) has been performed to solve the eigenproblem with the damping matrix, whereas there have been few studies on a technique to calculate the number of eigenvalues in this case in the open literature.

Tsai and Chen (1993) proposed the extended Sturm sequence property that can determine the root distribution of a polynomial on some specified lines of the complex plane. However, this extended property can not be applied to the non-proportionally damped system because the calculated lowest eigenvalues are ordered according to their distances form the origin in the complex plane and also because the extended Sturm sequence property needs a region surrounded by lines in the complex plane. So to apply this property, we must divide the boundary of the region into many lines to reshape the boundary to a circle. And also, the extended Sturm sequence property needs some symbolic operations, which are very difficult to realize numerically. Jung et al. (2001) proposed a technique of checking missed eigenvalues for eigenproblem with damping matrix using argument principle. This method requires the selection of checking points and the $\mathbf{LDL}^T$ factorization of the characteristic polynomial at those points. The accuracy of the method increases with the number of checking points, so it needs more factorization processes to get more exact results.

In this study, Gleyse's theorem (Gleyse and Moflih 1999), which can count the number of zeros of a characteristic polynomial inside an open unit disk, is used to calculate the number of eigenvalues for eigenproblem with a damping matrix. The characteristic polynomial of an eigenvalue problem is determined by using Rombouts' algorithm (Rombouts and Heyde 1998) which is considered as both stable and efficient algorithm. The determinants (minors) of the leading principal submatrices of order i in the Schur-Cohn matrix can be easily calculated by applying the $\mathbf{LDL}^T$ factorization process and the obtained final result is very similar to the Sturm sequence property for the undamped systems.

1.2 Literature Survey

The well-known Sturm sequence property has hitherto been applied to check the missed eigenvalues (Meirovitch 1980; Hughes 1987; Petyt 1990; Bahte 1996). The technique for checking the missed eigenvalues using the Sturm sequence property is very important in that it is the only means to make sure that indeed the required number of eigenvalues has been evaluated. As discussed in the background of this study, this checking process is required to obtain the exact dynamic response. The technique using the Sturm sequence property is used in the commercial FEM programs such as ADINA. However, this technique can only be applied to the eigenproblem without damping matrix such as the case of the undamped and proportional damped system (Newland 1989).

In most real systems, the damping matrix is non-proportional (Caughey and O'Kelly 1965). Even when a proportional damping is assumed for each sub-system in the analysis of soil-structure interaction problem, structural control problem, composite structures, etc., the resulting damping matrix for the complete structure will be non-proportional. In these cases, the eigenvalue problem with the non-proportional damping should be analyzed for the exact dynamic response. By using the subspace iteration method (Olson and Vandini 1989; Leung 1995), the Lanczos method (Chen and Taylor 1988; Rajakumar 1993), the Arnoldi method (Arnoldi 1951; Ren and Zheng 1997) and the method developed by Lee et al.(1998) and Kim and Lee(1999), one can easily obtain the solution of this eigenproblem. However, although the eigenvalue analysis is performed well, the accuracy of the dynamic response is not guaranteed because the missed eigenvalues may exist in the required ones. To guarantee the exact dynamic response, a technique that can check the missed eigenvalues of structures with non-proportional damping (i.e., the eigenproblem including the damping matrix) is required.

In the field of the structural dynamics, the study on a technique for checking the missed eigenvalues in the eigenvalue analysis including the damping matrix has been rarely carried out up to now. On the other hand, in the field of the control system engineering, relevant researches have been carried out a few (Locher 1993; Tsai and Chen 1993; Yamada et al. 1998). In the above literature, for stability test of a system having complex zeros, the root distribution of the characteristic polynomial of the system was investigated. However, the techniques presented in the above literature are difficult to apply to an eigenproblem including damping matrix of a system, because all the techniques calculate zeros of a polynomial by using rigorous symbolic algebraic operations. Recently, Jung et al. (2001) proposed a technique of checking missed eigenvalues for eigenproblem with damping matrix using argument principle. This method requires a selection of checking points and the $\mathbf{LDL}^T$ factorization of the characteristic polynomial at those points. The accuracy of the method increases with the number of checking points, so it needs more factorization processes to get more exact results.

As seen from the above review, a technique for checking the missed eigenvalues that is applicable to the eigenvalue analysis for the non-proportionally damped system has not been established yet. An appropriate technique for this case should be, therefore, developed to get the exact dynamic response as soon as possible.

1.3 Objectives and Scope

The purpose of this study is to develop an efficient numerical method for counting the number of eigenvalues inside an interested region for non-proportionally damped systems.

The objectives and scope of the study can be summarized as follows:

(1) <u>Selection of an appropriate mathematical property to be applicable to a technique for checking the missed eigenvalues of structures with non-proportional damping</u>:

(2) <u>Development of a technique for checking the missed eigenvalues that can be applicable to the non-proportionally damped structures by using the above appropriate mathematical property</u>:

Since an analytical solution by symbolic operations cannot be calculated, the numerical solution by complex arithmetic operations is considered. And, by analyzing the numerical examples, it is verified that the proposed method can check exactly the number of the missed eigenvalues.

1.4 Organization

This dissertation consists of four chapters. Chapter 1 discusses the background, the literature review, and the objectives and scope of this study.

In Chapter 2, the argument principle based method by Jung et al. (2001) is reviewed. The conventional equations of motion of damped system are reviewed in Section 2.1. In Section 2.2, the argument principle for a characteristic polynomial is reviewed. The effects of discretization of the closed contour are explained in Section 2.3. The evaluation of the arguments for the characteristic polynomial along the closed contour is explained in Section 2.4. In Section 2.5, the evaluation procedure and considerations for argument principle based method are presented.

In Chapter 3, the modified Sturm sequence property for damped systems is proposed. Three algorithms for calculating the coefficients of characteristic polynomial are reviewed in Section 3.1. In Section 3.2, a theorem for calculating the number of eigenvalues in an open disk of unit radius by Gleyse and Moflih (1998) is explained. In Section 3.3, the modified Sturm sequence property for counting the number of eigenvalues in an open disk of arbitrary radius from the origin is proposed.

In Chapter 4, to verify the effectiveness of the proposed method, two numerical examples of a simple spring-mass-damper system and a plane frame structure with lumped dampers are considered.

Finally, the conclusions of this study are summarized and some recommendations for the further studies are also presented in Chapter 5.

CHAPTER 2
REVIEW OF THE ARGUMENT PRINCIPLE-BASED METHOD

2.1 Equations of Motion of Damped Systems

In the analysis of dynamic response of structural system, the equation of motion of damped systems can be written as:

$$\mathbf{M}\ddot{\boldsymbol{u}}(t)+\mathbf{C}\dot{\boldsymbol{u}}(t)+\mathbf{K}\boldsymbol{u}(t)=0, \tag{2.1}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the ($n\times n$) mass, non-proportional damping and stiffness matrices, respectively, and $\ddot{\boldsymbol{u}}(t)$, $\dot{\boldsymbol{u}}(t)$ and $\boldsymbol{u}(t)$ are the ($n\times 1$) acceleration, velocity and displacement vectors, respectively. To find the solution of the free vibration of the system, the following quadratic eigenproblem is considered:

$$\lambda^2\,\mathbf{M}\,\phi+\lambda\,\mathbf{C}\,\phi+\mathbf{K}\,\phi=0, \tag{2.2}$$

in which λ and ϕ are the eigenvalue and eigenvector of the system respectively. There are 2*n* eigenvalues for the system with *n* degrees of freedom and these occur either in real pairs or in complex conjugate pairs, depending upon whether they correspond to overdamped or underdamped modes.

In general, the mass matrix $\mathbf{M}$ is non-singular, that is $\det(\mathbf{M}) \neq 0$, and we can reformulate the quadratic system of equation to a state-space form by doubling the order of the system (Meirovitch 1990; Rajakumar 1993; Kim and Lee 1999) such as:

$$\mathbf{A}\psi = \lambda\psi, \tag{2.3}$$

where

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}, \ \psi = \begin{Bmatrix} \phi \\ \lambda\phi \end{Bmatrix}. \tag{2.4}$$

The equation (2.3) is a standard eigenproblem, and the form of the matrix $\mathbf{A}$ in equation (2.4) is widely used in control engineering field (Meirovitch 1990).

2.2 Argument Principle for a Characteristic Polynomial (Jung et al. 2001)

Using the relationship between the eigenvalues of an eigenproblem and the zeros of the corresponding characteristic polynomial, the eigenvalues of the quadratic eigenproblem as equation (2.2) are equal to the zeros of the following characteristic polynomial:

$$p(\lambda) = \det(\lambda^2\mathbf{M} + \lambda\mathbf{C} + \mathbf{K}) = a_{2n}\lambda^{2n} + a_{2n-1}\lambda^{2n-1} + \cdots + a_1\lambda + a_0, \qquad (2.5)$$

where λ is a complex value and a_k $(k = 0,1,...,2n)$ are real coefficients. The value of λ that satisfies $p(\lambda) = 0$ is called an eigenvalue of the system.

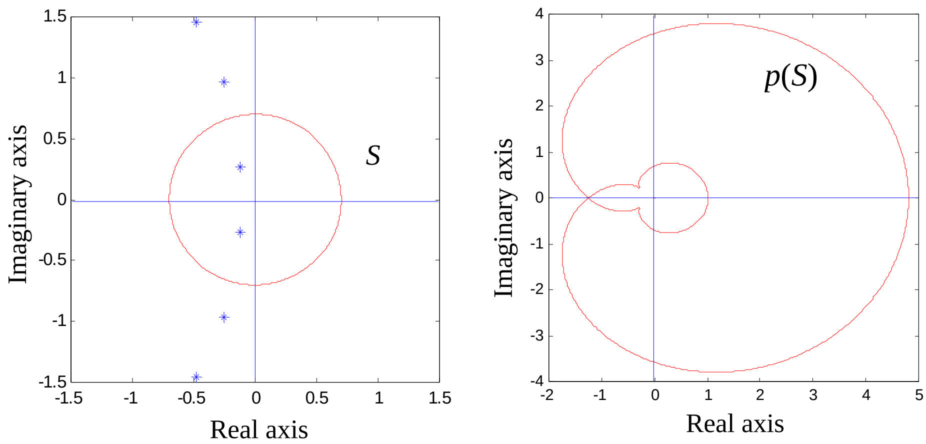
If the characteristic polynomial $p(\lambda)$ is analytic inside and on a simple closed contour S, the following argument principle can be applied (Franklin et al. 1998):

$$N = \frac{1}{2\pi i}\oint_S \frac{p'(\lambda)}{p(\lambda)} d\lambda = \frac{\Delta\theta}{2\pi}, \qquad (2.6)$$

where N is the number of zeros of $p(\lambda)$ inside the contour S, $\Delta\theta$ is the variation of the argument θ of $p(\lambda)$ around the contour S and i is defined as $\sqrt{-1}$.

Because $\Delta\theta/2\pi$ in the right side of equation (2.6) can be interpreted as the integer number of rotations, the characteristic polynomial $p(\lambda)$ that maps a moving point λ describing the contour S into a moving point $p(\lambda)$ encircles the origin of the $p(\lambda)$-plane N times if the polynomial $p(\lambda)$ has N zeros inside the contour S in the λ-

plane. An example that the characteristic polynomial $p(\lambda)$ has two zeros inside a contour S is shown in Figure 2.1. In Figure 2.1(a), the number of eigenvalues inside the contour S is 2, and in Figure 2.1(b), the number of rotations of the characteristic polynomial $p(\lambda)$ about the origin is 2.



(a) λ-plane (b) $p(\lambda)$-plane

Figure 2.1 Argument principle

However, since it is difficult to directly evaluate equation (2.6) using the symbolic algebraic operations, the numerical, or the iterative, approach was developed to apply the aforementioned argument principle to the non-proportionally damped systems.

2.3 Discretization of the Contour *S*

The characteristic polynomial $p(\lambda)$ in equation (2.5) at point $\lambda = \lambda_j$ can be factorized as follows:

$$p(\lambda_j) = a_{2n} \prod_{k=1}^{2n} (\lambda_j - z_k) = r_j \angle \theta_j \quad (2.7)$$

where z_k is the k-th zero of the $p(\lambda)$, and r_j and θ_j are the magnitude and argument of the value $p(\lambda_j)$ in polar form, respectively.

If we consider λ_j and z_k as a vector on the complex plane, $\lambda_j - z_k$ is also a vector from z_k to λ_j. The polar form of $\lambda_j - z_k$ can be written as:

$$\lambda_j - z_k = r_{j,k} e^{i\theta_{j,k}} \quad (2.8)$$

where $r_{j,k}$ is the length and $\theta_{j,k}$ is the argument of the $\lambda_j - z_k$. A graphical representation for the polar form of $\lambda_j - z_k$ is shown in Figure 2.2.

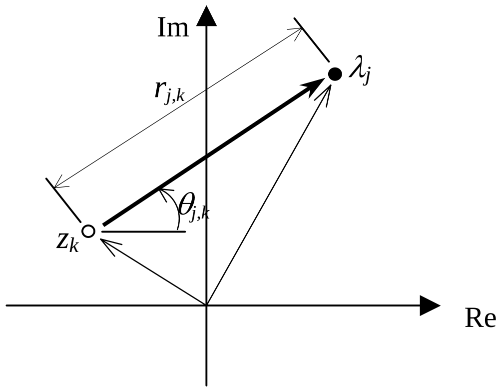


Figure 2.2 Graphical representation of λ_j-z_k

Using the polar form of $\lambda_j - z_k$ as in equation (2.8), the $p(\lambda_j)$ in equation (2.7) can be evaluated as:

$$p(\lambda_j) = a_{2n} \prod_{k=1}^{2n} (\lambda_j - z_k) = a_{2n} r_{j,1} r_{j,2} \cdots r_{j,2n} e^{i(\theta_{j,1}+\theta_{j,2}+\cdots+\theta_{j,2n})} \tag{2.9}$$

So, r_j and θ_j of the $p(\lambda_j)$ in equation (2.7) have the following relationships.

$$r_j = a_{2n} r_{j,1} r_{j,2} \cdots r_{j,2n} = a_{2n} \prod_{k=1}^{2n} r_{j,k} \tag{2.10}$$

$$\theta_j = \theta_{j,1} + \theta_{j,2} + \cdots + \theta_{j,2n} = \sum_{k=1}^{2n} \theta_{j,k} \tag{2.11}$$

To consider the effect of the discretization of the contour *S*, the values of $p(\lambda)$ at two consecutive discrete points $j=l$ and $j=l+1$ are evaluated as:

$$p(\lambda_l) = a_{2n} \prod_{k=1}^{2n} (\lambda_l - z_k) = r_l \angle \theta_l \tag{2.12}$$

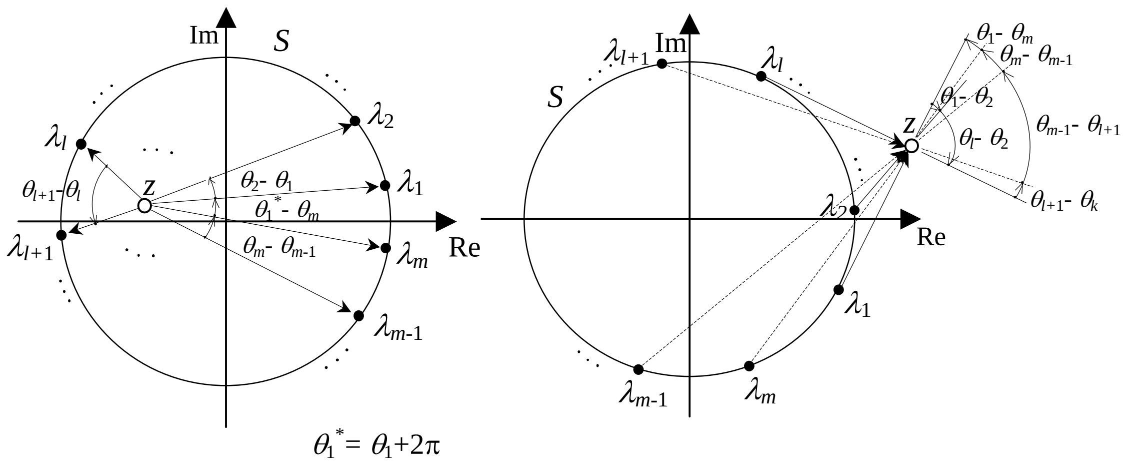
$$p(\lambda_{l+1}) = a_{2n} \prod_{k=1}^{2n} (\lambda_{l+1} - z_k) = r_{l+1} \angle \theta_{k+1} \tag{2.13}$$

Then, the change of argument $\Delta\theta_{l+1,l}$ from $p(\lambda_l)$ to $p(\lambda_{l+1})$ can be obtained as:

$$\Delta\theta_{l+1,l} = \theta_{l+1} - \theta_l = \sum_{k=1}^{2n} (\theta_{l+1,k} - \theta_{l,k}) \tag{2.14}$$

Since the argument change $\Delta\theta_{l+1,l}$ in equation (2.14) is simply sum of the effects of each zero of the characteristic polynomial $p(\lambda)$, we consider the simplest case that only one zero is inside the contour S as shown in Figure 2.3, where *m* is the number of discrete points on the closed contour S.

The Figure 2.3(a) represents the case that a zero is inside the closed contour *S*, and Figure 2.3(b) the case that a zero is outside the closed contour *S*. When a zero is inside the closed contour *S*, the sum of the argument change along discrete points on the *S* is 2π, and when a zero is outside the closed contour *S*, the sum of the argument change is 0. So, if we evaluate $p(\lambda)$ at some finite discrete point along the closed contour S, the sum of the change of the argument of $p(\lambda)$ divided by 2π is exactly equal to the number of zeros inside the contour S. From this figure we can also find that the argument change is abrupt when the zero is close to the contour S.



(a) zero inside contour *S* (b) zero outside contour *S*

Figure 2.3 Arguments at discrete points

2.4 Evaluation of the Arguments for $p(\lambda)$

The relationship between the characteristic polynomial and the factorized matrices by the $\mathbf{LDL}^T$ factorization process can be used to evaluate the arguments of $p(\lambda)$. The contour S is considered as the set of the discrete checking points as shown in Figure 2.4.

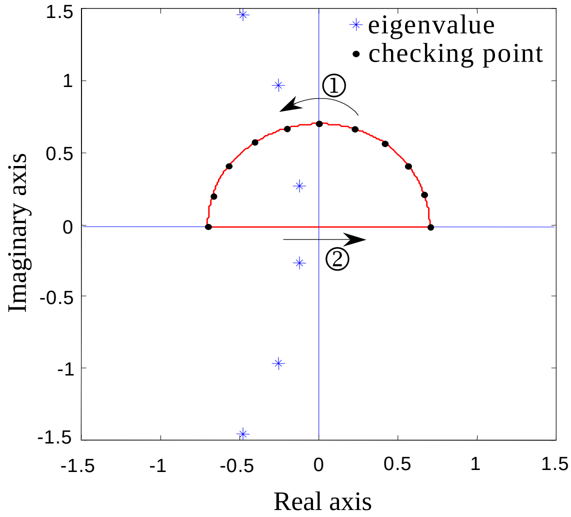


Figure 2.4 Process of checking missed eigenvalues

The $\mathbf{LDL}^T$ factorization process is performed at each checking point. Then, the argument at each checking point can be calculated as follows (Korn 1968; Peason 1974):

$$p(\lambda_j) = \det(\lambda_j^2\mathbf{M} + \lambda_j\mathbf{C} + \mathbf{K}) = \det\mathbf{L}\,\mathbf{D}\,\mathbf{L}^T = \prod_{k=1}^{n} d_{kk} = r_j \angle \theta_j, \qquad (2.15)$$

where d_{kk} are the diagonal elements of the diagonal matrix $\mathbf{D}$, and r_j and θ_j the are magnitude and argument of the value $p(\lambda_j)$ in polar form, respectively. The number of the eigenvalues inside the contour S is calculated by summing the variations of the argument of each checking point in equation (2.14).

2.5 Evaluaton Procedure and Considerations

In the implementation of the method for a practical problem, it is very important to properly choose the shape, the size and the number of discrete checking points of the closed contour S. The simplest shape of the contour is a disk of given radius about the origin as shown in Figure 2.5(a). This shape can be applied to various damping cases such as underdamped, critically damped, and overdamped cases. Most of practical systems are underdamped and eigenvalues of the system are complex conjugates, so a half-circle and a line on the real axis as in Figure 2.5(b) are sufficient to check the missed eigenvalues in this case.

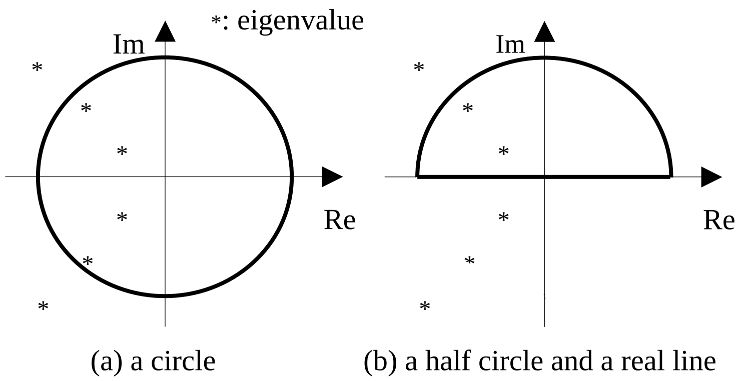


Figure 2.5 Shapes of the contour *S*

Because the argument change along the real line is 0, the number of checking points for the contour in Figure 2.5(b) is about half of that for the complete circle shown in Figure 2.5(a).

The size of the contour, i.e., the radius of a half-circle as in Figure 2.5(b) should be only a little bit larger than the largest eigenvalue to be considered to ensure that the next largest eigenvalue is not within the contour. The size of the contour recommended is

1.005 times the magnitude of the largest eigenvalue. This size is also used in the proposed method in the following section.

The number of checking points recommended is six times the number of considered eigenvalues (r). After the contour is equally divided into checking points, the part of the contour close to the largest eigenvalue is subdivided because the argument jump occurs in the part of the contour close to an eigenvalue. And, if the drastic change of the variation of the argument between two adjacent checking points occurs, the extra checking points between two adjacent checking points should be added. The detailed algorithm of the method is explained in Table 2.1.

The shape and size of the contour can be selected before applying the counting processes. However, the number of checking points varies during the processes when the drastic change of the variation of the argument occurs. And sometimes it is difficult to detect drastic change of the variation of the argument because the range of arguments is limited between 0° to 360° as well as it does not contain information about the number of rotations. If checking points are chosen sufficiently enough, the missed eigenvalues can exactly be checked by the method. However, the $\mathbf{LDL}^T$ factorizations of the characteristic polynomial at those points significantly increase the computational time.

Table 2.1 Algorithm of the argument principle based method

Step 1: Calculate the maximum magnitude, ρ.

- Select 1.005 times the absolute value of the *q*th eigenvalue ($\rho = 1.005|\lambda_q|$).

Step 2: Determine the number and the location of the initial checking points.

- *Number*: At least, select six times the number of the considered eigenvalues ($6r$).
- *Location*: Basically, divide into $6r$ equal parts.

Step 3: Perform the checking process.

1) Check the imaginary axis (the origin → ρi).
   - Perform the $\mathbf{LDL}^T$ factorizing process at each checking point.
   - Calculate the argument θ_j at each checking point.
2) Check the second quarter-plane ($\rho\angle 90°$ → $\rho\angle 180°$).
   - Perform the $\mathbf{LDL}^T$ factorizing process at each checking point.
   - Calculate the argument θ_j at each checking point.

Step 4: Analyze the variations of the arguments.

- If a decrease or aggressive variation of the argument occurs at a checking point, then go to Step 5 and if not, go to Step 6.

Step 5: Add extra checking points.

- Go to Step 3.

Step 6: Check the missed eigenvalues and stop.

- Calculate the total variation of the argument and the total rotation number.
- Compare the total rotation number (N in equation (2.6)) with the number of the considered eigenvalues (r).

CHAPTER 3

MODIFIED STURM SEQUENCE PROPERTY FOR DAMPED SYSTEMS

3.1 The Coefficients of the Characteristic Polynomial

The characteristic polynomial in equation (2.5) can be obtained another way using the matrix $\mathbf{A}$ in equation (2.4) as:

$$p(\lambda) = \det(\mathbf{A} - \lambda\mathbf{I}) = \hat{a}_{2n}\lambda^{2n} + \hat{a}_{2n-1}\lambda^{2n-1} + \cdots + \hat{a}_1\lambda + \hat{a}_0 = \sum_{k=0}^{2n} \hat{a}_k \lambda^k, \qquad (3.1)$$

where λ is a complex value and $\hat{a}_k \; (k = 0,1,...,2n)$ are real coefficients. The coefficients $a_k \; (k = 0,1,...,2n)$ in equation (2.5) are same scalar multiples to each $\hat{a}_k \; (k = 0,1,...,2n)$ in equation (3.1).

There are several methods for calculating the coefficients of the characteristic polynomial of a real square matrix. The most famous one is Le Verrier’s algorithm (Faddeev and Faddeeva 1953), which is often described as a standard method in text books (Chen 1984; Franklin 1998). Wang and Chen (1982) pointed out the numerical instability and inefficiency of Faddeev-Le Verrier’s method, and proposed a numerically stable method to compute the characteristic polynomial based on Frobenius form of a matrix. This method needs to prescribe a small value to prevent some elements be divided by this and this value should be guided by error analysis and/or experience. Recently, Rombouts and Heyde (1998) presented an algorithm for calculating the coefficients of the characteristic polynomial of a general square matrix for the evaluation of canonical traces in determinant quantum Monte-Carlo methods. This algorithm does not include division, so it is stable and also known as being efficient and accurate.

3.3.1 Le Verrier's Algorithm

Le Verrier (1840) was the first to consider the coefficient of the characteristic polynomial of a given matrix by using Newton's Formulae for symmetric functions. Except for some very short notes by Host (1935) and by Souriau (1948), nothing seems to have been added to what was known as Le Verrier's algorithm until the late fifties of the present century. Recently, there appeared some papers using Le Verrier's method, such as by Barnett's (1989). So Le Verrier's algorithm can be considered as the most widely used method for calculating the coefficients of the characteristic polynomial of a given matrix.

Each coefficient of the characteristic polynomial in equation (3.1) can be calculated as:

$$\begin{aligned}
\hat{\alpha}_{2n} &= 1 \\
\hat{\alpha}_{2n-1} &= -\frac{trAR_0}{1} \quad , R_0 = I \\
\hat{\alpha}_{2n-2} &= -\frac{trAR_1}{2} \quad , R_1 = AR_0 + \hat{\alpha}_{2n} I = A + \hat{\alpha}_{2n} I \\
&\vdots \\
\hat{\alpha}_1 &= -\frac{trAR_{2n-2}}{2n-1} \quad , R_{2n-2} = AR_{2n-2} + \hat{\alpha}_3 I = A^{2n-2} + \hat{\alpha}_{2n} A^{2n-3} + \cdots + \hat{\alpha}_4 A + \hat{\alpha}_3 I \\
\hat{\alpha}_0 &= -\frac{trAR_{2n-1}}{2n} \quad , \quad 0 = AR_{2n-1} + \hat{\alpha}_1 I
\end{aligned} \tag{3.2}$$

where *tr* stands for the *trace* and is defined as the sum of all the diagonal elements of a matrix. If the order of a matrix is k, then this algorithm requires a total of k times of multiplication of two k×k matrices . Hence the number of operations is proportional to k^4. Furthermore, this algorithm may encounter some numerical difficulties (Aplevich, 1974).

3.3.2 Chen's Algorithm

Chen (1988) suggested a numerically stable algorithm to obtain the coefficients of the characteristic polynomial of a real square matrix.

A given matrix $\mathbf{A}$:

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1,2n-1} & a_{1,2n} \\ a_{21} & a_{22} & \cdots & a_{2,2n-1} & a_{2,2n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{2n-1,1} & a_{2n-1,2} & \cdots & a_{2n-1,2n-1} & a_{2n-1,2n} \\ a_{2n,1} & a_{2n,2} & \cdots & a_{2n,2n-1} & a_{2n,2n} \end{bmatrix}, \tag{3.3}$$

can be transformed to $\overline{\mathbf{A}}$:

$$\overline{\mathbf{A}} = \begin{bmatrix} -a_{2n-1} & -a_{2n-2} & \cdots & -a_1 & -a_0 \\ 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}, \tag{3.4}$$

by applying a sequence of Gauss-elimination like similarity transformations. Detailed algorithms are shown in Table 2.2. Since $\overline{\mathbf{A}}$ was obtained by applying similar transformations to $\mathbf{A}$, the eigenvalues and eigenvectors of both $\overline{\mathbf{A}}$ and $\mathbf{A}$ are same. The characteristic polynomial of $\mathbf{A}$ can be obtained by observing the first row in the transformed matrix $\overline{\mathbf{A}}$, and the characteristic polynomial is

$$\begin{aligned} p(\lambda) &= \det(\mathbf{A} - \lambda\mathbf{I}) \\ &= \det(\overline{\mathbf{A}} - \lambda\mathbf{I}) \\ &= \lambda^{2n} + a_{2n-1}\lambda^{2n-1} + \cdots + a_1\lambda + a_0 = 0, (a_{2n} = 1). \end{aligned} \tag{3.5}$$

When some numerical instabilities due to very small off-diagonal elements occur during the transformations, the modified algorithm by Chen (1988) can be used.

Table 3.1 Chen's algorithm to transform an 2n×2n matrix of $\mathbf{A} = (a_{ij})$

Step 1: Choose two positive small numbers ε_1 and ε_2.

- $\varepsilon_1 < \varepsilon_2$.
- These two numbers are used to test small off-diagonal elements.

Step 2: Set j=1.

Step 3: Find a_{rj}***, the largest in absolute value element of the set***
$\{a_{ij},\ i = j+1, j+2, \cdots, 2n\}$

- If $r < j+1$, go to step 4.

Step 4: Interchange the (j+1)th and the rth rows, then the (j+1)th and the rth columns.

Step 5: If $|a_{j+1,j}| \le \varepsilon_1$***, go to Step 6.***

- Zero $a_{ij}\ (i = j+2, j+3, \cdots, 2n)$ by adding the product of the *(j+1)*th row with $(-a_{ij}/a_{j+1,j})$ to the *i*th row.
- Add the product of the *i*th column with $(a_{ij}/a_{j+1,j})$ to the *(j+1)*th column.

Step 6: If $|a_{j+1,j}| \le \varepsilon_2$***, go to Step 7.***

- Multiply the *(j*+1)th row by $(1/a_{j+1,j})$.
- Multiply the *(j*+1*)*th column by $a_{j+1,j}$.

Step 7: If $j < 2n, j \to j+1$***, and go to Step 2.***

- Set $i = 2n$.

Step 8: If $a_{i,i-1} \ne 1$***, go to Step 9.***

- Zero $a_{ij}\ (j = i, i+1, \cdots, 2n)$ by adding the product of the *(i-1)*th column with $(-a_{ij})$ to the *j*th column.
- Add the product of the *j*th row with a_{ij} to the *(j-1)*th row.

Step 9: If $i = 2$***, go to Step 10.***

- $i \to i-1$ and go to Step 8.

Step 10: Stop.

- The matrix **A** is transformed into the form $\overline{\mathbf{A}}$.

3.3.3 Rombouts' Algorithm

A general real square matrix $\mathbf{A}$ as in equation (3.3) can be transformed to upper Hessenberg form $\hat{\mathbf{A}}$:

$$\hat{\mathbf{A}} = \begin{bmatrix} \hat{a}_{1,1} & \hat{a}_{1,2} & \cdots & \hat{a}_{1,2n-1} & \hat{a}_{1,2n} \\ \hat{a}_{2,1} & \hat{a}_{2,2} & \cdots & \hat{a}_{2,2n-1} & \hat{a}_{2,2n} \\ 0 & \hat{a}_{3,2} & \cdots & \hat{a}_{3,2n-1} & \hat{a}_{3,2n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \hat{a}_{2n,2n-1} & \hat{a}_{2n,2n} \end{bmatrix}, \tag{3.6}$$

by applying Householder reduction or sequence of Gaussian elimination like similarity transformations (Press et al. 1988).

Because the matrix $\hat{\mathbf{A}}$ was obtained by applying similarity transformations to $\mathbf{A}$, the eigenvalues of both $\hat{\mathbf{A}}$ and $\mathbf{A}$ are the same and the characteristic polynomials are different from each other by a scalar ratio to each other. For the purpose of calculating eigenvalues of the system, therefore, the characteristic polynomial can be considered as:

$$p(\lambda) = \det(\hat{\mathbf{A}} - \lambda \mathbf{I}). \tag{3.7}$$

If we define $\overline{p}(\lambda)$ as:

$$\overline{p}(\lambda) = \det(\mathbf{I} + \lambda \hat{\mathbf{A}}), \tag{3.8}$$

then this polynomial is closely related to $p(\lambda)$ as:

$$\overline{p}(\lambda) = (\lambda)^{2n} \, p(-1/\lambda) \, . \tag{3.9}$$

The basic idea of the Rombouts' algorithm is to consider $\hat{\mathbf{A}} + \lambda\mathbf{I}$ as a matrix of polynomial in λ. We then calculate the polynomial $\overline{p}(\lambda)$ by evaluating the determinant in equation (3.8) using Gaussian elimination, with polynomials instead of scalars as matrix elements. As presented in equation (3.9) the coefficients of $\overline{p}(\lambda)$ are closely related to the coefficients of the $p(\lambda)$. The procedures of Rombouts' algorithm for calculating the coefficients of the characteristic polynomial of a 2*n*-by-2*n* matrix $\mathbf{A}$ are presented in Table 3.2.

Table 3.2 Rombouts' algorithm for calculating characteristic polynomial

***Step 1: Reduce the given matrix* A *(2n-*by*-2n) to upper Hessenberg form* $\hat{\mathbf{A}}$.**

- Use Householder reductions.

Step 2: Initialize a matrix* B *(2n-*by*-2n).

- Set all the elements of matrix **B** to 0.

Step 3:Calculate element* b_{ij} *of the matrix* B *using the element* $\hat{a}_{ij}$ *of the matrix* $\hat{\mathbf{A}}$ *as follows:

DO $j = 2n,1,-1$
    DO $i = 1,j$
        DO $k = 2n-j, 1, -1$
            $b_{k+1,i} = \hat{a}_{i,j} b_{k,j+1} - \hat{a}_{j+1,j} b_{k,i}$
        ENDDO
        $b_{1,i} = \hat{a}_{i,j}$
    ENDDO
    DO $k = 1, 2n-j$
        $b_{k,j} = b_{k,j} + b_{k,j+1}$
    ENDDO
ENDDO

Step 4: Calculate the coefficients of the characteristic polynomial* $\hat{a}_i$ *(i=0, ⋯,2n) and stop

- Using the first row of the matrix B, the coefficients of the characteristic polynomial can be computed as:

$$\hat{a}_i = (-1)^{2n-i}\, b_{2n-i,1}$$

3.2 The Number of Eigenvalues in an Unit Open Disk

Gleyse and Moflih (1999) suggested a method of calculating the number of eigenvalues of a real polynomial in a unit open circle as shown in Figure 3.1 by a determinant representation.

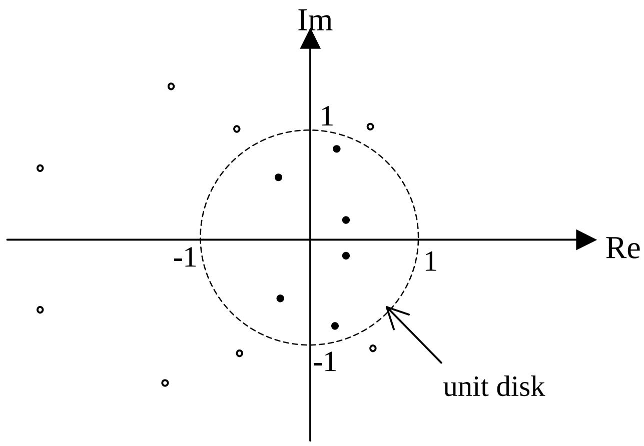


Figure 3.1 Eigenvalues in a unit open disk
(○:eigenvalues outside the open disk , •: eigenvalues in the open disk)

Let $p(\lambda)=\sum_{k=0}^{2n}\hat{a}_k\lambda^k$ ($\hat{a}_k$ is a real number) be the characteristic polynomial of a given matrix **A**, then the number of eigenvalues in a unit open circle can be determined as:

$$N_o = 2n - V[1, d_1, d_2, \cdots, d_{2n}] \qquad (3.10)$$

where N_o is the number of eigenvalues in a unit open circle, $2n$ is the degree of the polynomial, $V[d_0, d_1, d_2, \cdots, d_{2n}]$, $(d_0 = 1)$ is the number of sign changes in the sequence $d_i\ (i=0,1,\cdots,2n)$ and $d_i\ (i=1,2,\cdots,2n)$ are the determinants (minors) of the leading principal submatrices of order *i* in the Schur-Cohn matrix $\mathbf{T}$:

$$
\begin{array}{cc}
 & \begin{array}{ccccc} \mathbf{T}_1 & \cdots & \mathbf{T}_i & \cdots & \mathbf{T}_{2n} \end{array} \\
\mathbf{T}= & \begin{bmatrix} t_{11} & \cdots & t_{1i} & \cdots & t_{1,2n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ t_{i1} & \cdots & t_{ii} & \cdots & t_{i,2n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ t_{2n,1} & \cdots & t_{2n,i} & \cdots & t_{2n,2n} \end{bmatrix}
\end{array}
$$

$$t_{ij} = \sum_{h=0}^{\min(i,j)} \left(\hat{a}_{2n-i+h}\hat{a}_{2n-j+h} - \hat{a}_{i-h}\hat{a}_{j-h} \right) \quad (i,j = 1,2,\cdots,2n) \tag{3.11}$$

$$d_i = \det(\mathbf{T}_i) \tag{3.12}$$

The processes of counting the number of eigenvalues inside a unit open circle by the above method require calculation of the characteristic polynomial of a given matrix $\mathbf{A}$, the construction of the Schur-Cohn matrix $\mathbf{T}$ and the calculation of the determinants (minors) of the leading principal submatrices of order *i* in the Schur-Cohn matrix $\mathbf{T}$. The coefficients of the characteristic polynomial of a given matrix can be determined by Rombouts' algorithm described in the previous section 3.1.3, and each element of the Schur-Cohn matrix can be obtained using equation (3.11).

3.3 Proposed Method

Gleyse's theorem (Gleyse and Moflih 1999) considers only the number of eigenvalues in a unit open circle. To apply this theorem to an open circle of arbitrary radius ρ, we substitute $\lambda = \rho\overline{\lambda}$ (ρ is a real number) to equation (3.1), then the modified characteristic polynomial can be written as:

$$\begin{aligned} P(\overline{\lambda}) &= \hat{a}_{2n}\rho^{2n}\overline{\lambda}^{2n} + \hat{a}_{2n-1}\rho^{2n-1}\overline{\lambda}^{2n-1} + \cdots + \hat{a}_1\rho\overline{\lambda} + \hat{a}_0 \\ &= \tilde{a}_{2n}\overline{\lambda}^{2n} + \tilde{a}_{2n-1}\overline{\lambda}^{2n-1} + \cdots + \tilde{a}_1\overline{\lambda} + \overline{a} = \sum_{k=0}^{2n}\tilde{a}_k\overline{\lambda}^k , \end{aligned} \tag{3.13}$$

where $\tilde{a}_k = \hat{a}_k\rho^k \, (k = 0, 1, \cdots, 2n)$ are modified coefficients. Using the modified coefficients $\tilde{a}_k \, (k = 0, 1, \cdots, 2n)$ in equation (3.13), this theorem can be extended to calculate the number of eigenvalues in the open disk of arbitrary radius ρ as shown in Figure 3.2.

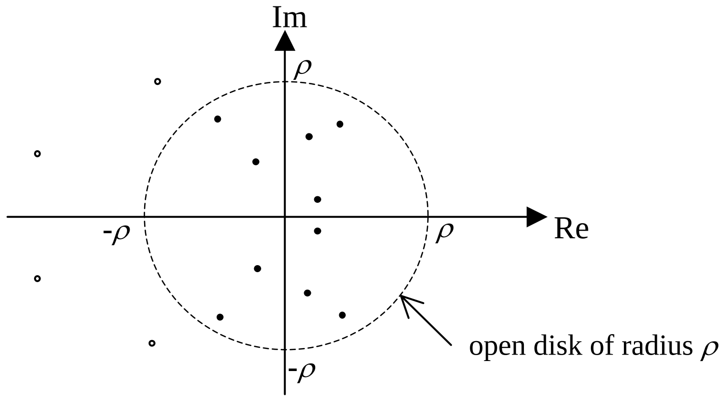


Figure 3.2 Eigenvalues in an open disk of radius ρ
(○:eigenvalues outside the open disk , ●: eigenvalues in the open disk)

The calculation of $d_i\ (i = 1, \cdots, 2n)$ can be easily performed by the $\mathbf{LDL}^T$ factorization of the Schur-Cohn matrix $\mathbf{T}$. If $\mathbf{T} = \mathbf{LDL}^T$, then:

$$\mathbf{T}_i = \mathbf{L}_i \mathbf{D}_i \mathbf{L}_i^T , \tag{3.14}$$

$$\begin{array}{c} \begin{array}{ccccc} \mathbf{L}_1 & \cdots & \mathbf{L}_i & \cdots & \mathbf{L}_{2n} \end{array} \\ \mathbf{L} = \begin{bmatrix} 1 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ l_{i1} & \cdots & 1 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ l_{2n,1} & \cdots & l_{2n,i} & \cdots & 1 \end{bmatrix} \end{array}, \quad \begin{array}{c} \begin{array}{ccccc} \mathbf{D}_1 & \cdots & \mathbf{D}_i & \cdots & \mathbf{D}_{2n} \end{array} \\ \mathbf{D} = \begin{bmatrix} d_{11} & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & d_{ii} & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & d_{2n,2n} \end{bmatrix} \end{array} \tag{3.15}$$

where the matrix $\mathbf{T}_i$ is the leading principal submatrices of order *i* in the Schur-Cohn matrix $\mathbf{T}$, the matrix $\mathbf{L}_i$ is the leading principal submatrices of order *i* in the factorized lower triangular matrix $\mathbf{L}$, and the matrix $\mathbf{D}_i$ is the leading principal submatrices of order *i* in the factorized diagonal matrix $\mathbf{D}$ as shown in equation (3.15). The values of $d_i\ (i = 1, \cdots, 2n)$ can be evaluated as:

$$\begin{aligned} d_i &= \det(\mathbf{T}_i) = \det(\mathbf{L}_i \mathbf{D}_i \mathbf{L}_i^T) = \det(\mathbf{D}_i) \\ &= \prod_{h=1}^{i} d_{hh} \quad . \end{aligned} \tag{3.16}$$

Therefore, each $d_i = \det(\mathbf{T}_i)$ can be obtained by multiplying from the first diagonal element d_{11} to the *i*-th diagonal element d_{ii} of the factorized diagonal matrix $\mathbf{D}$.

Considering equation (3.10), we only need to know the sign of each d_i because the unknown value of $V[1, d_1, d_2, \cdots, d_{2n}]$ depends on sign changes of each $d_i \ (i = 1, \cdots, 2n)$, and from equation (3.16) the sign change of d_i from d_{i-1} occurs when the diagonal element d_{ii} of the factorized diagonal matrix $\mathbf{D}$ is negative. So, the value of $V[1, d_1, d_2, \cdots, d_{2n}]$ is equal to the number of negative element in the matrix $\mathbf{D}$ which can be proven by mathematical induction. If we combine this result with equation (3.10), the number of eigenvalues inside an open disk of radius ρ and the number of positive elements the factorized diagonal matrix $\mathbf{D}$ have the following relationship:

$$N_\rho = \textit{the number of positive elements in the diagonal matrix}\,\mathbf{D} \qquad (3.17)$$

where N_ρ is the number of eigenvalues inside an open disk of radius ρ and $\mathbf{D}$ is the diagonal matrix obtained by factorization of Schur-Cohn matrix $\mathbf{T}$ constructed using the modified coefficients in equation (3.13). This relation is very similar to the Sturm sequence property for undamped systems. The algorithm of the proposed method is summarized in Table 3.3.

Table 3.3 Algorithm of the proposed method

Step 1:Change to a standard eigenproblem.

- Change the given eigenproblem to a standard form.

Step 2: Calculate the coefficients of the characteristic polynomial.

- Using Chen's algorithm, construct $p(\lambda) = \sum_{k=0}^{2n} a_k \lambda_k = 0$.

Step 3:Determine the radius ρ of an open disk.

- The radius ρ can be arbitrary, but select a little bit larger magnitude than the largest known eigenvalue to check missed eigenvalues.

Step 4: Modify the coefficients of the characteristic polynomial.

- Substitute $\lambda = \rho\bar{\lambda}$ to $p(\lambda)$ obtained at Step2.

Step 5: Construct the Schur-Cohn matrix.

- Construct the Schur-Cohn matrix **T** using the modified coefficients of the characteristic polynomial.

***Step 6: Perform the $\mathbf{LDL}^T$ factorization the Schur-Cohn matrix* T.**

Step 7: Calculate (check) the number eigenvalues inside the open disk and stop.

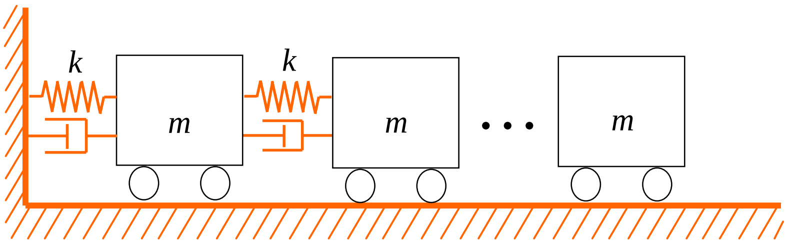
- Calculate the number of positive elements in the diagonal of **D**.
- The number of eigenvalues is equal to the number of positive elements in **D**.

CHAPTER 4
NUMERICAL EXAMPLES

To show the applicability of the proposed method, two numerical examples are analyzed. A simple spring-mass-damper system having the exact analytical eigenvalues is considered to verify that the method can exactly calculate the number of eigenvalues in the open disk of arbitrary radius for the eigenproblem with the damping matrix. The plane frame structure with lumped dampers is also considered to verify the method for the system with multiple eigenvalues.

4.1 Simple Spring-Mass-Damper System (Chen 1993)

A simple spring-mass-damper system is shown in Figure 4.1.



Mass : m= 1 Spring stiffness: k= 1
Rayleigh Damping Coeff.: α=0.05, β=0.5

Figure 4.1 Simple spring-mass-damper system

The finite element discretization of the system results in a diagonal mass matrix, a tridiagonal damping and stiffness matrices of the following forms:

$$\mathbf{M} = m\mathbf{I} \tag{4.1}$$

$$\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K} \tag{4.2}$$

$$\mathbf{K} = k\begin{bmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & \\ & -1 & \ddots & \ddots & \\ & & \ddots & 2 & -1 \\ & & & -1 & 1 \end{bmatrix} \tag{4.3}$$

The analytical solutions can be resulted through following relationships:

$$\lambda_{2j-1,2j} = -\xi_j\omega_j \pm i\omega_j\sqrt{1-\xi_j^2} \quad \text{for } j = 1,\ldots,n \tag{4.4}$$

$$\xi_j = \frac{1}{2}\left(\frac{\alpha}{\omega_j} + \beta\omega_j\right) \tag{4.5}$$

$$\omega_j = 2\sqrt{\frac{m}{k}}\sin\frac{2j-1}{2n+1}\frac{\pi}{2} \tag{4.6}$$

where ω_i and ξ_i are the undamped natural frequency and modal damping ratio, respectively and i is defined as $\sqrt{-1}$. A system with order 10 is used in analysis. All the eigenvalues and their radii from the origin in the complex plane are shown in Table 4.1.

The number of considered eigenvalues is 8 and the radius of the contour S of the open circle is calculated by the 1.005 times the magnitude of the eighth eigenvalue (ρ =

$1.005|\lambda_8| = 1.005$). The half-circle of the contour S is initially divided into 24 (r=4) equal points as shown in Figure 4.2.

Table 4.1 Calculated eigenvalues of example 4.1

Mode Number	Eigenvalues(λ) Real	Imaginary	Radius ($\rho = \|\lambda\|$)
1, 2	-0.0306	±0.1463	0.1495
3, 4	-0.0745	±0.4388	0.1495
5, 6	-0.1585	±0.7133	0.4450
7, 8	-0.2750	±0.9614	1.0000
9, 10	-0.4137	±1.1763	1.2470
11, 12	-0.5624	±1.3540	1.4661
13, 14	-0.7077	±1.4932	1.6525
15, 16	-0.8368	±1.5959	1.8019
17, 18	-0.9381	±1.6651	1.9111
19, 20	-1.0028	±1.7046	1.9777

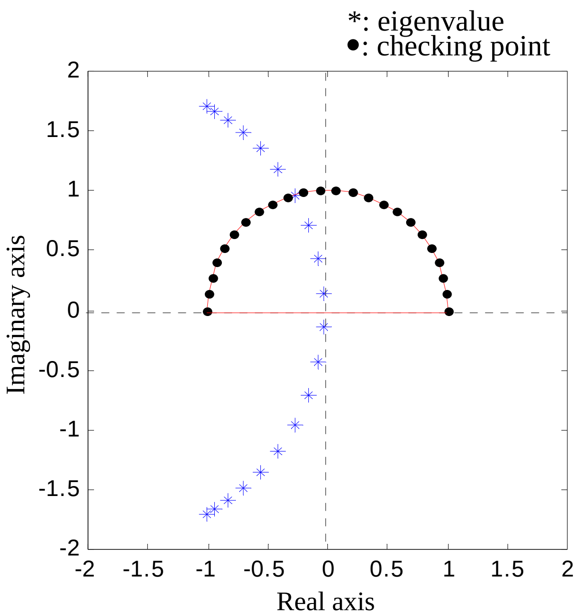


Figure 4.2 Contour of S of example 4.1

Since the argument of the largest eigenvalue is 105.96°, the part of the contour between 101.74° and 109.57° is subdivided into six equal parts. The total variation of the argument is 1440° as in Table. 4.2, and the number of rotations is

$$\boldsymbol{N} = \frac{\sum_{\boldsymbol{j}=1}^{2\boldsymbol{n}-1} \Delta\theta_{\boldsymbol{j}}}{2\pi} = \frac{1440°}{360°} = 4 \qquad (4.7)$$

So, the number of eigenvalues inside a circle of radius ρ (= 1.005$|\lambda_8$) is 8 (=4×2), which exactly agrees with the calculated value in Table 4.1. If we assume that the largest eigenvalue is unknown and use the same radius for the contour *S*, we can detect the checking point where the drastic change of the variation of the argument occurs in this case as shown in Table 4.2. In Table 4.2, the change of the variation of the argument is 187.684° at checking point $\rho\angle 109.57°$. Because this value is over 180°, we can conclude that new checking points are needed. The contour of $p(S)$ is shown in Figure 4.3. It is very difficult to count the number of rotations from Figure 4.3.

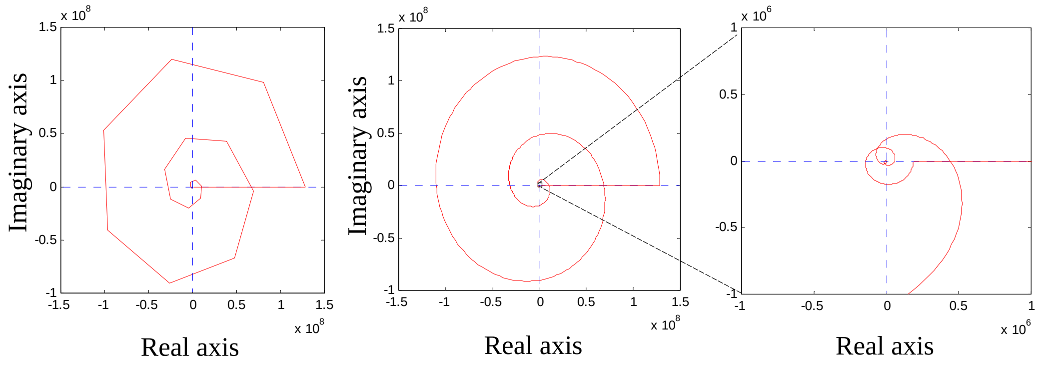
Because only the arguments are important to count the number of eigenvalues, the magnitude of $p(S)$ can be scaled to help the graphical interpretation as shown in Figure 4.4. The number of rotations for the contour $p(S)$ in the figure is 4.

The results for the modified Sturm sequence property are shown in Table 4.3. As shown at the last column in Table 4.3, the number of sign changes is 12. So if we use the equation (3.10), the number of eigenvalues inside the circle is 20-12 =8.

Table 4.2 Arguments and the variations of the arguments of example 4.1

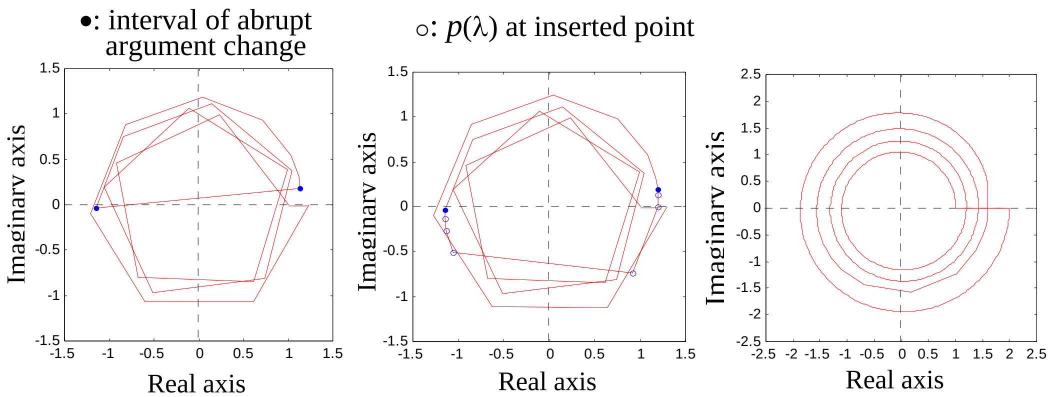
First checking process				Second checking process				$\sum \Delta\theta_j$
z_j	θ_j	$\Delta\theta_j$	*Y/N*	z_j	θ_j	$\Delta\theta_j$	*Y/N*	
origin	0.00	-	-					0.00
$\rho\angle 7.83°$	76.70	76.70	*N*					76.70
$\rho\angle 15.65°$	153.30	76.60	*N*					153.30
$\rho\angle 23.48°$	229.69	76.39	*N*					229.69
$\rho\angle 31.30°$	305.74	76.05	*N*					305.74
$\rho\angle 39.13°$	21.29	75.55	*N*					381.29
$\rho\angle 46.96°$	96.11	74.82	*N*					456.11
$\rho\angle 54.78°$	169.91	73.80	*N*					529.91
$\rho\angle 62.61°$	242.24	72.34	*N*					602.24
$\rho\angle 70.43°$	312.50	70.26	*N*					672.50
$\rho\angle 78.26°$	19.75	67.25	*N*					739.75
$\rho\angle 86.09°$	82.58	62.82	*N*					802.58
$\rho\angle 93.91°$	138.32	55.75	*N*					858.32
$\rho\angle 101.74°$	181.54	43.22	*N*					901.54
				$\rho\angle 102.86$	186.85	5.31	*N*	906.85
				$\rho\angle 103.98$	193.13	6.28	*N*	913.13
				$\rho\angle 105.10$	205.92	12.80	*N*	925.92
				$\rho\angle 106.21$	321.02	115.10	*N*	1041.0
				$\rho\angle 107.33$	359.83	38.81	*N*	1079.8
				$\rho\angle 108.45$	6.33	6.49	*N*	1086.3
$\rho\angle 109.57°$	9.23	**187.68 or –172.32**	*N*					1089.2
$\rho\angle 117.39°$	15.71	6.49	*N*					1095.7
$\rho\angle 125.22°$	28.00	12.28	*N*					1108.0
$\rho\angle 133.04°$	52.44	24.44	*N*					1132.4
$\rho\angle 140.87°$	88.12	35.68	*N*					1168.1
$\rho\angle 148.70°$	132.90	44.78	*N*					1212.9
$\rho\angle 156.52°$	184.437	51.53	*N*					1264.4
$\rho\angle 164.35°$	240.86	56.17	*N*					1320.6
$\rho\angle 172.17°$	299.83	59.02	*N*					1379.6
$\rho\angle 180.00°$	0.00	60.38	*N*					1440.0

where $0° \le \theta_j < 360°$, $\Delta\theta_j = \theta_j - \theta_{j-1}$, and '*Y*' means that the additional checking points are required and '*N*' means that the additional ones are not required.



(a) $p(S)$ at $6r$ (=24) points (b) $p(S)$ at 1000 points

Figure 4.3 Contour of $p(S)$



(a) $p(S)$ at 6 r(=24) points (b) $p(S)$ at $6r$+6 points (c) $p(S)$ at 1000 points

Figure 4.4 Scaled contour of $p(S)$ S of example 4.1

Using equation (3.17), the number of positive elements in the matrix **D** is 8 as shown at the third column in the Table 4.3.

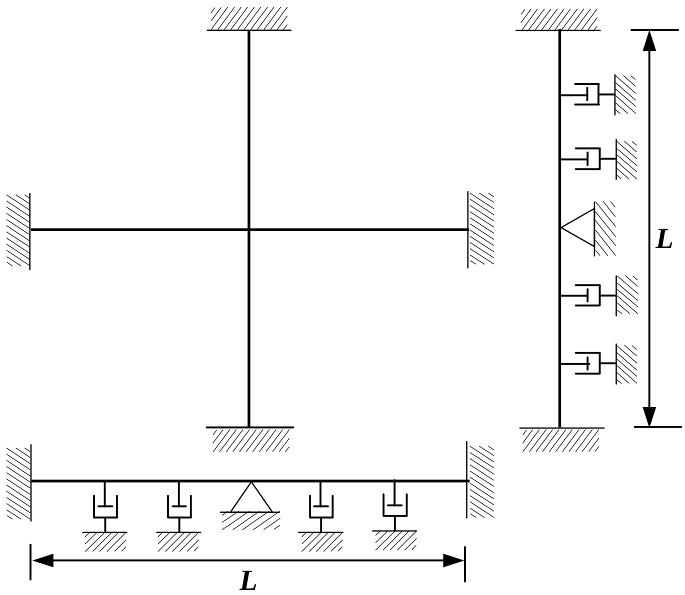
Table 4.3 Coefficients $\bar{a}_i$, diagonal elements d_{ii} of $\mathbf{D}$, and signs of d_i

i	ρ=1.005$\lvert\lambda_8\rvert$=1.005			
	$\bar{a}_i$	d_{ii}	sign of d_i	V
0	1.0000e+000	-	+	-
1	1.4500e-001	2.2079e-001	+	-
2	1.9009e+001	3.6950e-002	+	-
3	2.3853e+000	-4.4940e+004	-	1
4	1.5313e+002	-7.4052e+003	+	2
5	1.6424e+001	-2.0035e+003	-	3
6	6.8074e+002	-2.0029e+003	+	4
7	6.0218e+001	-1.1531e+003	-	5
8	1.8222e+003	-1.1531e+003	+	6
9	1.2972e+002	-2.7488e+002	-	7
10	3.0067e+003	-2.7473e+002	+	8
11	1.6522e+002	4.5463e+002	+	-
12	3.0065e+003	4.5454e+002	+	-
13	1.2015e+002	-1.0560e+003	—	9
14	1.7177e+003	-1.0556e+003	+	10
15	4.5550e+001	2.0436e+001	+	-
16	4.9538e+002	2.0433e+001	+	-
17	7.4260e+000	-6.6555e+000	—	11
18	5.5026e+001	-6.6555e+000	+	12
19	3.2500e-001	7.1550e-001	+	-
20	1.0000e+000	7.1549e-001	+	-

where V represents the number of sign changes of d_i, '+' means positive value and '—' means negative value. The d_0 is defined as 1.

4.2 Plane Frame Structure with Lumped Dampers (Kim and Lee 1999)

In this example, a plane frame structure with lumped dampers is presented. The geometric configuration and material properties are shown in Figure 4.5.



Young's Modulus: 1000 *Mass Density: 1.0*
Cross-section Inertia: 1.0 *Cross-section Area: 1.0*
Span Length : L= 6.0 *Concentrated Damping: 0.3*
Rayleigh Damping Coeff.: α=0.001, β=0.001

Figure 4.5 Plane frame structure with lumped dampers

The model is discretized in 6 beam elements with equal length for each direction resulting in the system of dynamic equation with a total of 18 degrees of freedom. Thus, the order of the associated eigenproblem is 36. The consistent damping matrix is derived from the classical damping given by $\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K}$ and concentrated dampers resulting in non-proportional damping matrix. All eigenvalues are calculated by the modified Lanczos method developed by Kim and Lee (1999) and their radii from the origin in the complex plane are calculated by $\rho_i = |\lambda_i|$ as in Table 4.4.

Table 4.4 Calculated eigenvalues of example 4.2

Mode Number	Eigenvalues(λ)		Radius ($\rho = \|\lambda\|$)
	Real	Imaginary	
1,2	-1.1369	±46.2187	46.2327
3,4	-1.1369	±46.2187	46.2327
5,6	-1.3731	±51.1333	51.1517
7,8	-1.3731	±51.1333	51.1517
9,10	-3.3902	±81.0872	81.1490
11,12	-3.3902	±81.0872	81.1490
13,14	-3.9407	±87.4771	87.5659
15,16	-3.9407	±87.4771	87.5659
17,18	-8.1642	±127.4394	127.7006
19,20	-8.1642	±127.4394	127.7006
21.22	-10.2629	±142.8367	143.2049
23,24	-10.2629	±142.8367	143.2049
25,26	-14.8662	±171.7301	172.3720
27,28	-14.8662	±171.7301	172.3720
29,30	-20.5387	±201.6249	202.6683
31,32	-20.5387	±201.6249	202.6683
33,34	-23.7699	±216.7332	218.0328
35,36	-23.7699	±216.7332	218.0328

The number of considered eigenvalues is 8 and the radius of the contour *S* is calculated by the 1.005 times the magnitude of the eighth eigenvalue (ρ = 1.005$|\lambda_8|$ = 51.4075). The half-circle of the contour *S* is initially divided into 24 equal points. And since the argument of the largest eigenvalue is 91.54°, the part of the contour between 86.09° and 93.91° is subdivided into seven equal parts as shown in Figure 4.6.

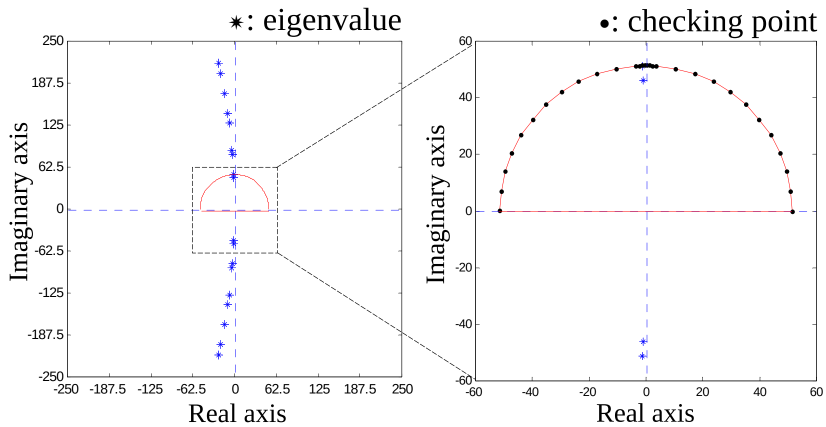


Figure 4.6 Contour of *S* of example 4.2

Since the total variation of the argument is 1440° as in Table 4.5, the number of rotations is

$$N = \frac{\sum_{j=1}^{2n-1} \Delta\theta_j}{2\pi} = \frac{1440°}{360°} = 4 . \qquad (4.8)$$

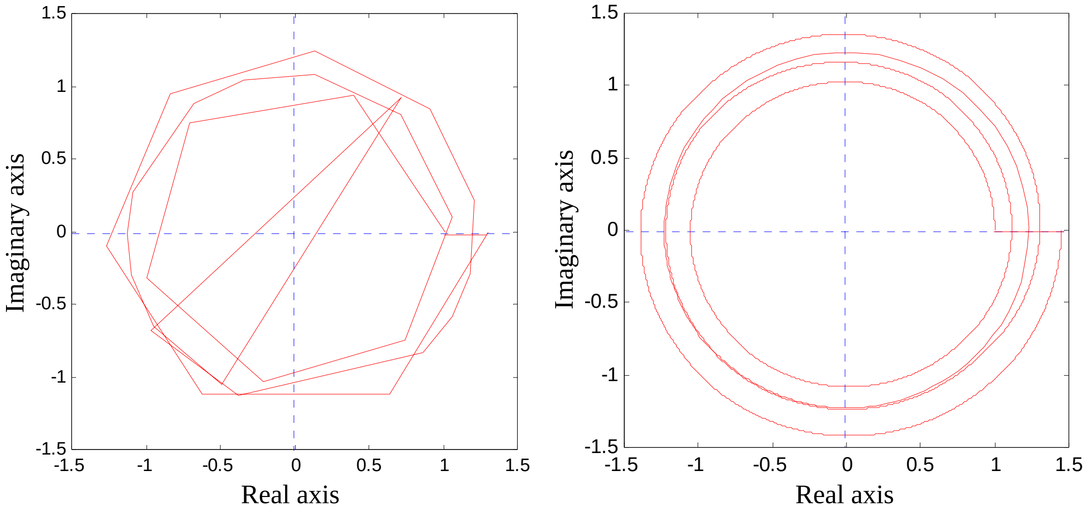
So, the number of eigenvalues inside a circle of radius ρ ($= 1.005|\lambda_8|$) is 8 (=4×2), which exactly agrees with that obtained in Table 4.4. If we assume that the largest eigenvalue is unknown and use the same radius for the contour *S*, it is very difficult to detect the checking point where the drastic change of the variation of the argument occurs in this case. In Table 4.5, the change of the variation of the argument is 85.44° at checking point $\rho\angle 93.91°$.

Table 4.5 Arguments and the variations of the arguments of example 4.2

First checking process				Second checking process				$\sum \Delta\theta_j$
z_j	θ_j	$\Delta\theta_j$	*Y/N*	z_j	θ_j	$\Delta\theta_j$	*Y/N*	
origin	0.00	-	-					0.00
$\rho\angle 7.83°$	67.26	67.26	*N*					67.26
$\rho\angle 15.65°$	133.49	66.23	*N*					133.48
$\rho\angle 23.48°$	197.60	64.12	*N*					197.60
$\rho\angle 31.30°$	258.47	60.86	*N*					258.47
$\rho\angle 39.13°$	314.83	56.36	*N*					314.83
$\rho\angle 46.96°$	5.32	50.49	*N*					365.32
$\rho\angle 54.78°$	48.47	43.15	*N*					408.47
$\rho\angle 62.61°$	82.89	34.42	*N*					442.89
$\rho\angle 70.43°$	108.09	25.20	*N*					468.09
$\rho\angle 78.26°$	127.76	19.66	*N*					487.76
$\rho\angle 86.09°$	166.22	38.46	*N*					526.22
				$\rho\angle 87.21°$	180.76	14.54	*N*	540.76
				$\rho\angle 88.33°$	195.28	14.52	*N*	555.28
				$\rho\angle 89.44°$	214.36	19.08	*N*	574.36
				$\rho\angle 90.56°$	245.10	30.75	*N*	605.10
				$\rho\angle 91.68°$	52.05	166.94	*N*	772.05
				$\rho\angle 92.80°$	215.33	163.28	*N*	935.33
$\rho\angle 93.91°$	251.66	**85.44 or –274.56**	***Y***					971.66
$\rho\angle 101.74°$	315.81	64.14	*N*					1035.8
$\rho\angle 109.57°$	330.99	15.19	*N*					1050.9
$\rho\angle 117.39°$	346.18	15.18	*N*					1066.1
$\rho\angle 125.22°$	9.90	23.72	*N*					1089.9
$\rho\angle 133.04°$	42.84	32.94	*N*					1122.8
$\rho\angle 140.87°$	83.89	41.05	*N*					1163.8
$\rho\angle 148.70°$	131.55	47.66	*N*					1211.5
$\rho\angle 156.52°$	184.34	52.79	*N*					1264.3
$\rho\angle 164.35°$	240.86	56.53	*N*					1320.8
$\rho\angle 172.17°$	299.83	58.97	*N*					1379.8
$\rho\angle 180.00°$	0.00	60.17	*N*					1440.0

where $0° \le \theta_j < 360°$, $\Delta\theta_j = \theta_j - \theta_{j-1}$, and '***Y***' means that the additional checking points are required and '*N*' the additional ones are not required.

Because this value is smaller than 180°, we cannot conclude whether new checking points are needed or not. The scaled contour of $p(S)$ is shown in Figure 4.7. The number of rotations for the contour $p(S)$ in the figure is 4.



(a) $p(S)$ at $6r+6(=30)$ points (b) $p(S)$ at 1000 points

Figure 4.7 Scaled contour of $p(S)$

The results of the modified Sturm sequence property are shown in Table 4.6. As shown at the last column in Table 4.6, the number of sign changes is 28. So if we use the equation (3.10), the number of eigenvalues inside the circle is 36-28 =8. Using equation (3.17), the number of positive elements in the matrix **D** is 8 as shown at the third column in the Table 4.6. Therefore, we verify that the method can exactly check the number of eigenvalues inside some open disk of arbitrary radius for the system with multiple eigenvalues.

Table 4.6 Coefficients $\bar{a}_i$, diagonal elements d_{ii} of $\mathbf{D}$, and signs of d_i

i	ρ=1.005| λ_8 |=51.4075			
	$\bar{a}_i$	d_{ii}	sign of d_i	V
0	2.5017e+006	-	+	0
1	2.3595e+006	-6.2584e+006	–	1
2	1.8523e+007	-6.2584e+006	+	2
3	1.5836e+007	-6.2584e+006	–	3
4	6.0360e+007	-6.2584e+006	+	4
5	4.6688e+007	-6.2584e+006	–	5
6	1.1462e+008	-6.2584e+006	+	6
7	8.0016e+007	-6.2584e+006	–	7
8	1.4176e+008	-6.2584e+006	+	8
9	8.9075e+007	-6.2584e+006	–	9
10	1.2111e+008	-6.2584e+006	+	10
11	6.8279e+007	-6.2584e+006	–	11
12	7.4043e+007	-6.2584e+006	+	12
13	3.7327e+007	-6.2584e+006	–	13
14	3.3137e+007	-6.2583e+006	+	14
15	1.4882e+007	-6.2576e+006	–	15
16	1.1017e+007	-6.2563e+006	+	16
17	4.3882e+006	-6.2388e+006	–	17
18	2.7450e+006	-6.2202e+006	+	18
19	9.6439e+005	-6.0086e+006	–	19
20	5.1461e+005	-5.9203e+006	+	20
21	1.5833e+005	-4.8338e+006	–	21
22	7.2533e+004	-4.7180e+006	+	22
23	1.9357e+004	-2.4454e+006	–	23
24	7.6408e+003	-2.4029e+006	+	24
25	1.7448e+003	-5.4284e+005	–	25
26	5.9414e+002	-5.3645e+005	+	26
27	1.1381e+002	-6.1754e+003	–	27
28	3.3379e+001	-4.1125e+003	+	28
29	5.1996e+000	2.4923e+004	+	-
30	1.3074e+000	1.3928e+004	+	-
31	1.5712e-001	3.0169e+003	+	-
32	3.3582e-002	2.8778e+003	+	-
33	2.8092e-003	2.5915e+000	+	-
34	5.0338e-004	2.5924e+000	+	-
35	2.2413e-005	2.8473e-004	+	-
36	3.2943e-006	1.6343e-004	+	-

where V represents the number of sign changes of d_i, '+' means positive value and '–' means negative value. The d_0 is defined as 1.

CHAPTER 5

CONCLUSIONS AND FURTHER STUDIES

Methods of counting the number of eigenvalues for non-proportionally damped systems have been presented. The method based on argument principle requires many factorization processes at many checking points and sometimes it is difficult to detect drastic change of the variation of the argument. On the other hand, the proposed method needs only one factorization of the Schur-Cohn matrix. The final checking of the method is done by counting positive diagonal elements of the factorized Schur-Cohn matrix, which is very similar to the well known Sturm sequence property. As demonstrated in two numerical examples, the proposed method can exactly calculate the number of eigenvalues in an open disk of given radius.

The proposed method is based on well-proven algorithms and theorems, but during calculation of the coefficients of the characteristic polynomial, some small numerical errors may be accumulated mainly due to memory limitation. To apply the proposed method to large structures, therefore, further research to reduce the effects of numerical errors should be performed.

요 약 문

감쇠시스템의 개선된 Sturm 수열 성질

부분공간 반복법, Lanczos 방법과 같은 대부분의 고유치해석 방법들은 어떤 구조물의 전체 고유벡터 집합을 구하는 것이 아니라 이 집합에서 저차의 일부분만을 계산하기 때문에 고유쌍 중의 일부를 누락시킬 수 있다. 따라서, 모드중첩법을 이용하여 정확한 동적응답을 얻기 위해서는 고유치해석 수행 후에 누락된 고유치를 검사하는 과정이 반드시 수행되어야만 한다. 비감쇠 또는 비례 감쇠 시스템의 경우, 즉 감쇠를 고려하지 않은 고유치해석의 경우에는 Sturm 수열 성질을 이용하여 누락된 고유치를 검사할 수 있으며 대부분의 고유치해법들이 이 기법을 채택하고 있다. 그러나, 지반-구조물 상호작용 문제, 구조물의 진동제어 문제, 복합재료 구조물과 같은 비비례 감쇠 시스템의 경우, 즉 감쇠를 고려한 고유치해석의 경우에는 누락된 고유치를 검사하는 기법이 아직까지 확립되어 있지 않다. 본 연구에서는, Sturm 수열 성질, Gerschgorin 정리, 편각의 원리 및 Gleyse 의 정리와 같은 다양한 수학 성질들을 자세히 검토한 후, 감쇠를 고려한 고유치 해석에서 누락된 고유치를 검사할 수 있는 효율적인 기법을 제안하였다. 이 기법은 Gleyse 의 정리를 이용하여 Schur-Cohn 행렬을 행렬분해한 후 대각행렬의 대각요소 중 양의 부호를 가지는 요소들의 개수를 조사하여 누락된 고유치를 검사할 수 있으므로 최종결과가 기존의 Sturm 수열 성질과 유사하다. 제안방법의 효용성을 검증하기 위하여, 단순 스프링-질량-감쇠기 시스템과 집중 감쇠기가 부착된 평면 뼈대 구조물에 대해서 수치해석을 수행하였다.

LIST OF REFERENCES

1. Aplevich, J. D. (1974). “Direct computation of canonical forms for linear systems by elementary matrix operations.” ***IEEE Transactions on Automatic Control***, 19, 126-129.

2. Arnoldi, W. E. (1951). “The principle of minimized iterations in the solution of the matrix eigenvalue problems.” ***Quarterly Journal of Mechanics and Applied Mathematics***, 9, 17-29.

3. Barnett, S. (1989). “Le Verrier’s algorithm: A new proof and extensions.” ***Journal of Matrix Analysis and Application***, SIAM, 10, 551-556.

4. Bathe, K. J. (1996). ***Finite Element Procedures***. Prentice-Hall, Inc., Englewood Cliffs, N. J.

5. Benhamadou, M. (1999). “On the Calculation of the Multiplicity of a Real Eigenvalue of Hessenberg Matrix.” ***Advances in Engineering Software***, 30**,** 210-210.

6. Caughey, T. K. and O’Kelly, M. E. J. (1965). “Classical Normal Modes in Damped Linear Dynamic Systems.” ***Journal of Applied Mechanics***, ASME, 32, 583-588.

7. Chen, C. T. (1984). ***Linear System Theory and Design***. Oxford University Press, Inc., 1984.

8. Chen H. C. and Taylor R. L. (1988). “Solution of Eigenproblems for Damped

Structural Systems by Lanczos Algorithm.” ***Computers and Structures***, 30**,** 151-161.

9. Faddeev, D. K. and Faddeeva, V. N. (1953). ***Computational Methods of Linear Algebra***. Freeman.

10. Franklin, G. F., Powell, J. D. and Workman, M. (1998). ***Digital Control of Dynamic Systems***. Addison Wesley Inc.

11. Gleyse B. and Moflih M. (1999). “Exact Computation of the Number of Zeros of a Real Polynomial in the Open Unit Disk by a Determinant Representation.” ***Computers and Mathematics with Application***, 38**,** 257-263.

12. Guckhenheimer, J. and Holmes, P. (1983). ***Nonlinear Oscillations in Dynamical Systems and Bifurcation of Vector Fields***. Springer, Berlin.

13. Host, P. (1935). “A method for determining the coefficients of a characteristic equation.” ***The Annals of Mathematical Statistics***, *6***,** 83-84.

14. Hughes, T. J. R. (1987). ***The Finite Element Method; Linear Static and Dynamic Finite Element Analysis***. Prentice-Hall, Inc.

15. Jung H. J., Kim D. H. and Lee I. W. (2001). “Technique of Checking Missed Eigenvalues for Eigenproblem with Damping Matrix.” ***International Journal for Numerical Methods in Engineering***, 50**,** 55-66.

16. Kim M. C. and Lee I. W. (1999). “Solution Eigenproblems for Non-Proportional Damping System by Lanczos Method.” ***Earthquake Engineering and Structural Dynamics***, 28**,** 157-172.

17. Lee, I. W., Kim, M. C. and Robinson, A. R. (1998). “An efficient solution method

of eigenproblems for damped systems using modified Newton-Raphson technique." ***Journal of Engineering Mechanics***, ASCE, 124, 576-580.

18. Leung, A. Y. T. (1995). "Subspace iteration method for complex symmetric eigenproblems." ***Journal of Sound and Vibration***, 184, 627-637.

19. Le Verrier, U. J. J. (1840). "Seculaires des elements elliptiques des sept planets principales." ***Journal des Mathematiques Pures et Appliques***, 5, 220-254.

20. Li, L. (1999). "Estimation for Matrix Singular Values." ***Computers and Mathematics with Applications***, 37, 9-15.

21. Locher, F. (1993). "A stability test for real polynomials." ***Numerische Mathematik***, 66, 33-40.

22. Meirovitch, L. (1980). ***Computational Methods in Structural Dynamics***. Sijthoff & Noordhoff.

23. Meirovitch, L. (1990). ***Dynamics and Control of Structures***. John Wiley and Sons.

24. Newland D.E. (1989). ***Mechanical Vibration Analysis and Computation***. Longman Scientific & Technical.

25. Olson, L. and Vandini, T. (1989). "Eigenproblems from finite element analysis of fluid-structure interactions." ***Computers and Structures***, 33, 679-687.

26. Petyt, M. (1990). ***Introduction to Finite Element Vibration Analysis***. Cambridge University Press.

27. Press, W. H., Flannery, B. P., Teukolsky, S. A. and Vetterling, W. T. (1988). ***Numerical Recipes in C***. Cambridge University Press.

28. Rajakumar, C. (1993). “Lanczos algoritm for the quadratic eigenvalue problem in engineering applications.” ***International Journal for Numerical Methods in Engineering***, 35, 1-22.

29. Ren, G. and Zheng, Z. (1997). “A reformulated Arnoldi algorithm for non-classically damped eigenvalue problems.” ***International Journal for Numerical Methods in Engineering***, 40, 3537-3555.

30. Rombouts, S., and Heyde, K. (1998). “An Accurate and Efficient Algorithm for the Computation ot the Characteristic Polynomial of a General Square Matrix.” ***Journal of the Computational Physics***, 140, 453-458.

31. Souriau, J. M. (1948). “Une Methode pour la decomposition spectrale et I’inversion des matrices.” ***Publications Mathématiques de l'IHÉS***, 227, 1010-1011.

32. Tsai, J. S. H. and Chen, S. S. (1993). “Root distribution of a polynomial in subregions of the complex plane.” ***IEEE Transactions on Automatic Control***, 38, 173-178.

33. Wang, J. and Chen, C. T. (1982). “On the Computation of the Characteristic Polynomial of a Matrix.” ***IEEE Transaction on Automatic Control***, 27**,** 449-451.

34. Yamada, I., Kurosawa, K., Hasegawa, H. and Sakaniwa, K. (1998). “Algebraic multidimensional phase unwrapping and zero distribution of complex polynomials – characterization of multivariate stable polynomials.” ***IEEE Transactions on Signal Processing***, 46, 1639-1664.

감사의 글

늦은 나이에 올바른 학문의 길로 들어설 수 있도록 인도해주셨을 뿐만 아니라 인생의 길에 있어서도 커다란 은혜와 사랑을 베풀어주신 이인원 교수님께 존경 가득한 감사를 드립니다. 세심한 심사와 조언으로 논문을 풍부하게 해주신 김진근 교수님, 이승래 교수님, 기계공학과 윤성기 교수님과 특히 멀리서 오셔서 논문의 부족한 점을 조목조목 지적해주신 이병구 교수님께 깊은 감사드립니다. 훌륭한 강의로 많은 가르침을 주신 최창근 교수님, 윤정방 교수님, 신항식 교수님, 박희경 교수님, 김동수 교수님, 곽효경 교수님께 감사드립니다. 매주 세미나에 참석하셔서 유익한 조언을 해주신 한남대학교 오주원 교수님, 경일대학교 이종헌 교수님, 공주대학교 고만기 교수님, 성균관대학교 박선규 교수님 그리고 한경대학교 김운학 교수님께 감사드립니다. 저희 실험실을 위해 아낌없는 지원을 해주신 현대건설 이지송 사장님께도 감사드립니다.

잠시나마 동고동락하다가 먼저 사회로 나가신 동옥이형과 만철이형 또 오랫동안 동고동락하며 나이든 선배를 위해 애쓰다 조금 일찍 사회로 진출한 동현, 형조, 현택이 그리고 연구실에서 부대끼며 고운 정, 미운 정이 쌓인 여러 후배들, 항상 성실한 병완이, 언제나 힘이 되어준 상원이, 책임감 강한 규식이, 멀리 베트남에서 와서 고생하는 탄, 가끔은 친구같기도 한 영종, 신혼살림을 하며 실험실 재정을 보느라 바쁜 강민, 조금 용감한 헌재, 가끔 썰렁한 준식, 부족한 선배를 잘 믿어준 동두, 갖가지 실험실 일에 고생인 성진, 종우 그리고 새로 들어온 현우와 지은이에게도 감사드립니다.

석사과정을 마친 후 아무것도 모르는 사회초년병으로 직장생활을 시작하였을 때 잦은 실수와 잘못을 따뜻한 사랑으로 감싸주셨던 SK 건설의 많은 상사님들 특히 허양 상무님과 노문환 소장님 그리고 차광찬 사장님께 감사드리며, 현장에서 언제나 솔선수범 하시는 모습을 보여주셨던 구재완 고문님과 건축구조의 실무에 관한 여러 가지를 느낄 수 있도록 해주신 센구조 이창남 소장님께도 언제나 건강하시길 바라며 감사의 말씀을 올립니다.

언제나 저를 사랑해 주시는 어머니. 항상 어머님의 걱정만 끼쳐드리는 이 못난 자식이 자그마한 결실을 보게 되었습니다. 그 동안에 베풀어 주신 은혜에 보답하지는 못하겠지만, 저의 이 보잘 것 없는 작은 영광을 부모님께 돌리고자 합니다. 또한 부족한 저를 언제나 대견한 사위로 맞아주시고 지켜봐 주신 장인, 장모님께도 너무 감사하다는 말씀을 올립니다. 더 좋은 사위가 되도록 노력하겠습니다. 그리고 멀리서 말없이 응원해준 동생과 제수씨 그리고 형운이, 누나와 매형, 처남 및 처제 내외분들께도 깊이 감사 드립니다. 같은 신성동에 살며 힘들 때 우리 가족에게 큰 힘이 되어준 많은 어린이집 및 방과후집 식구들에게도 감사드립니다.

마지막으로, 가장 힘들 때 가장 큰 힘이 되어주었고 어려운 시간 속에서 서로 격려하며 같이 꿋꿋이 참아준 저의 사랑하는 아내, 순희에게 그 동안 표현하지 못하고 간직해 둔 고마움과 사랑의 마음을 보내며, 우리 큰 딸 승연이와 작은 딸 해연이와 함께 이 작은 행복을 언제까지나 소중히 간직하고 싶습니다.

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Journal Papers:

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2. "Eigenvalue-counting Methods for Non-proportionally Damped Systems." with Hyung-Jo Jung, Man-Gi Ko and In-Won Lee, *International Journal of Solids and Structures*, 2003.

3. "CMAC Neural Network for Real-Time Identification of Structures." with Dong-Hyawn Kim and In-Won Lee, accepted for publication in *Journal of Engineering Mechanics*, *ASCE*, 1999.

<u>Conference Papers:</u>

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2. "Modified Bang-Bang Control of Seismically Excited Structures Using MR Damper." with Sang-Won Cho and In-Won Lee, *KAIST-Kyoto University Joint Seminar on Earthquake Engineering,* KAIST, Korea, 125-132, 2002.